Warthog Delta '01

Third Southern Hemisphere Symposium on Undergraduate Mathematics Teaching

Gearing for Flexibility

Kruger National Park
1 - 5 July 2001

COMMUNICATIONS

PREFACE

The Warthog Delta'01 conference is jointly presented by the International Delta Committee, SAMERN (Southern African Mathematics Education Reform Network) and the African Mathematical Union. The Conference is the third in a series of conferences on the undergraduate teaching and learning of Mathematics, as part of a collaboration between Southern Hemisphere countries.

The first conference Delta'97 (delta implying change) took place in Brisbane, Australia in November 1997. The second conference, Delta'99, took place in the stimulating location of Laguna Quays, also in Queensland, Australia. Since then, this informal collaboration between Southern Hemisphere countries, at the moment including Australia, New Zealand and South Africa, has grown into the formation of the International Delta Committee, with current members: Johann Engelbrecht. (South Africa) (chair), Milton Fuller (Australia) (co-chair), Carol Bohlman (South Africa), Pat Cretchley (Australia), Ansie Harding (South Africa), Derek Holton (New Zealand), Ivan Reilly (New Zealand), Michael Sears (South Africa), Walter Spunde (Australia) and Christina Varsavsky (Australia). The Delta conferences take place bi-annually and the next one, Delta'03, is planned for Queenstown, New Zealand in November 2003.

The theme for Delta'97 was "What can we do to improve learning?" For Delta'99 it was "The Challenge of Diversity" and for Delta'01 the theme is "Gearing for Flexibility".

One of the objectives at the present conference is to develop a structure that can expand cooperation between countries in Africa (Southern Africa in particular) on the topic of undergraduate mathematics teaching. For this purpose a regional steering committee has been appointed. It is envisaged that this conference will greatly contribute towards closer collaboration between these countries in future.

The 120 delegates, attending from 24 different countries worldwide, are responsible for close to 80 contributed presentations, panel discussions and round table discussions. The conference has two publications: the Proceedings, consisting of peer reviewed research papers, and the Communications, consisting of reports on teaching experiences and research in progress.

We believe that the deliberations at this meeting will influence the course of future tertiary mathematics education worldwide. It is broadly accepted that skills and training in the quantitative sciences will be crucial to success in the future. Thus our belief is that this meeting will be one of the most important of 2001.

Thank you to Carol Bohlmann who was responsible for compiling (but not editing) the Communications.

Johann Engelbrecht

Chairperson
Warthog Delta 01

CONTENTS

EXPLORING POLYNOMIAL APPROXIMATIONS WITH "*MATHEMATICA*"

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ABSTRACT. In this paper, we present work done in the study of infinite series and polynomial approximations using "*Mathematica*". The students involved are computer science and engineering students in Calculus III and computer science majors in the Numerical Analysis class. The challenge was to design projects involving the visual, the numeric as well as the analytical in order help the students gain an understanding of infinite processes and the concept of the limit. I present two case studies, one involving infinite sequences and series, another involving the Taylor approximations to a function.

1. Introduction. This paper is the result of work done with two groups of students in the fall of 2001, one group taking a third semester course in Calculus, the other group taking a course in Numerical Analysis. The students in Calculus are engineering and computer science majors, whereas the other course is made up of computer science students only, many of whom resent the fact that they have to take more math as their main interest is computer science. The challenge was to design projects using *Mathematica* where the students could see the relationship between the numerical, the visual and the theory done in class. We included the verbal by asking the students to describe their observations. This was done to encourage students to express themselves verbally in accordance with the recommendations concerning the teaching of a reformed Calculus course that every topic should be presented geometrically, numerically, and algebraically, and which was extended to the "rule of four" by emphasizing the verbal, or descriptive, point of view as well. From our experience, visual information presented in class is completely lost to students as they never do anything with it and rarely are graphs and their interpretation included on tests The topics that we choose to describe here include infinite sequences and series and the approximation of functions by polynomials using Taylor series. These are topics with which students face a great deal of difficulty and even though they may learn to use the methods for testing convergence, after a lapse of time when asked about these topics, it is clear that all that they recall are some of the words.

2. Project Description.

2.1 Project 1. The first project we describe is about sequences and series, and involved making a table of values of the terms of the sequence as well as a list of the values of the sequence of partial sums. They were also asked to evaluate the terms to a number of significant digits to see that as n becomes large the error becomes closer and closer to zero. To say that a sequence has a limit means that the limit can be approximated to as many significant digits as we want. The sequences considered are:

$$\{1/n\}, \{(-1)^n(1/n)\}, \{1/n^2\}.$$

We give the students the *Mathematica* and we illustrate with an example:

```
<<Graphics'MultipleListPlot'
a[n_] = 1/n^2
list1[n_]:=Table[ a[k], {k,1, n}]
list2[n_] := Table[Sum[a[k], {k,1,j}], {j,1,n}]
```

The output to N[list1[10]] and to N[list2[10] is:

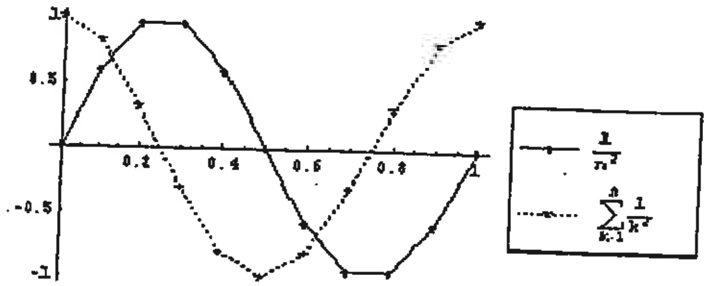
{1.,0.25,0.111111,0.0625,0.04,0.0277778,0.0204082,0.015625,0.0123457,0.01}

and

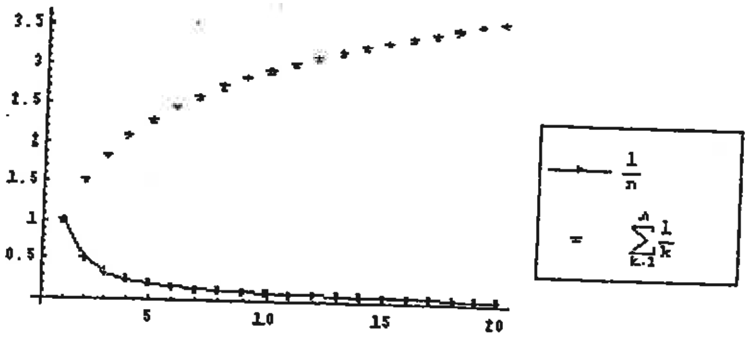
{1.,1.25,1.36111,1.42361,1.46361,1.49139,1.5118,1.52742,1.53977,1.54977}

We now plot the these points ðn the same graph, first a simple list plot and then joining the points b segments and including a legend indicating explaining the plots. To obtain the graphs, the following *Mathematica* code is used:

```
MultipleListPlot[list1[10], list2[10]]
MultipleListPlot[list1[10], list2[10],PlotLegend->{"1/n²","∑_1^n 1/k²"},PlotJoined->True}
```



The students are asked to do the computation to 10 significant digits and are asked for an upper bound the error if that term is taken as an approximation to the limit. The experiment is repeated with differe values of n. Students were asked to relate the results obtained to the theoretical work done in class. Next t same is done with the two series $\Sigma(-1)^{k+1} 1/k$ and the harmonic series $\Sigma 1/k$. When they plot the sequence terms and the sequence of partial sums the following diagrams is produced:



From the work they have done in class they know that the harmonic series diverges and here when asked to find the sum of the first 1000 terms, they enter:

Sum[1/k, {k,1, 1 000}]//N

and the output is 7.48547, whereas trying a value of 10 000 they get an answer of 9.78761. So the sequence of partial sums is increasing. To know that the series diverges, using the integral test yields that the integral diverges.

Discussion. Here the objective of the exercise is to let students clearly distinguish between the terms of the series and the sequence of partial sums. Since the students do very little computations with sums of the series and most of the work in class and in the text emphasizes the testing the convergence of the series most students do not get a feeling as to what the series represents. Here both the numeric and the visual representation help clarify what the sequence of partial sums represents. From the graphical representation or from the numeric values they can conjecture that the series converges, however they can see that this not always obvious as is illustrated with the series $\sum_{n=1}^{\infty} \frac{1}{n}$ which grows so slowly that from the computations and the graphical representation, it is not clear what is the behavior of the series.

2.2. Project 2. This problem was given to both the Calculus class and the numerical Analysis class with minor variations. For the students in numerical analysis, they had completely forgotten what the Taylor series was all about, so this exercise served as a way of reviewing the material. An example was developed in the notes handed out, and then they were required to do the same exercise for another function. Following is the *Mathematica* code to be inputted:

```
f[x_] = E^(x^2)
{f1[x_],f2[x_], f3[x_],f4[x_]}= Table[Normal[Series[f[x], {x, 0,k}]],{k,1,4}]
```

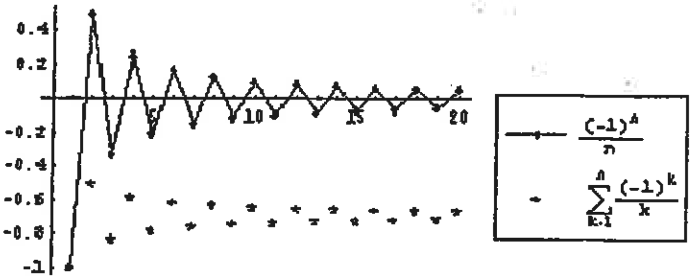
and the output produced will be:

```
(1, 1 + x^2, 1 + x^2, 1 + x^2 + x^4/2}
```

Here, they observe that the Taylor polynomials f2[x] and f3[x] are the same, and students have to explain the reason for that, and if they could generalize what is happening. To plot the polynomial functions of degree less than or equal to 4, the following is entered:

```
Plot[{f[x],f1[x],f2[x], f3[x],f4[x]}, {x,-1,3}, PlotStyle->{Dashing[{.01,.01}],GrayLevel[0],
Dashing[{0.02, 0.02}]}]
```

and the output will be:



Next, they are given a *Mathematica* function which will allow them to animate a function f_n and its polynomial expansion up to degree n at some point x_0, and here they have to specify the plotRange. Here, they have to think about what the y-range needs to be.

```
animateTaylor[fn_, x_, x0_, n_, plotRange_] := Module[{polys},
        polys= Table[ Normal[Series[fn, {x, x0,i}]], {i,1,n}];
        Map[Plot[{#, fn}, {x, plotRange[[1,1]], plotRange[[1,2]]},
        PlotRange->plotRange,
        PlotStyle->{GrayLevel[0], Dashing[{0.02, 0.02}]}]&, polys]]
```

Calling animateTaylor[f,0, 10, {{0, 3}, {1, 1000}}] produced a series of frames of the plots of e^x and the polynomial $f_n(x)$ for n =1 to 10, which could then be animated to show the relationship between the polynomial approximation and the function.

