Tertiary Students' Changing Views on Mathematical Creativity

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Why Creativity?

- World Economic Forum: Creativity is "one of the most important and in-demand skills in the next 5 years" (Schöning & Witcomb, 2017)
- **Mathematical Association of America's CUPM 2015 Guidelines**
  - A successful major offers a program of courses to gradually and intentionally leads students from basic to advanced levels of critical and analytical thinking, **while encouraging creativity** and **excitement about mathematics.**
- Creativity ⇒ Gaining content knowledge (Leikin, 2014)
- Creativity is an important aspect of professional mathematicians' work (Borwein et al. 2014)

Theoretical Perspective

- Developmental perspective of creativity (Kozbelt, Beghetto & Runco, 2010)
  - creativity develops over time
  - emphasizes the role of the environment
  - students are provided authentic tasks and opportunities to interact with others

What is Mathematical Creativity?

- Over 100 definitions (Mann, 2006)

- **A process** of offering **new solutions** or insights that are **unexpected** for the student, **with respect to his/her mathematics background** or the problems s/he has seen before

Unpacking the Definition

- **A process…**
  - Not necessarily a/the end product
- **…of offering new solutions or insights that are unexpected…**
  - Originality and surprise
- **…for the student, with respect to his/her mathematics background or the problems s/he has seen before.**
  - Relative to the student instead of to his/her peers or mathematics in general

Creativity Literature

- Considerable amount of literature on mathematical creativity at the primary and secondary levels (e.g., Silver, 1997; Lev-Zamir & Leikin, 2011).
  - However, few studies at the undergraduate level
  - Also, few studies of students beliefs about mathematical creativity
- Torrance (1966, 1978) created testing for creativity and giftedness in K-12 education
- Silver (1997) expanded three aspects of K-12 mathematical creativity
  - Flexibility - An ability to look at a problem from new perspective
  - Originality - Using an unexpected or unusual approach
  - Fluency - Applying ideas, tools of one area in a different area

Research Questions

- What are tertiary students' perceptions of mathematical creativity?

- In what ways do these views evolve in an introduction-to-proofs course which emphasized mathematical creativity?

Methods

- Transition to proofs course
  - Small, liberal arts college in the SW United States
- Inquiry-based learning (IBL) pedagogy
- Creativity was fostered and valued by making use of the Creativity-in-Progress rubric
- 4 female and 3 male students were interviewed at the end of term
- Hypothesis coding (Saldana, 2013)

Creativity-in-Progress Rubric (CPR) on Problem Solving

- Categories
  - Making Connections
  - Taking Risks

- Levels (Continuum)
  - Beginning
  - Developing
  - Advancing

Making Connections:
The ability to connect the proving task with definitions, theorems, multiple representations, and examples from the current course as well as possible experiences from previous courses.

Taking Risks:
The ability to actively attempt a proof, demonstrate flexibility in using multiple approaches or techniques, pose questions about reasoning within the attempts, and evaluate those attempts.

CPR on Proving

MAKING CONNECTIONS:

	Beginning	**Developing**	**Advancing**
Between Definitions/Theorems	Recognizes some relevant definitions/theorems from the course or textbook with no attempts to connect them in their proving	Recognizes some relevant definitions/theorems from the course and attempts to connect them in their proving	Implements relevant definitions/theorems from the course and/or other resources outside the course in their proving
Between Representations[1]	Provides a representation with no attempts to connect it to another representation	Provides multiple representations and recognizes connections between representations	Provides multiple representations and uses connections between different representations
Between Examples	Generates one or two specific examples with no attempt to connect them	Generates one or two specific examples and recognizes a connection between them	Generates several specific examples and uses the key idea synthesized from their generation

[1] We define a *mathematical representation* similar to NCTM's (2000) definition. It includes written work in the form of diagrams, graphical displays, and symbolic expressions. We also include linguistic expressions as a form of lexical or oral representation. For example, a student can use the lexical or oral representation, "the intersection of sets A and B"; a Venn Diagram to depict his/her mathematical thinking; a symbolic representation $A \cap B$; or set notation $\{x | x \in A \text{ and } x \in B\}$ (which is also a symbolic representation). Note the last two representations are in the same category, e.g. symbolic, but they are still considered two different representations.

CPR on Proving

TAKING RISKS

TAKING RISKS:

	Beginning	Developing	Advancing
Tools and Tricks[2]	Uses a tool or trick that is algorithmic or conventional for the course or the student	Uses a tool or trick that is model-based or partly unconventional[3] for the course or the student	Creates a tool or trick that is unconventional for the course or the student
Flexibility[4]	Begins a proof attempt (or more than one proof attempt), but uses only one approach	Acknowledges and/or uses more than one proving approach, but only draws on one proof technique	Uses more than one proof technique
Posing Questions	Recognizes there should be a question asked, but does not pose a question[5]	Poses questions clarifying a statement of a definition or theorem	Poses questions about reasoning within a proof
Evaluation of Proof Attempt	Examines surface-level[6] features of a proof attempt	Examines an entire proof attempt for logical or structural flow	Examines and *revises* an entire proof attempt for logical or structural flow

[2] Based on the Originality category from Leikin (2009).
[3] Learned in a different context.
[4] A proof attempt is a continuous, sustained line of reasoning focused on a single theorem or conjecture. A proof approach is a proof attempt in which a new or different (to the prover) idea is introduced. Finally, a proof technique is a proof approach that addresses the overall logical structure of the proof. Common proof techniques include induction, proof by cases, direct proof, contradiction, and contrapositive.
[5] For example, a student writes a "?" next to something.
[6] Surface-level features include technical, computational, and line-to-line logical details.

Results: Students Views on Mathematical Creativity

- Some students had views on mathematical creativity similar to the literature (Flexibility, fluency, and originality)

Being creative in mathematics is the same as being creative in anything else. It's taking the road less traveled. It's not just doing what the herd is doing but finding your own way to get to where you need to be.

- The rubric categories were also reflected

[M]y personal definition of creativity, and I guess to just really sum it up in one statement is just really thinking outside of the box and being able to be comfortable or at least willing to take risks.

So, in that I think it [referring to IBL] forces you to really try to make connections and it forces you to get creative because you have, um, very little like understanding of the right way to do it, so it kind of throws that out of a student's mind, out of my mind. And so it makes anything possible.

Results: Students Views on Mathematical Creativity

- However, some students also talked about creativity as an inherent trait

Um, to me creativity, that's kind of like, born with—is like being able to come up with like a nifty idea for like a creative like art project that will make it like simple or like being able to- I know art takes like a lot of practice and a lot of work, like, itself to do—to be able to like draw or like paint. (Alice)

- Students also saw creativity as akin to efficiency

So, the one guy I was telling you about before, he was very efficient. He would make these algebraic tricks up, and then another person would come up with an algebraic trick to use. So, his creative moment, I could then use to expand on and do something a little different with to have my own creative moment. (Stephanie)

Results: Evolution of Students Views

- Three students reported explicit shifts in their creativity views
  - Learning community
  - Having more mathematical tools to work with
  - *I think I started to look at creativity a little bit different through this course...Prior to this it's been all very applied mathematics...So before, just using the trig equations to solve geometry was creative for me. Whereas now, this has just opened up a whole new door of opportunities for it because I can solve a proof using a contradiction, while somebody else used a contrapositive and somebody else used a direct proof and somebody else used induction, and we all do it completely different.*
  - Seeing each other's work
  - *I really, I really did not feel like I was being creative at all throughout the course. It really was just things in my head, it makes sense that led to a conclusion that made sense. But, considering that I thought other people were exceptionally creative, I kind of thought that maybe they though that about me too.*

Discussion & Future Work

- It is possible to affect students views on mathematical creativity through teaching practices
- NSF funded grant: ”Reshaping Mathematical Identity by Valuing Creativity in Calculus”

Thank You!

Questions?

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