To get an approximation to the integral over the interval [0,1], two methods are used; one by evaluating the integral of the polynomial over the interval, and using the left and the right Riemann sums to give bounds for the value of the integral of the function.

```
lsum[ f_,a_,b_,n_]:= Sum [f[a + k(b-a)/n] (b-a)/n, {k, 0, n-1}]
rsum[ f_,a_,b_,n_]:= Sum [f[a + k(b-a)/n] (b-a)/n, {k, 1, n}]
```

Using 30 subdivisions,

lsum[f, 0, 1, 30]//N yields 1.42083

and

rsum[f, 0, 1, 30]//N yields 1.49179

We deduce that the integral is between 1.42083 and 1.49179.

So if we want the answer correct to 3 significant digits, we need to divide the interval into a larger number of intervals, which we can find by trial and error.

lsum[*f*, 0 , 500] // N yields an answer of 1.46094

and

rsum[[*f*, 0 , 500] //N yields 1.46437.

The answer that *Mathematica* gives is 1.46265.

We ask the students to evaluate the error and the relative error, and to repeat the process when the interval of integration is the interval [0,2] and ask them to plot the graphs of the function and the Taylor polynomials to relate the results to the graphs of the function and the approximating polynomial as the integral represents the area under the curve. Here they are asked to find by trial and error what degree polynomial should they take in order that the answer is correct to 3 significant digits.

3. Conclusion. The topic of infinite sequences and series is a difficult one for students to understand as is documented by many studies. To develop projects using a CAS to help students better understand the subject is a challenge. Animation of the graph of a function and its Taylor polynomial of degree n, helps in visualizing how the approximation is local at the point at which the expansion is being done, and how the approximation improves as the degree of the polynomial increases. Seeing the animation dynamically as well as the seeing the frames statically helps give insight into what is happening. Including a numerical element into the picture, by including an integral over an interval a concrete idea that students understand helps in tying together the visual and the numerical.

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READING SKILLS AND MATHEMATICS

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ABSTRACT. This paper considers some of the linguistic difficulties experienced by students whose mother tongue is not English as they study mathematics. The pilot phase of a project investigating these difficulties is discussed, as well as plans for the future that are emerging from the second phase of this project.

1. Introduction. It is self-evident that if we can't read we can't study. Many studies have been undertaken on the role of language in mathematics, and we know a lot about the ways in which poorly developed language skills undermine students' mathematical performance. But to what extent does reading ability (not just specific knowledge about the structure of the language) influence our ability to comprehend and do maths? And how do we know that our ideas are valid?

Consider the following example (from a 2000 first-year exam)

"Consider the function f defined by

$$y = f(x) = \frac{x^2 - 5x}{(3 - x)(x + 1)}.$$

Write down the zeros of the function f? What do the zeros of f tell us about the graph of f? Does the graph of f cut the y-axis? If so, where? If not, why not? " They were also asked "Does the graph of f cut the y-axis? If not, explain why not. If so, what is the y-intercept?"

Some of the answers (verbatim) were:

1. The graph of f does not cut the y-axis, this is shown by calculating the y-intercepts. But if zero should be included the graph will cut the y-axis at zero and since this is not true we find that the graph actually does not cut the y-axis. It only touches it in other words that is the turning point.
2. $x(x - 5)$ tells us that the graph passes through the origin.
3. $y = 0$ No because is one to one function
4. No! $x = 0$ is not defined
5. No, the graph does not cut the y-axis but it passes through the origin
6. No, the graph of f does not cut the y-axis because a vertical asymptote never cuts the y-axis, it does not exist

These responses identify a variety of problem areas.

In 2000 the Departments of Mathematics and Linguistics at the University of South Africa (Unisa) began a joint research project to investigate these issues. Many different and inter-related parameters need to be investigated. It may be impossible to sufficiently differentiate between the linguistically and mathematically weak students to isolate the effects of reading; however, we felt it was worth trying. ESL students are often required to learn two conceptually difficult and different concepts at the same time, one related to language and the other to mathematics. From a Vygotskian perspective [31], while the text may be written with the target audience in mind, implementing sound instructional design principles, it is unlikely that the linguistic and mathematics will be simultaneously in the same "zones", and always synchronised with one another and with students' needs. Focus on reading skills essential for constructing meaning may help to bridge the gap.

2. Background to the study. Language, mathematics and Western Culture are inseparable [12]. In South Africa, many students who have difficulty learning mathematics are from non-Western, non-English speaking, rural backgrounds, although they may live in an urban, Western environment. Since English is the most prevalent written language of mathematical instruction in South Africa at this stage, this gives rise to a wide variety of problems, in particular in distance learning (or contact-distance combinations). The creation of foundation-level, user-friendly distance learning mathematics study material, in English, for learners for whom English is a second (or third or fourth) language (ESL learners), embraces issues of academic and mathematical literacy. Many students are first-generation learners and lack support for learning in their social environment. When mathematics, essentially also a "language", is mediated through English, by educators whose primary language is not English, to learners whose primary language is also not English, the potential for misconceptions is vast.

Initially we decided to concentrate only on the difficulties experienced in reading mathematical texts. What do we mean by "reading"? Frobisher [14] discusses two distinct mathematical processes, reasoning processes and operational processes. He considers reading to be both a reasoning process, as well as an operational processes (involved in "collecting" information). Without effective reading, guessing and pattern searching are not possible. Reading is more than fluency in articulating what is written; it is also more than understanding the meanings of individual words. We decided to focus on *reading as a way of constructing meaning*. In print-based distance education this is particularly important, and knowledge of effective facilitative processes needs to inform the development of written study material.

Much of the research done in this direction identifies the problems, but very little seems to point to a solution or to the development of any instructional strategy that may facilitate the process of learning from text. For many years research has shown high positive correlations between reading ability and mathematical achievement. These investigations have dealt with monolingual English speakers. Recently more research involving mathematics learning and ESL learners has been undertaken.

Language allows thoughts to surface to a level of awareness where individuals can review them and clarify or modify their ideas. (Gibbs & Orton, [16]). It is the medium by which teachers introduce and convey concepts and procedures, through which texts are read and problems are solved. Language skills, particularly the reading skills needed to comprehend mathematics texts and word problems, are the tools with which students access, learn and apply mathematical concepts and skills. See [4].

At the outset it must be noted that we need to avoid over-simplification of the problem. Many people with poor language skills are good mathematicians; many people with excellent linguistic skills do not cope with mathematics. There are obviously many different variables involved, not least of which is the issue of motivation. It is also often the case that poor mathematical comprehension is the cause of poor expression; i.e. inadequate language proficiency is a symptom, rather than a cause, of poor understanding.

Given the significant number of problems associated with learning mathematics in English, an obvious issue is whether it may not be more sensible to develop indigenous mathematical languages, rather than remediate the English language difficulties. However, many such efforts to enrich the indigenous language with translations of Western terms have not met with much success (see [3], [17], 24]), not because the speakers of these languages have a conceptual deficit, but because their language is structured differently, to accommodate a different world view [15]. Also, at some stage in their academic careers, students will have to cope with the demands of mathematical and other scientific texts. Over-simplification of text to avoid linguistic problems is thus not a viable long-term solution.

2.1. Some issues which affect ESL learners as they read mathematics.

2.1.1. Oral and written language traditions, early language acquisition, bilingualism. In many South African language communities, the home language has not been significantly developed in written form; children often have limited exposure to the written word, either to read it or to write it. The level to which the home language has been verbally developed does not allow for inclusion of academic aspects of the spoken word. As a result of poor schooling and limited linguistic development at home, many ESL learners entering tertiary institutions are only *semilingual* [21], rather than bilingual or multilingual, as ESL learners in a first world country may be.

We are aware of the difficulties that arise when the *primary language is an oral rather than a written language*. The establishment of the printing press resulted in an "information explosion" which necessitated classification schemes, hierarchies, relationships and order [15], and in an oral tradition these structures are largely invisible. Vygotsky [30] proposed that even a minimal level of writing requires a high level of abstraction, and Zepp [32] considers whether written language can thus be considered a major factor in conceptualisation.

We know that there is a relationship between *early language acquisition* and future mathematical ability [11]. In the Papua New Guinea Indigenous Mathematics Project [29], it was reported that English language competence (the language of instruction) exhibited the strongest relationship with mathematics achievement.

The relationship between *bilingualism* and the ability to learn mathematics is also well established. For adult Hispanic bilinguals there appeared to be a relationship between the degree of bilingualism and logical reasoning ability [10]. This is locally substantiated by Prins [26]. *Competence* in a language different from the language of instruction appears to be a potential advantage, likely to assist in mathematics learning. (See [10], [27], [12]). Several studies of bilingual learners ([1], [2], [18])have shown cognitive benefits associated with bilingualism.

Cummins and Gulutsan [7] proposed the "threshold hypothesis", whereby "cognitive functioning of those who have achieved a certain level of competence in two languages is assisted, both by their present access to two languages and by the cumulative effects of their bilingual learning experiences." Dawe [9] noted that mathematical logic correlates more highly with first language ability than with second language ability. This suggests that under-development of a learner's first language hinders not only the development of the second language, but also the development of other skills (such as mathematics) that must be learned in the second language, significant factors with respect to academic and mathematical reading skills for many ESL learners whose first language is largely a spoken rather than a written language.

2.1.2. Everyday language, academic and mathematical language. Language used for academic tasks differs from that used for basic interpersonal communication. Linguists distinguish between BICS (basic interpersonal communication skills) and CALP (cognitive academic language proficiency). With CALP, meaning is inferred essentially from the linguistic features of the text. Students can acquire CALP via any language as long as they have reached a certain threshold of proficiency in that language. Cummins (1981) contends that there exists a minimal level of linguistic competence - a threshold - which students must attain in order to perform cognitively demanding tasks. Dawe (see [4]) has postulated that students also need to have reached a threshold level of proficiency in what he calls CAMP: Cognitive Academic Mathematics Proficiency. Dawe holds that the underlying proficiency needed to complete mathematical tasks involves cognitive knowledge (mathematical concepts and their application) embedded in a language specifically structured to express that knowledge. Research has shown that ESL learners may acquire BICS fairly quickly, but the acquisition of academic language skills takes an average of five years ([5], [15]).

2.1.3. Mathematical discourse. Apart from all these issues, mathematics is taught and understood via the *sub-language of mathematical discourse*, or the *mathematical register* [13]. In mathematical discourse the following features arise.

- **Specialised academic vocabulary**: words such as "whereas", "however", "consider"
- **Specialised mathematical vocabulary**: In the function example given earlier: there is evidence of confusion with words/concepts such as "intercept", intersect", "cut", "touch", "cross", "pass through". These words belong to normal English vocabulary, but also have specific mathematical meaning; some derive meaning from the context ("turning point" in a parabola, and in this rational function *f*, are conceptually different).

  Many words are *specific* to mathematics (e.g. denominator); other words have a different meaning in everyday English to their mathematical meaning (e.g. square root, base, real, function; such words or phrases need to be relearned. Various (possibly unfamiliar) *technical terms* are also employed, such as units of measurement; also words formed from Greek or Latin words, such as "triangle", "equilateral", "hypotenuse". One frequently misinterpreted word is "variable" [13]. Mathematical discourse involves *more complex relationships* than normal discourse. It also uses *specialised terminology* (e.g. if and only if) and includes words whose mathematical meanings differ from their normal English meanings.

  In mathematical discourse *meaning is often related to context*: e.g. the word "by" (and this applies to many other prepositions): we have

  3 multiplied by 10, which signifies multiplication;
  increase 3 by 10, which signifies addition
  8 divided by 2, where 2 is the divisor
  8 divided into 2, where 8 is the divisor.

Different words/phrases denote the same operation: for example addition can be denc terms such as increase, add, sum..

- **Special symbols** which denote processes and concepts.
- **Specialised semantic structure**: it is "a characteristic of many mathematical words th represent *concepts* and not *objects*" [12]. For example, apparently simple words, such as "tl "one third", have no unique, unambiguous representation in the world outside of mathem: word like logarithm is an even more obscure representation of a concept unrelated to any s object.

Making inferences in "mathematics language" often hinges on the ability to identify key wo to determine the other words in the problem to which the key words are linked. In the exampl earlier we saw that the word "it" may have been linked in the student's mind to the correct re but the student was unable to make this reference clear.

Anaphoric referencing: Phrases in the students' answers to the function question me earlier such as " ...if zero should be included the graph will cut the y-axis at zero and since th true ..."; "... it only touches it ..."; " ... because (?) is one to one ..."; "...a vertical asy never cuts the y-axis, it does not exist" show that they need to become more aware of refe mechanisms.

Specialised syntactic structure: the syntax of *comparative structures* [22] 197 is an inevital of mathematical language. We often encounter expressions like "greater than", "... times a as". Particularly problematic is the phrase "as ... as", which implies equality between th being compared.

The *pre-articles (*e.g. "one of"), *indefinite articles* (e.g. "a") and *definite article* ("the") ha specific linguistic applications [22].

Cardinal and ordinal numbers follow certain basic grammatical patterns [22].

Other important *quantifiers/modifiers* common to mathematical texts are "specific", "ce "particular". These terms are also absent from certain indigenous languages.

There is also frequent use of *logical connectors*: "if ... then", "because", "such that", "consequently", "that is", "for example", "thus": students must be able to *recognise* connectors and the situations in which they appear: i.e. do the connectors signify *sim contradiction, cause/effect, reason/result, chronological sequence* or *logical sequence*. Th to recognise where the logical connectors appear in a sentence, and what their purpose is [8

Consider the following relatively simple problem [4]: "Find a number such that 7 less t number is equal to twice the number minus 23". Forgetting for the moment that this may be contrived problem, consider the linguistic skills required in order to solve the problem. Fi must understand that "such that" relates "7 less than the number" to "twice the number min Also, "a number" and "the number" refer to the same number. Finally, the problem refers to than the number". Syntactically students would be likely to read, or write, or understand whereas we know that $n - 7$ is implied. (Confusion between *surface syntax*, and *real syntax* frequently.)

Another example: "There are 6 times as many students as professors at a University". S tend to write $6S = P$, rather than $6P = S$. Apart from the syntax error involved, this relate difficulties with variables, as S and P are seen to represent the objects themselves, and number of objects.)

Conjunctives such as i.e., e.g., may be problematic.

- The use of **passive voice**: for example "x is divided by three".
- Language at the **receptive/productive** level.

The responses in the function example given earlier, as well as in general, in assignme discussions with students, suggest that students find it more difficult to recall and express mathematical terminology than to respond to it. This holds true for most of us in an everyda we can respond to a wider vocabulary than we can actually use. Within a linguistic dime mathematical term can be used at the "receptive" level, and the "productive" level. However the mathematical register items have cognitive and contextual dimensions as well. The cognitive dimension relates to the level of complexity of the mathematical concept involved and the contextual dimension relates to the level of contextual support given. This provides a three-dimensional model with which we could analyse mathematical language tasks. [16].

Dimension	**Range**		
Linguistic	Receptive	⟷	Productive
Cognitive	Undemanding	⟷	Demanding
Contextual	Supported	⟷	Unsupported

This model creates eight domains within which we can analyse tasks. Consider the example given earlier and suppose that the first part of the question had been asked differently, for example: "Consider the graph of the function *f*. (For the purposes of this discussion we assume the graph is provided). Identify the points on the graph where the function value is zero. Identify the *x*-intercepts of the graph." The formulation of the question now falls within the linguistically receptive, cognitively demanding (at least for some students at this level) and contextually supported. The original formulation falls within the domain of the linguistically productive, cognitively demanding and contextually unsupported domain. Of course it may not be possible or desirable to keep students at a linguistically receptive, cognitively undemanding and contextually supported level; however, analysis of the introduction of new concepts within this framework is helpful. Would it be helpful to highlight these dimensions so that students could learn to paraphrase questions into a linguistically less demanding form; provide additional contextual support for themselves?

- Creation of **mental representation** (visualisation) of problems or concepts – this relates to the previous paragraph: if graphics or tables must be provided or interpreted by the student, the difficulties involved in this type of "reading" must be overcome.
- **Redundancy** of words in ordinary language (more prevalent in certain languages than in others: Prins [26] provides a graph (based on Kaplan, [20]) showing four kinds of discourse structures contrasted to English linearity, and mathematics discourse is even more "linear" if we consider the precise meaning of every word in a mathematical definition.
- **Lack of equivalent words** in the mother tongue: apart from certain articles and modifiers there are in many languages no words to express concepts such as "perpendicular", "exponential".
- **Reading rate adjustment** because of conceptual density; multiple readings are required as well as an understanding of symbolic devices such as charts, graphs. This has an impact on poor readers who consequently experience increased anxiety when completing time-dependent tasks.
- **Physical factors**: processing mathematical text requires up-down as well as left-right eye movement.

2.1.4. Methodology and the strategies used. During 2000 a group of students taking the mathematics access module took part in a series of pilot tests designed to identify problem areas in the reading of mathematics. During this phase we identified test items that were problematic, and refined them to avoid any ambiguities when we administered a large-scale, consolidated test to all access students in August 2000.

Out of an initial group of about 30, 23 students wrote all the tests. We had previously identified four specific features of reading, and designed the tests accordingly. It was postulated that students would not be able to make sense of what they were reading if

- their general English reading ability was weak
- they could not make anaphoric references
- their general and academic vocabulary was deficient

- they were unable to perceive semantic/logical relations such as temporal, causal, conditiona contrastive, and whole-part relations
- they could not respond to visual representation of information
- they could not identify the main concept in a passage, or distinguish between main and secondary concepts.

The final test was sent to approximately 950 students; 280 responded.

2.1.5. Analysis of 2000 results. Using SPSS we obtained performance on individual test sections and a comparison between the mathematics exam mark and the reading skill test results. We found, as expected, correlations between linguistically and mathematically weak students. See the cluster analysis results in the table below.

Table 1:Cluster analysis: reading skills and mathematical performance, 2

(n) = total for item	Cluster 1	Cluster 2	Cluster 3	Cluster 4
contrast (7)	4.61	3.76	3.50	3.15
sequence (12)	6.98	6.31	6.52	4.34
graphics (17)	14. 48	13.21	12.79	9.62
proanaphora (10)	8.30	6.48	6.61	5.59
detanaphora (10)	6.72	5.43	5.35	4.65
acad/tech vocab (10)	8.13	7.21	6.94	6.05
cause/condition (17)	11.76	10.88	10.84	7.99
exam average	69.09	50.26	31.41	18.14

Note: the categories are briefly described as follows:
contrast = however/but relations
sequence = sorting out the order in scrambled paragraphs (following an argument)
graphics = tables, graphs
proanaphora = it/them/they relations
detanaphora = this/these relations
acad/tech vocab = words such as "consider", "determine", "hypothesis"; also specific mathematical terminology
cause/condition = causal relations, including conditionals

(Similar experiments with first year psychology students at Unisa [25], have also shown a clear correlation between students in the following categories: "fail " (< 40%), "at risk" (40% - 60%) "pass" (60% - 74%), "distinction" (>75%) and their reading behaviour as indicated by similar tests.) W know that correlations unfortunately do not establish cause and effect, and additional work is required.

3. Activities planned for 2001 and 2002.

3.1. Decisions for 2001. The tentative conclusions drawn from the 2000 pilot study have lead to the following decisions for 2001. We decided to:

- establish a volunteer group of not more than 40 on-campus access module students to participa in the reading skills enhancement project
- administer to this group the comprehensive test used in 2000
- set up a control group using students who register for tutorial classes at Unisa's Learning Centi in Johannesburg - for logistical reasons only this centre will be involved: students at the Pretori

Learning Centre will be excluded since they may be part of the volunteer group). The same or a similar test will be administered to the volunteer and control groups at the end of phase two (end August 2001)

- work with the students for a period of 22 weeks (testing: 3 weeks, 19 weeks for reading strategies.

During the 3 weeks of **testing**, apart from the importance of the information obtained, we hoped that raising awareness of problems might motivate students to work at solving the problems. The testing involved

- pre- and post tests, as described above., and a reading speed test, so that students can compare the required speed for academic proficiency with their actual speed
- a cloze test, to determine relative reading difficulty: is the student reading at an independent, instructional or frustration level [23]?

During the **reading strategy** phase we planned to introduce a variety of tasks. The "Reading to Learn Mathematics" project ([28]) explored the potential of improving reading skills to assist students to become active and "generative" learners so as to develop a deeper understanding of mathematics. In the Siegel and Fonzi project [28], reading was classified into five different categories, two of which (reading to comprehend, reading to remember) are relevant to the teaching mathematics through distance education. The following reading components were identified in the two categories, and we will consider possible mathematical applications.

- *Reading generatively to make sense of the text.* For example, readers are grouped into pairs and are instructed to "say something" as they read the selected text. In mathematics, students could read a definition and "say" after each phrase, condition, etc., what they understand the text to mean, comment on aspects that are confusing, etc. Students would read and talk their way through passages of text. In terms of distance learning this would later need to be experienced as a conversation with oneself, although many peer-learning groups exist in which discussion is possible.
- *Reading to understand and follow directions* (e.g. understanding conditions that apply and procedures to follow when applying algorithms).
- *Skimming* to make a decision (e.g. moving quickly through a section to find an example of a particular method).
- *Reading the teacher's comments* to get the message (e.g. reading the comments made to an assignment)
- *Reading to make sense of graphics* (e.g. reading a graph or visual representation of a problem to grasp relationships, special characteristics).
- *Reading critically and reflectively* (e.g. reading carefully and focusing on issues such as whether the text is consistent with previous text or previously held concepts, whether cause and effect are clearly distinguished).
- *Reading to extract specific information* (e.g. reading precise values when doing a calculation, ensuring that the actual question asked has been answered, determining from the context the meaning of words: e.g. "time" meaning "when" (i.e. at what time) or "how long" (i.e. time travelled); "since" in a temporal or consequential sense).
- *Reading to make notes of one's own* (e.g. reading through a topic in order to make a summary for future reference).
- Use of the PQ4R technique: (P) *preview*: try to obtain the main ideas, (Q) *question*: phrase own questions while reading; (R) *read* with concentration on detail; (R) *reflect*: go over ideas again to be sure you're still in control; (R)*rewrite*: construct mathematical statements from language statements; (R)*review*: check your reasoning in appropriate ways)

Specific activities planned for the weekly reading strategy sessions are the following.

- Vocabulary skills, both academic and mathematical
- Identification of semantic difficulties: ordinary language terms that have specific mathematical meaning, or multiple meanings; mathematical terms that have no meaning in ordinary language, use of articles.
- Recognition of syntactic difficulties, especially the role of prepositions (such as *to*, *by*, *from*,

into), articles (*a, an, some*) and modifiers (*each of, any, every, all of, particular, certain, specific, various*); the difference between *surface* and *real syntax.*

- Anaphoric referencing.
- Identification of logical relations, especially cause and result.

Apart from these strategies which should facilitate the reading of mathematical texts, we plan to encourage students to read more. Since 2001 is "The Year of the Reader" we will provide mathematical stories, articles, etc., written in layman's terms, as well as other items of general interest in which mathematics features, such as sports magazines, on the assumption that English reading of this nature will facilitate the attainment of the elusive "threshold" of proficiency .

3.2 Preliminary results.

3.2.1. Activities undertaken so far. Every week the students are encouraged to submit book/magazine reports. They have access to magazines and newspapers, and submit their reports (on a prepared report form). There are similar forms to complete when they have finished reading a book. At the beginning of each session they work in pairs and share 10 new words with their partners, to improve vocabulary and dictionary skills.

At the time of writing we are at the end of week 10. The following activities have been carried out.

Week 1: Introduction: aims and commitments set out

Week 2: Pretests (reading skills) - also administered to control group

Week 3: Reading speed tests (related to comprehension)

Week 4: Feedback on reading speed; the implications of below-average speed for academic purposes; March maths quiz: two problems, one language-dependent; the other language-free, but requiring attentioı to detail regarding the form in which the final answer was required

Week 5: Feedback on the March maths quiz (language-dependent problem): explanation of the strategy of identifying, and changing, linguistically productive, contextually unsupported activities to linguistically receptive, contextually supported activities

Week 6: "Know your textbook": the need for previewing and other strategies to create a framework withir which to construct meaning from printed text

Week 7: Vocabulary skills: what to do when words are unfamiliar (apart from using a dictionary) e.g. how affixes (prefixes and suffixes) are used to create new words (how "linear" is derived from "line"; "definition" from "define", etc.)

Week 8: use of the PQ4R technique (described above) related to their mathematics study guides (followin from the lack of precision displayed in the second problem in the March maths quiz)

Week 9: Logical relations: cause and effect in a concrete context

Week 10: Logical relations in a more abstract context; examples from the mathematics study guides; explanation of a causal chain and the importance of this in reading mathematics

3.2.2.Observations. The results gathered during the weekly activities have not yet been analysed. Observations of the students during these sessions have shown the following:

- The initial group of over 50 has decreased to a group of about 30 who seem enthusiastic and committed. In discussion with individual students they appear to be very enthusiastic about the impact the sessions are having.
- Punctuality, attention to detail (i.e. words neatly written in books, handouts filed) seems to be improving.
- Magazine/newspaper articles: most students seem to be taking this seriously; standard of work handed in for marking generally low
- Book reports: very few so far
- Word sharing: the "dictionaries" are growing in size; even with an awareness of the problems experienced by ESL learners we were surprised at the number of words that we regarded as commonplace, that were included in these lists
- Mathematics attitude: currently positive, and fairly positive about previous mathematics

experiences (i.e. at school); unrealistic about mathematics performance (as is the case with many under-prepared students, they expect excellent exam results

- March Maths Quiz: overall performance very poor
- Logical relations: students find it difficult to determine reason and result; also the logical chain involved in most mathematical deductions; this is particularly important for students in a distance-learning context where they have to "deconstruct" solutions presented in study material.

3.3 Plans for the 2002. We realise that even though we will be interacting with the volunteer group of students in a contact situation, the strategies we develop must be applicable in a distance-learning context. Based on the results of the various strategies outlined above and used during 2001, we will develop a set of strategies that are relevant to distance learning, and can be applied for all access students. In 2002 these strategies will be incorporated into the submitted and self-assessed assignments that these students will be required to do. To determine whether the strategies have an impact, we will compare the final results of the 2001 students (who will not have been exposed to any specific reading skill enhancement programme, apart from the volunteer group who participate in the project) and the 2002 students. We will consider their final mathematics mark, and their performance on the reading skill test currently being used. All other parameters will remain constant (i.e. same study material and processes, same methods of assessment). To determine whether there are any significant differences in general ability of the 2001 and 2002 students we will compare the performance on the first assignment set for the two years, and only begin with the reading enhancement strategies after the assignment has been submitted. If there is a marked difference in ability between the two groups we will not be able to attach the same significance to any other results we obtain.

4. Conclusion. We hope to be able to determine whether a reading intervention programme has any significant impact on the mathematics performance of our ESL access level students.

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PRESSURES AND OPPORTUNITIES IN MATHEMATICS EDUCATION IN ZAMBIA

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ABSTRACT. Despite several schemes devised to bring more mathematics teachers into schools, the situation of shortage remains obstinately with us. Some decisions taken by educational administrators and politicians over the years have not helped to attract and to hold all those who could have made a valuable contribution to mathematics teaching.

Financial implications both for the production of appropriate materials and for the in-service training of teachers is another aspect exerting pressure on the teaching of mathematics. There is also the decline in confidence in mathematics teaching expressed particularly by employers of certain industries. The pressure to achieve a specific mathematical target regardless of its appropriateness for the children concerned cannot be ignored too.

This paper addresses the issue of the present state of the teaching of mathematics in Zambian schools, and prospects for the future. Despite the pressures on the teaching of mathematics, we have, at the present time, opportunities for reshaping the mathematics curriculum in our schools to respond to the changing needs of society. This will encourage us to view our subject as an extension of the language of human communication, with all that this implies for the total curriculum and the place of mathematics within it. This paper argues for the importance of leadership in mathematical education both within schools and in the wider community of mathematics teachers and it brings out the need to bridge the gap between research and the reality of the mathematics classroom; and above all, points the way towards constructive debate and sensible decision-making.

1. Introduction.

1.1 Zambia and its Economic Setting. Zambia is a land-locked country, covering an area of about 753,000 square kilometres. It shares boarders with the Democratic Republic of Congo and Tanzania in the north; Malawi and Mozambique in the east; Zimbabwe and Botswana in the south; Namibia in the south-west and Angola in the west. Zambia achieved its independence from the British rule in 1964. The population of Zambia was estimated to be about 10 million in the 2000 Census and the average annual growth rate is about 3.2 per cent. This population growth rate has in recent years imposed considerable strain on the Government in providing resources for the social sector, including education.

The interaction of low prices of copper, Zambia's leading export commodity, and the rising oil prices, creating a foreign-exchange problem, together with the deterioration of the international economic situation, have put the Zambian economy under serious pressure. The financial trend in the last decade has demonstrated growth in public expenditure significantly exceeding revenue growth, thus resulting in budgetary deficits which now exceed 6 per cent of the Gross Domestic Product (GDP). These budgetary pressures have impinged greatly on social services including education, forcing the Government to introduce a cost sharing principle in public expenditure as a strategy for economic sustenance and recovery.

1.2 The Education System. Zambia's progress in the field of education since independence in 1964 has been tremendous. During the 1970's, enrolment in primary, secondary and technical education increased by 43, 69 and 46 per cent, respectively. Enrolment in teacher training colleges and in the University of Zambia increased by 84 and 119 per cent, respectively. Primary and secondary education was free until the advent of the past decade when the principle of cost sharing was introduced.

The system of education currently in place in Zambia comprises five major sub-sectors: pre-primary, primary, secondary and high, and post secondary (tertiary). Though Government does not fund the construction and re-current costs for pre-schools, it has been able to control the nation's pre-school curriculum through the Ministry of Education. Ownership of pre-schools is left to communities, individuals and non-governmental organizations and companies. Most of the rest of the education sub-sector is funded by government to the tune of about 90 per cent, which constitutes about 25 per cent of the national yearly budget.

The formal education sub-sector consists of primary, secondary and tertiary education, pyramidal in nature, as is the case with most developing countries. A broad primary level at the bottom and a narrow tertiary level at the top characterize it. The primary school cycle starts at the age of seven, and comprises seven years that lead up to the Composite examination at Grade 7. The Composite examination is used for selection to Secondary school. Primary education is further divided into two stages, the lower consisting of the first four grades and the upper primary stage comprising the last three grades.

There are limited places in Grade 8 to cater for primary school graduates. The progression rate between Grade 7 and Grade 8 is below 20 per cent.

Secondary education is segmented into lower (Grades 8 and 9) and upper (Grade 10 - 12), and only about 50 per cent of the lower secondary graduates are able to continue to upper secondary school. At the end of Grade 12, a School Certificate Examination is administered locally by the Examinations Council of Zambia and supervised by the Cambridge Overseas Examining Board. Advanced level education was re-introduced in 2000 at some selected schools and caters for selected subjects only including mathematics.

A wide rage of fields of study such as advanced specialized programs leading to diplomas in teaching, agriculture, technology, nursing, etc., are found at tertiary education level as well as programs at the University of Zambia and the Copperbelt University. Adults who have never completed or never entered the formal education system can take up part-time studies. Literacy education, distance education courses, and in-service education aiming at increasing competence in vocational and professional skills are offered at various institutions in the country.

1.3 The Curriculum in the Formal School System. The monitoring of national educational standards throughout the formal school system necessitated the establishment of the Curriculum Development Centre in the early 1970's, whose responsibility is to design and issue curricular materials for primary and secondary levels. While all curricular subjects at primary and junior secondary school levels are adequately covered by the Curriculum Development Centre in terms of material production, for senior secondary school curriculum, syllabuses designed by Cambridge are being used except for the local languages; the materials for these are locally designed and eventually assessed and approved by Cambridge. Textbooks for most subjects are still being imported from abroad.

Three subjects form the core in all the sub-sectors of the formal school system at primary and secondary levels. These are English, Science and Mathematics. A wide range of subjects are offered as electives at all levels to add to the core. English and mathematics are compulsory subjects at all levels.

2. The Status of Mathematics Education in Zambian Schools. It is generally accepted that closely linked to the socio-economic development of a nation or society is progress in science and technology, whether indigenous or adopted from others. Science and technology has played a major role in the development of the established industrialized nations and the rapid industrialization of Asian countries. This can be seen through the impact on society of advances in diverse fields such as telecommunication, information technology, energy, industry, medical science, biotechnology, agriculture, earth sciences and the environment.

A major challenge confronting Zambia today is the need to ensure sustainable development. Mathematics in Zambia has not progressed adequately in the post-independence era to the extent that it could contribute sufficiently to the required rate of development. The experiences of other continents and certain nations in Africa show that the contribution of mathematics to science and technology can spur the development process. However, science and technology indicators in Zambia, such as enrolment in mathematics, physics, chemistry and other basic sciences, and the national budget in support of mathematics and science education infrastructure, research and development are among the lowest in the world. In view of the symbiotic relationship between mathematics and development, there is a need to further nurture mathematics and provide the necessary support, to enable mathematics play its proper role in contributing to the social and development aspirations of the Zambian people. This implies strong support at all levels for the basic prerequisites in mathematics subjects.

3. Trends in Mathematics Education in Zambia. A solid foundation in mathematics is essential, f example, to follow studies in the mining sciences, engineering, medicine, economics and several other fields. It is also essential for vocational training and for systematic research in the applied sciences required for long-term development. Unfortunately, for several reasons, development of mathematics has fallen far below expectation, and has contributed in some way to the current unsatisfactory state of development in Zambia.

3.1. Historic Trends. There are many factors why development of mathematics has fallen below expectation in Zambia. At the time of independence in 1964, the emerging government could only build or expand on what the colonial administration had established. The main investor in education has been the government. However, as the overall investment priority has been on defence, security and training in general administration, inadequate resources have been assigned to education, thus negatively impacting on the quality of education. At that time Zambia had no university but had a few institutions of higher learning. These institutions offered limited opportunities for advanced training in mathematics, and far less in the applied sciences such as engineering. Many of the higher-level mathematics personnel in these institutions of higher learning or those in government services, as well as doctors, architects and other professionals, were citizens of metropolitan countries. As there was inadequate infrastructure and conducive environment for higher education in mathematics, the few Zambian mathematicians had to acquire their advanced degrees abroad.

Unfortunately, the expansion of education at primary, secondary and tertiary levels came about at a time when Zambia had to face a number of challenges, which had major social and economic implications. These were due to a combination of factors, such as:

- poor agricultural policies and decline in world copper prices which set back the Gross Domestic Product
- political instability and civil strife in the neighbouring countries
- the oil crises of 1973/74 and 1979
- unfavouable terms of trade and an increasing debt burden.

All these factors have contributed negatively to the ability of the Government to realize the required resources that could have been channelled to education and particularly mathematics.

4. Pressures in Mathematics Education in Zambia. Teacher's selection, in-service training, staff utilization, school organization, curriculum construction, pupil counselling, community relations, parent concern, and budget considerations, are to be cumulatively viewed as a factor to developmental programmes of mathematics education.

One can point an accusing finger at a number of causes, for the dismal state of affairs in the present Zambian educational set-up; over-crowded classes (over 50 pupils per class), over-ambitious syllabus, anachronistic teaching pattern, tuition rackets, examination orientedness, student unrest, abuse of authority, mushroom growth of coaching classes, influence of guides, student politics, bad environmental set-up, negative influence of modernism on pupils (drug addiction?), increase in the growth of problem-children, poor time tables for schools/colleges, over-loading of academic institutions, lack of parental co-operation, testing procedures which do not reflect the goals of the syllabus, a pompous and irrelevant educational system, and poor salaries to teachers are some of the general factors. Whatever may be the cause, students' knowledge in mathematics is minimal and sometimes, less than minimal competence, when compared to the expected objectives.

The shortage of mathematically trained teachers at all levels of mathematics education i has contributed to the poor development of mathematics in Zambia. This shortage is a sit remained obstinately with us despite several schemes devised to bring some mathema schools. It has become normal to begin the year short of one or two mathematics teachers, the timetable as best we can. This is mainly due to business and industry, who need ma attract them with good conditions of service. Teaching positions cannot be filled bec applicants with mathematical qualifications.

This shortage has forced most schools to use teachers who are inadequately trained or mathematics. Teaching methods in most of these schools are still formal, emphasizing dril of information, instead of stressing understanding and creativity. In other words, pupils s school time accumulating and storing inert ideas that are merely received into the mind wi or tested, or thrown into fresh combinations.

Unfortunately, some decisions taken by educational administrators and politicians in re awarding extra allowances to teachers involved in afternoon classes, empowering teache houses, giving hardship allowances to those in rural areas, have not helped to attract and to could have made a valuable contribution to the teaching of mathematics. What does debilitating effect on the teaching of mathematics to the more able young people, if their te of the incentive to maintain an interest in the subject beyond the demands of the examina

Another shortage, which exerts its pressures on teachers of mathematics, is the shorta difficult to know what one can usefully say about that, in the current economic climate. We reached a point at which financial constraints are seriously threatening the quality of math Zambian schools. Most Government schools can hardly afford new teaching materials replace the old ones, if at all there are any.

A national pupil - teacher ratio of about 35 - 1 sounds fine, but most of the primary and teachers I meet are trying to cope with classes closer to 50. Moreover, whereas a decade ag been a non-teaching head of department, a part-time assistant or a supply teacher availabl unheard of. Thus, teachers have no time to devote to the needs of the individual child or s the class, which is essential if sound mathematical foundations are to be laid in the early y

There is also the challenge of in-service training. Whether this takes place within teacher's college, it is pointless to provide the resources if teachers do not have the o advantage of them. One has to appreciate the time and effort which teachers (c responsibilities) are prepared to give in the interests of their pupils, their colleagues and

Another challenge worth mentioning is the lack of mathematical inspectors of school standards have been falling for some time now. This has lead to the decline in confiden teaching, which has been publicly expressed in some quarters, particularly by employers i

One of the more depressing features of the present day scene is the small number of articles in journals and reports which are relevant to their work; perhaps this also reflect mathematics teachers, though I suspect it tells us more about their opportunities.

This brings me to one of the most important and immediately applicable pieces of res which has to be undertaken in Zambia.

The overriding impression that comes through is that for many children it takes far lo relationships essential to the successful learning of mathematics than is generally reali arithmetic, algebra or trigonometry, one sees time and again evidence of the mathematica take readily to a wide range of ideas, and quickly learn to perform successfully.

The most damaging of all pressures on teachers of mathematics in Zambia at all level achieve a specific mathematical target regardless of its appropriateness for the children c pressure that comes from both without and within the education system. The trouble is tha are always expressed in terms of an ability to carry out certain arithmetical or algebraic p evidence is that this is not the real problem.

Children's ability to carry out routine processes in a conventional manner already outs associate meaning with these processes. But there are many other children who, if we do r at an early age, will be ready to learn to use algebraic notation meaningfully in their last f

5. Opportunities in Mathematics Education in Zambia. Aggrieved by the great dissatisfaction that is prevailing in the teaching of mathematics in Zambian schools, the increasing number of failures in this subject, the compunction of teachers, students and parents alike, I have tried to find some remedial measures and I feel confident, that if the suggestions could be implemented in the day-to-day teaching of mathematics at all levels, they could help to retrieve the present situation.

5.1. Nature of the Curriculum. The curriculum, which was suitable a few years ago, is unable to satisfy our present-day needs. In recent years, radical changes have occurred in social, cultural, technological, scientific and other fields. As a result, the outlook of mathematics must also change to meet new challenges. The framing of such a curriculum should not be a one-man show. Mathematicians, methodologists, psychologists, researchers, technicians and teachers should come together to frame the different items of the curriculum. It is necessary for the syllabus to be sequential and scientific right from elementary level to university level. Each topic selected in it should motivate the child for further study.

5.2. Objectives of the Curriculum. The Ministry of education, teacher training colleges and the Curriculum Development Centre in Zambia, have not clearly stated the "objectives" of mathematics, standard-wise, within the framework of the prescribed syllabi.

Mathematics is now object-oriented rather than a subject of accumulated facts. Thus, each textbook chapter, topic or concept, must be explicitly expressible in terms of desirable behavioural objectives.

5.3. Continuity of the Curriculum. The syllabus of senior secondary schools should prepare the pupil for competitive examinations and entrance tests to professional courses like actuarial studies. As a matter of fact, the student who leaves school should be in a position to earn his bread and butter with vocational value of mathematics, or else, this value is reduced in the curriculum.

5.4. Modification of the Curriculum. It is a fashion nowadays, to change the curriculum frequently and to delete certain topics, which have already been covered, just before the examination. It causes great confusion and anguish among the students and teachers. Sometimes, textbooks are prepared and then the syllabus is modified to suit these books. At every stage of teaching, teachers are doubtful whether the topics they are teaching will be retained or not.

Curriculum formation is based on the principle that the syllabus will be constant for a reasonable length of time. It is the textbook, teacher's notes, work sheets and dictionaries, which need suitable modification every year, after thoroughly testing them at different orientation courses, and in classroom situations. They should be up-to-date, in order to reflect the current trends in mathematics education.

New examples, problems, diagrams and the similar items, when infused, increase the enthusiasm among the learners, since they are free from the confinement of textbook pages of stereotyped version.

I stress this because we have, at the present time, fresh opportunities for reshaping the mathematics curriculum in schools. We should be equally concerned to construct the right framework for children of average ability, one which puts the main emphasis on the use, and the meaning of mathematics rather than formal skills.

5.5. Availability of Textbooks. There should be a variety of textbooks in every school; text books which show an insight into the needs, capabilities and background of the majority of students. Among the books, we need to find some which deal with the fundamental concepts of mathematics, with practical examples that would make day-to-day teaching easy. So, it is necessary that each textbook is accompanied by "resource material" for teachers and parents, containing specifications, mathematical back-ground, precautions, teacher's activities, pupil's activities, laboratory equipment and important applications, in accordance with our Zambian requirements and conditions. It is also necessary for teachers to consult foreign books as references. School libraries could procure these.

5.6. Computer Courses. The other area of fresh opportunity is the presence of the electronic calculator and the personal computer. This technology needs to be popularised further before it can exert more than a peripheral influence on mathematics education. But the calculator alone should fundamentally change th objectives of mathematics as taught to many of our pupils. It forces us to rethink the balance between verb and written skills, between fractions and decimals, between algebraic and arithmetical thinking, between exac and inexact numeration, between computation and problem solving. Coming to terms with the implications c the calculator must surely have top priority in mathematics education for the next few years.

6. Conclusion. It is true that, devoid of mathematics, science would be hard put to solve problems. W therefore find that we have arrived at a situation, where having regard to the exigencies of "tomorrow' mathematics must be made so as to enable the student to apply and expand the knowledge he has. This is process that started in the countries of the west, where abstract principles of mathematics have been applie in the solution of problems, realizing them under "applicable", as opposed to "applied mathematics" - continuous process of becoming and not a static bedrock of being.

The growth of mathematics is inherent in its very nature. It is perennial, both in its theoretical and applie fields. It stands to reason therefore, that in response to the change, curriculum needs framing and re-framin at all levels of teaching. It is the considered opinion of many notable mathematicians, that only by creatin mathematics anew, can it be best learnt. It is this, which is at the root of a large number of pilot projec which favour active, subjective and creative learning.

Against this background, it would appear that there can be no validity in any criticism, directed against tl subject, unless the very subject is proved wrong.

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THE TEACHING OF MATHEMATICAL ANALYSIS TO UNDERGRADUATE STUDENTS

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It is universally agreed that, the best method of teaching mathematical analysis is based on the teacher's own philosophy and reflects his/her own decision in the light of that philosophy. If one introspects, one could ask oneself the question: "Is my technique of teaching really a true reflection of the nature, psychology, philosophy, anatomy and architecture of mathematics?"

The answer in general is "no". So, philosophical views alone are not sufficient; but they should be framed within the true perspective of mathematics.

Present-day teaching techniques are product-oriented and not process-oriented. That is how the short-cut methods are designed by the teacher to obtain correct answers. To my mind, there is no royal road to learning mathematical analysis through short-cut methods.

The other question that arises, is: "Is the task of learning a subject such as mathematics, different in some way from that of learning other subjects of Natural Sciences? In this subject concepts have only mental existence. So, to learn this subject, one has to turn away from the world of sensory objects, to an inner world of purely mental objects.

In order to get a sound knowledge of the subject, the characteristic components of mathematical analysis are to be isolated first and the students should be trained there of; and then, one should proceed to situations of problem-solving nature, where all components are applied in a synthetic way, for comprehensive study.

Mathematical analysis should not be taught as a set of isolated facts and formulae. It has to be taught as a "subject of Mathematical Science," in which relationship of facts is as important as the facts themselves, and within the facts are arranged in sequences.

On a similar hypothesis, one may invent one's own techniques of teaching, however, in the initial stages, it is good for students, to follow their own methods for self reliance. The teacher should help to bring forth the ideas conceived in the students mind, by prudent guidance.

I close this very brief note with the remarks of Albert Einstein: "I dislike the straight-jacket, regimental methods of teaching mathematics. To me, teaching should be such that what is offered is perceived as a valuable gift and not as hard duty."

Bridging the Gap: The Swinburne Strategy

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ABSTRACT. The School of Mathematical Sciences of Swinburne University of Technology some years ago devised a bridging strategy to allow a more flexible approach for first year students. This strategy was aimed at supporting those students who were making the transition into undergraduate courses containing significant mathematical content, but who lacked the usually accepted level of mathematical expertise. The aim of the program was to provide on-going individual and group support to students on a request basis. Intensive preparatory and remedial programs supplemented this support. In addition, individual weekly assignments, designed to assist students in diagnosing their specific needs, were incorporated into the first year mathematics subjects. The development of this year-long bridging program, together with its philosophy and its mode of operation are outlined in this paper.

1. Introduction. The last decade has seen a number of authors writing about the diversity of background students entering tertiary education, in particular of those embarking upon courses containing substan mathematical content. This diversity is demonstrated in a wide range of abilities, educational ; mathematical experiences and personal attitudes. The reasons for such diversity are many and varied nevertheless result in many students requiring extra support if they are to succeed. Taylor [1] discusses nature and history of support for mathematics learning in undergraduate courses in Australian Universitie the late eighties, Swinburne University of Technology was no exception to this trend and the Schoo Mathematical Sciences reacted by introducing its own integrated solution.

2. The Swinburne Solution. In 1989, having identified the need for some form of intervention on the pa many students, the School of Mathematical Sciences of Swinburne University of Technology introduce bridging program. This was an effort to improve and assist student learning in certain first-year mathema subjects. Initially, students undertaking the first year of a Mathematics and Computer Science degree wer focus. Since then the program has been progressively expanded and modified, until today it extend support all students undertaking a first-year mathematics subject in the university. In the year 2000 amounted to in excess of 1250 students covering the areas of Applied Science, Business, Computing Information Technology, Engineering and Technology, Multimedia, Health and Human Services, So Sciences and Arts.

The program arose in response to the awareness of difficulties encountered by a number of first mathematics students. At the time of its inception it was not unusual to find students admitted to hig education courses in science and technology, who were inadequately prepared in mathematics. This may resulted from inadequate or inappropriate preparation at the school level or as a result of being away f scholarly pursuits for some time. Today this is still true. In addition, there are always a number of stud who lack confidence in their mathematical ability and are studying the subject only because it is a neces part of their chosen course, not because of any predilection to the subject. In any case a traditional first mathematics course does not have the time or resources necessary to address this situation. In respons these problems, a new program was devised at Swinburne, built around the idea of early identificatio those students thought to be at risk of failing or not performing to a reasonable standard. It concentrate first year students as it has been found that because of the particular difficulties of transition from scho tertiary study this is the critical time for intervention.

Regular assessment in the form of individual weekly assignments was introduced. The aim of this twofold - students were encouraged to keep up to date with their mathematics, and secondly t assignments were designed to help diagnose any apparent weaknesses. Students were then in a position t on this information by seeking help. To this end the Mathematics Resource Centre was set up, a forum w students could confidently obtain extra help.

The Mathematics Resource Centre is a room equipped with books and learning resources housed wi the precinct of the School of Mathematical Sciences. As such it is seen very much as an integral part of the School and not as a separate entity. It is staffed by a Coordinator (.8 fractional full-time) and an Assistant Coordinator (.5 fractional full-time), and from time to time extra staff are called upon to provide support. The principal mode of operation is as a student-initiated drop-in facility, providing one-on-one tutorial assistance. From the outset it has been popular with students, positively viewed and well utilized. An average of 1663 consultations per year have been conducted over the last seven years with an average of 38% of eligible students visiting the Centre over the last four years.

With many bridging programs, students with perceived weaker backgrounds are given an intensive remedial course before the course proper begins in an attempt to remedy their deficiencies. Another approach is to stream students at the start of a course into different groups, which are then taught at different paces, or with different syllabuses or different expectations. The first strategy adopts the premise that knowledge deficit is the sole cause of the problem, whereas the second strategy limits the opportunities of the "weaker" students, emphasizing the idea that students with weak backgrounds are unlikely to catch up with their more capable counterparts. This stifles any chance of growth in confidence or in positive feelings towards mathematics on the part of the students.

Swinburne has long had a reputation for a personal approach to scholarship. This caring, positive environment required nurturing and so Swinburne's strategy was to instigate a program based on integrated bridging. This strategy differs from most other strategies in two significant ways. The provision of remedial assistance is considered to be an integral part of first-year courses and secondly it is available to all students if they or their teachers feel the need for such assistance.

The integrated bridging strategy was spread over the entire first year. Initially, students believed to be at risk of poor performance in mathematics were offered the opportunity to take part in a two-week revision program prior to the commencement of classes. These revision classes were not mandatory and students could select those classes from which they felt they could most benefit. Once the semester commenced, individual weekly assignments were incorporated into the mathematics courses. These acted as a catalyst for many students, who upon recognizing their deficiencies, were prompted to take steps to remedy the situation. After all, they were being assessed on these assignments (20% of the semester's mark). The students were encouraged to visit the Mathematics Resource Centre for assistance. Here help was provided on a one-to-one basis or given in small groups. At times the staff of the Resource Centre ran larger classes on topics believed to be useful to some students. For example, prior to linear algebra being covered in lectures, the Resource Centre staff might teach elementary work on matrices to students who had never covered this topic before or who felt they needed to brush up on it.

As well as personal assistance being provided to students in a supportive environment, course support materials that would allow a student to start at a level appropriate to his or her own level and to work independently if required were developed. Videos on particular background areas relevant to the course were also provided for student use.

Table 1 shows the way in which the first year mathematics program was structured to allow for differential rates of learning and the integration of remedial support.

A radical feature was introduced between semesters as an integral part of the bridging program. Once the first semester examinations were marked, those students who fell short of a pass mark were offered the opportunity to attend an intensive revision program, which ran for two weeks. At the end of this time they were able to sit another exam. The higher mark obtained was the accepted one.

This proved to be a highly successful innovation. The students benefited from this program in a number of ways. Generally they approached it diligently and derived a much clearer understanding of the first semester's mathematics. This, of course, provided them with an improved basis for their second semester's work. As well, a large proportion of students improved their semester mark to a pass or certainly to a better result and this had a positive influence on their attitude to their mathematics subject. To discourage students from taking unfair advantage of this scheme, certain conditions were imposed. Students had to demonstrate that they had made a concerted effort during the semester before they were given the opportunity to participate in the revision program. They needed to have attained a particular minimum level on the semester's work, i.e. a mid-semester test and weekly assignments. This semester's work comprised 40% of the semester mark. There was also a ceiling imposed on the mark from the second examination. This, it was felt, was a reasonable approach, both for the students who had performed creditably during the semester and hence had no need of chance, and for those who genuinely required extra time and understanding to make the grade.

The second semester proceeded in much the same way with the exception of the opportunity fo to participate in an intensive revision program once examinations were finished. The sole reason fo of this revision program was due to university administrative constraints.

Weeks	Normal	Additional
1-2	Introductory program for target group.	Assessment of initial needs & bridgin instruction on an individual basis whe necessary.
3-16	Normal instructional program but using a range of strategies aimed at meeting a wide range of needs.	On-going diagnosis of mathematical deficiencies & individual remedial support where necessary.
17-21	Formative and summative assessment of student performance and remediation prograi semesters for those not making the grade.	
22-36	Normal instructional program but using a range of strategies aimed at meeting a wide range of needs.	On-going diagnosis of mathematical deficiencies & individual remedial support where necessary.
37-39	Summative assessment of student performance.	

Table 1: Structure of the first year mathematics program allowing for differential rates of and the integration of remedial support.

Today the program is still functioning, having been expanded and modified for various reaso been adopted to a degree by each first year mathematics course. Every student studying a mathematics and/or statistics subject may avail themselves of the Resource Centre, either by refe teaching staff or on their own initiative. A number of courses utilize regular individual assignmen prepared by the Resource Centre staff, as are the solutions. In addition, many students continue to the Resource Centre seeking help in their second and later years. While not officially covered by the these students are assisted if time permits. Unfortunately, due to university constraints the intensiv programs which were run both pre and post semesters are no longer functioning.

3. Effects of the bridging program. To gauge the effects of the integrated bridging program o performance and progression a detailed study was made. Barling and Jones [2] found that impleme the bridging strategy was associated with a significant improvement in pass rates (48%) and compl (33%) in first year mathematics for the target group, compared to the equivalent student group in th year. This also translated to a significant increase in first year mathematics pass rates (20%) for intake as a whole when compared to the previous year's intake. As well, the improvement in pe continued into second year, with completion rates being 40% higher for the target group compa equivalent group of students in the previous year.

No further formal studies have been conducted on the effect of the bridging program, but infor felt that the effect is a positive one. Both staff and students hold this view. In an independent engineering students [3], the Resource Centre rated with the Sports Centre as the two University most used and recognized by the students. Without this widespread belief in the program its underl would not have extended from one Mathematics subject in 1989 to every first year Mathemati taught in the University in 2001.

4. Conclusion. The bridging program at Swinburne has proved successful as is evidenced by the subjects serviced and the steady increase over the years in the number of consultations conduct Resource Centre. With the apparent continuing trend of student diversity the need for such a progra abate. We should not become complacent or too rigid, however, but remain vigilant to students' ne prepared to modify and/or adapt support programs to suit our student population.

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PROMOTING STATISTICAL MODELLING IN THE WIDER COMMUNITY

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ABSTRACT. With declining interest from secondary school students in mathematical based subjects, it is important that researchers and teachers take every opportunity to promote applications of mathematics and statistics to the wider community. The author has been successful in gaining widespread publicity for his research into sport in the press, radio and television. This publicity has promoted the statistics profession as well as the author's institution to possible users and financiers of the research, future students and the general public. Similar publicity is possible for many staff research or student projects. This paper discusses examples and strategies that have been employed.

1. Introduction. Although mathematics and statistics are playing an increasingly important role in many areas of study, there has been a steady decline in enrolments in mathematics-based courses in both secondary and tertiary education. In Australia during the nineties there has been a decline from about 50000 to 30000 in the number of year 12 secondary students studying advanced mathematics subjects. With this declining interest from students in mathematical based subjects, it is more important that researchers and teachers take every opportunity to promote the applications of mathematics to the wider community.

My research area is modelling in sport, and sport is one area the general public do associate with statistics. By increasing awareness of the many interesting applications of mathematics, we are raising the profile of mathematicians and their discipline. This hopefully results in more students taking up the challenge of learning the methods. . This paper explores ways in which the results of mathematical modelling can be brought to the attention of the general public.

2. Articles in popular magazines. As a researcher I tend to publish in statistics or mathematics journals. In most cases, this is preaching to the converted. However most statistical or mathematical analysis takes place in an area of application, and there may be alternative sources of publication relevant to that area. It is possible a less technical paper, concentrating on the results rather than the statistical techniques, could be written. A paper on tie point strategy was published in a refereed journal [1], but my co-author John Norman also organised a version in a squash magazine. The upshot was a surprisingly technical article [2], complete with tree diagram and subscript notation, but that also conveyed the main message - play to 10 unless much weaker than the opponent.

3. Weekly predictions in the daily press and television. In 1980, as a result of a student project, I wrote a computer program to automatically predict the results of Australian rules football matches [3]. I contacted The Sun, a major daily newspaper in Melbourne, and suggested they set up the program in competition with their major tipster. They took up the suggestion, and the tips have appeared in newspapers and television for 20 years. They have also resulted in many related research topics such as home advantage and draw difficulty, which have also generated publicity for statistical techniques. I often wonder at the far-reaching consequences of a couple of letters sent to newspapers.

4. Relationship with a journalist. Pick up any newspaper and you will usually find several regular columns. The authors of these columns have to come up with ideas and generate text each day or week. The columnist is usually grateful for any ideas you might have. I built up an excellent relationship with Ted Hopkins, a freelance journalist for The Australian Financial Review. Ted wrote a weekend column analysing football player statistics, which was so successful, they continued into the summer season, and extended into scientific findings in a range of sports. Because of a previous association, Ted often used me as a sounding board for ideas. In many cases this resulted in an article based on previous research by myself [4,5]or others. For example Hopkins [6] describes the results of a masters by coursework in Statistics, which investigates why girls do not take up golf at the same rate as boys. In other cases [7], we did some extra analysis that allowed comment on a current event.

This sort of relationship can be quite voracious. Studies that might have taken years to complete and publish were dispensed with in a single weekly article. In fact 20 years of my research was consumed in one summer of weekly articles. We probably had input to well over 20 articles, and like the columnists this can stretch your capacity for ideas. A good idea is to have some reasonably standard technique that you can apply to a regularly occurring event. For example, a simulation of a tennis draw based on probabilities derived from official rankings [8] can be used to predict outcomes each time a tennis major is played.

These articles required a varying degree of input. In some I just discussed a possible topic, in some I wrote a draft for the article, which Ted then rewrote in journalise. In fact the partnership was so successful, we even had our sports statistician write several of the articles and get his byline on the article while Ted was on holidays. In some cases we could stray slightly from the main topic of sports statistics and venture in to education. The column [9] discussed the job opportunities for sports Statisticians, the need for education and the Swinburne Computing and Applied Statistics course. In most of these articles we always tried to work in a copy of our Web address www.swin.edu.au/sport.

5. Web site. Researchers now have an easy way to go to the public directly via the world wide web. In our case we maintain a web site with several topical applications. In addition to regular computer predictions of our major Australian rules football competition, we have predictions of most major Tennis tournaments based on logistic regressions and simulations, predictions and analysis of major events like the world cups of cricket and soccer, and the Olympics. Web sites have the advantage that technical information can be provided for those that are interested. In our case several pages have clocked up over 30000 hits in a couple of years. Usually, to receive a lot of hits, you need your web page address published regularly in some other media outlet.

6. Press release. A common way of gaining exposure for some project or analysis you may have worked on is a press release. In my case the corporate marketing section at Swinburne usually uses this strategy. Conferences may also send out press releases, and would usually mention those papers they think would be of interest to the media. A one-page summary of the main results or point of the article/research is made, and faxed to a variety of media outlets. For example, one year prior to the football season we issued a press release on the Computer prediction of the final ladder. This resulted in an invitation for a group from Swinburne to appear on the then top rating TV Australian rules football show. A press release on a conference paper on home advantage in the Olympic games [10] ultimately resulted in two television, six print [11], a www article and over ten radio interviews.

It is probably better to get a third party to write the release. It is very difficult to put yourself up as a world expert in something, whereas journalists seem to have no compunction in sensationalising research. As the author of a study, you are familiar with all the complications and clever analysis you have done, and would probably put too much detail in the release. The press is often interested in an attention-grabbing headline, and someone trained in the media can often pick this out. It is then up to you, once you have the interview, to discuss those aspects you think are important.

The degree of detail required also diminishes as you go from print to radio to TV. In a recent six-minute TV appearance, the interviewers covered optimal batting strategy in one day cricket, demonstrating corruption by analysing cricket scores, optimal number selection in lotteries, home advantage in the Olympics and forecasting the number of medals Australia would obtain. There is not much time for any detail when so little time is available for each topic.

In terms of generating further publicity, an article in the daily press is usually the best outcome. This often results in contact by radio stations that require live or taped interviews. While press articles have a way of transforming themselves into other articles for other papers independently of any input from the original interviewee, each radio station has its own show, which requires a separate interview.

7. Respond to requests. A simple strategy is just to respond to requests. About once every six months I receive an e-mail via our general university system, which asks for contributions to a magazine or media program. For example, Science Magazine Australia asked for scientists to submit popular articles based on their research (I still haven't written the one I promised). One I did respond to was Quantum, an Australian television show on Science. This resulted in a four day interstate trip to watch the Australian cricket team train and play, interviews with players, the coach and support personnel, and ultimately a 10 minute segment on the show which highlighted my research into one-day cricket batting strategy [12, 13]. As a spin-off, I have found students are strangely much more attentive to a TV replay of me explaining the results of my dynamic programming research, than to me doing it personally.

8. Conclusion. There are many reasons for seeking publicity. In my case the University has certainly been pleased with putting Swinburne's name in the public eye. In the same way, the public has been exposed to results of statistical and mathematical studies. Of course results are variable and you should be prepared for failure. Sometimes what you think should catch the attention of the media does not. However it is certainly not as difficult as having refereed papers accepted, and offers a sometimes exciting alternative means of recognition. It is certainly pleasing to know that results are read widely, and in most cases it takes little extra effort.

While my application area is one that has some advantages in gaining publicity, most other areas are also suitable. Studies in health, social statistics, politics, finance, transport etc would all be of interest to the general public. A little extra effort could bring some exciting rewards for both the individual and the statistics profession.

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Of Paw-paws and Peano
or:
Why the Formula Makes No Sense

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ABSTRACT. We have both taught courses on number systems to different audiences over a period of time. This led us to a realization of the tension that exists between the elegance or simplicity of abstraction and the need to ground the mathematics in real-life examples or experience. We believe that achieving a balance between abstraction and every-day experience has decided benefits both for moving to higher levels of abstraction and in building confidence in the usefulness of mathematics.

In this paper we will illustrate our ideas by referring to cases that have occurred in the various courses that we have taught. These include courses for primary school teachers, secondary school teachers and tertiary level students.

1. Declaration. We, as mathematics teachers, want our students to use mathematics with confidence and to feel that mathematics makes good sense and in so doing can be used to solve real problems.

On the other hand, we appreciate that all mathematics involves some level of abstraction.

As a result, we feel that common sense, abstraction and the links or connections between abstraction and common sense form an absolutely crucial part of the teaching of mathematics. As the student investigates higher and higher levels of abstraction, the links between common sense and the abstract become more difficult to maintain.

In our view, the links are supplied by *proofs* of some kind. The omission of appropriate proofs leads to a breakdown of the sense of meaningfulness of mathematics.

We attempt to provide definitions for some of the terms used above.

Definition1. Good sense, common sense: Any diagram, example, real world scenario or argument that conveys a sense of nearly incontrovertible reality

Definition2. Abstraction: Any argument or statement that uses mathematical symbols. (Like 1, 2, x, symbols for division, infinity and the set of natural numbers.)

Definition3. Appropriate proof: Any argument using good sense that will convince you - and that you can use to convince reasonably sceptical others - of the value or truth of your abstract assertion.

2. Why are we making such a declaration? We see the process of learning mathematics as one in which the learner progresses through increasing levels of abstraction. At each stage of this progress, we would ideally like the learner to have a sense of conviction that what is being learnt is securely based on previous experience (mathematical and otherwise). In this way, a meaningful, coherent and usable body of knowledge could be constructed.

Apart from our normal academic teaching duties, we have both been involved in teaching the algebra of natural numbers, integers and rationals, to widely differing audiences over the past years. Our students in these courses included practising primary and secondary teachers as well as students in their second year of study in the Science Faculty at the University of Cape Town (UCT).

Our experiences in these courses suggest that, for most people, there is a point in the process of learning mathematics at which the conviction and security cease. At this stage an uncritical acceptance of further abstraction may occur. The abstraction is adopted as a meaningless but acceptable relief from the complexity of the situation.

We feel that one factor in this breakdown is the lack of appropriate links, or, in our terminology, appropriate proofs.

3. A Disclaimer. We wish to point out that we are not professional mathematics educationalists. We teachers of mathematics at a tertiary level, both undergraduate and postgraduate.

Our interest in Mathematics Education can therefore be regarded as naive and we acknowledge be possibly uninformed as regards the debates that constitute the realm of the professional mathema educationalist.

In the light of the above, this paper cannot claim to be developing knowledge or breaking new ground the field of mathematics education. What we are asserting may well be folklore to a professional Mathematics Education. We do claim, however, that as teaching mathematicians, our experiences in teaching environment put us in a position to reflect on our students' responses to abstraction and that th reflection may be of interest to other teachers of mathematics.

We were fortunate to teach to a much wider spectrum of students and at more levels than ma mathematicians. This has, we feel, enhanced our understanding of the way that mathematics becomes meaningless and the reasons for this.

4. A first example. We opt for simple illustrations of our ideas in practice. Our illustrations are drawn fro out teaching experience. In the first example, we sketch briefly what we did in a class for primary school mathematics teachers. We choose to look at division of rational numbers. We do not claim that ou teaching method is in any way original (on the contrary, see for example [3]), nor do we wish to engage in general discussion of teachers' and situation where being able to divide one fraction by another would b useful?

[At this stage we feel that a slight digression is needed. We needed to put our students in a situation that would force them to see the necessity for reverting to common sense. How could we do this for those (o students) steeped in a history of algorithms and who believe in the irrational effectiveness of abstra mathematics? We tried to achieve this by setting up a situation where the students were required to "teach" mathematics on a desert island stripped of resources (books, other teachers, higher authority) in a different base (base 5, as it turned out) to a sceptical bunch of learners. We called these ideal learners "Islanders" (See [1] for more details.)]

Responses to the above questions were slow in coming. It was some time before a suggestion along the lines of "If you have 23 paw-paws and wish to serve a one third slice of paw-paw to each guest, how man guests can you invite for dinner?" arose.

We then challenged the students to find practical solutions to such problems that would be acceptable to their student islanders. This meant that they could not appeal to any machinery learned on the mainland. We used this strategy to build up our students' belief in their common sense and confidence that their solutions were meaningful.

The next step was to move in the direction of abstraction: specifically, we wanted them to consider finding for their islanders an algorithm that would cover all problems of the kind that involve dividing a whole number by a monic fraction.

Finally, we insisted that they be able to explain their algorithm in terms of the examples dealt with earlier. They were required to be able to convince each other and the islanders that their algorithm would really produce the correct answer under all conditions. We were asking them here to link their common sense solutions to the problems and the algorithm. This, to our way of thinking, means finding appropriate proofs for their discoveries.

In lectures we progressed in a similar way to deal with division of arbitrary fractions, but we feel that the above illustrates the essence of our methods and ideas.

5. A questionnaire. We were reminded of the extent to which an acceptance of the meaninglessness of mathematics obtains at a tertiary level by the results of a questionnaire administered (at the beginning of the course) to science students last year. The students were all enrolled for a course, at second year level, called Number Systems. We illustrate this problem with one question from the questionnaire.

What do you think is the reason for the rule "When you are dividing fractions, just invert the second fraction and multiply?"

We list a few (edited) responses:

- It makes getting the answer easier.
- You can't divide a number with two, three or four numbers at a time. It's clumsy.
- It is easier to deal with multiplication than division.
- By inverting the fraction you must invert the sign and the inverse of $\div$ is $\times$.
- Multiplication and division are defined as inverse of each other, so you must invert.
- According to BODMAS we divide, then multiply.
- I really do not know.

It was disconcerting that at least a quarter of the students either did not answer this question or answered "I do not know".

6. A second example. Our second example is drawn from the new course on number systems referred to in the previous section. One of the major motivations for the introduction of the course was the substantial difficulty experienced by students when confronted with the sudden increase in the level of abstraction typically encountered in a first course in linear algebra. In the light of our experiences, we felt that students' perception of abstraction as meaningless was a largely responsible for these difficulties. Our aim was to try to bring back a belief in the meaningfulness of abstraction, and to do this in the (hopefully) familiar context of number systems.

We relied heavily on our experiences and the materials developed n the courses for teachers in designing and presenting this course. We adopted a two-pronged approach. On the one hand we took pains to develop the arithmetic of integers and rational numbers in such a way that it was clear that this was a meaningful response to practical needs and was firmly rooted in reality. On the other hand we felt free to introduce more sophisticated mathematical terminology and more formal definitions once we felt that the basics had been understood. We hoped in this way to show up the familiar number systems as motivation for the more abstract mathematical structures that they would encounter later.

Our second example comes from this context. Division of rational numbers was initially treated from a practical perspective (as with the teachers). In the process we also explored the link with multiplication, and emphasized the fact that a problem involving division could always be rephrased in terms of multiplication. Rather than adopt a purely formal approach (inverses, inverse operations), we wanted the natural duality between division and multiplication to be shown up. To give students more practice in this principle in a less familiar context, we introduced them to multiplication of natural numbers modulo 5, and asked them to set up a multiplication table for Z_5. (They had seen addition modulo 5 earlier as well.) After some questions on the properties of this type of multiplication, they were asked to try to define a division operation $\ominus$ in Z_5. There were two types of responses:

- Some students attempted to modify the template for multiplication:
  "Define $m \ominus n$ to be the remainder when m/n is divided by 5". They had not explored the links of this question with what they had seen before, nor reflected on the meaninglessness of their own definition.
- The remainder of the students, who realised that they could bring their previous experience with division to bear here, defined division in terms of the multiplication given. They succeeded in drawing up a division table and were well on their way to showing that (in sophisticated terminology) Z_5 is a field.

7. Our reactions and observations. We were surprised at the time it took our students, even those at university, to find examples illustrating division of fractions. They clearly found this a novel and difficult exercise, and even though they were working in groups, it took up a substantial proportion of classroom time. We were further surprised that the teachers preliminary attempts to find examples illustrating division of fractions often produced examples which needed multiplication rather than division.

In the process of getting the students to discover and formulate an algorithm we met with considerable resistance. There seemed to be a perception that you cannot discover a rule. Their attitude seemed to be that rules were simply handed down from teacher to teacher (and that it was morally indefensible to withhold a rule from a learner!). Linked to this was a deep distrust that their own thoughts and experience could lead to the discovery of a useful rule.

Once the resistance mentioned above had been overcome, it took students a relatively short time to use their examples to concoct what amounted to proofs of the newly discovered rule. It was gratifying to see that they began to enjoy this activity.

It was disconcerting to see the confusion concerning the notion of proof that prevailed amongst the students. In their view a proof would necessarily be terse, very formal, use symbols in preference to prose and be unrelated to their experience. They did not see their informal attempts as valid proofs.

There is an acute sense of meaninglessness which colours the responses of our students to the question posed in the questionnaire mentioned above. One gets the impression that mathematics to them is a somewhat arbitrary and capricious activity. The rule proposed for division of fractions, for example, is viewed as just one of many that might have been chosen.

The acceptance of meaninglessness at school level has a marked impact on the attitude of students to abstraction at tertiary level. It was distressing to see the number of students who were satisfied with their definition of division for Z_5, even when the merest attempt to use their definition would have resulted in obvious confusion. The provision of a definition was seen as unrelated to the use to which the definition might be put.

8. Our Conclusions. In this section we will present conclusions based on our reactions and observations in the previous two sections.

In general we feel that the stock of common sense is low. (This is no insult; look at the definitions!) It is no use believing that everyone has an adequate store of everyday experiences that are waiting to illustrate all sorts of mathematical ideas. In fact the reverse seems to be the case. There seems to be a major difficulty in seeing mathematics in life. While all people do have a large store of everyday experiences, there is no practice in seeing any mathematics in it. The skill of seeing mathematics in life needs to be actively grown. Moreover students must be challenged to find *their own* examples. This, we believe, is not necessarily the case. Such examples must come first, the definitions and algorithms later.

A corollary is that the stock of abstraction is relatively too high. We are producing a population that has exposure to a relatively large amount of abstraction which, because it is ill understood and unmotivated, is largely useless. We are not, however, advocating a reduction in the amount of abstraction that should be covered. A growth in the stock of common sense will, we think, make the acquisition of abstraction more efficient (see the antepenultimate paragraph of section 7).

It is clear that the role and nature of proofs is misunderstood. They are not thought of as explanations and aids to understanding. Proofs are seen as peripheral to the business of mathematics and an unnecessary imposition. ("If I can see why it works, do I need a proof?") Please see [4] for a perceptive discussion of the nature and role of proofs in mathematics.

Once meaninglessness has become the norm, it is difficult, at a tertiary level, to restore a belief that mathematics can make sense. A profound lack of confidence results and survival strategies become the aim of learning.

9. Our Beliefs and Flexibility. It seems that mathematics teaching and learning is in the grips of a false dichotomy: to be abstract or not to be abstract. To our way of thinking this is a most unhelpful. One cannot escape abstraction in mathematics; it is part of the power that successful mathematics at any level provides. One also cannot have mathematics that is not built on good common sense; to do so is to move into the realm of arbitrariness and uselessness.

To marry abstraction and concreteness or reality, appropriate links, which we have called proofs, must be developed. Without these links, mathematics becomes inaccessible, meaningless and abhorrent to many. The study of fractions seems to be particularly prone to this, and for many people this is the point at which mathematics becomes meaningless.

Based on our interaction with teachers and students, we are unhappily convinced that such a meaningless and abhorrent state is not uncommon. We feel very strongly that the principal ingredient that contributes to such a state is not the overemphasis of abstraction but rather the tendency to ignore the common sense aspects of mathematics and the way they inform one's view of the abstract.

We suggest that the principle reason for this tendency is that the mathematical elements of one's daily experiences are seldom considered or pondered in class. Seeing the mathematics in one's daily life is a skill that needs to be developed. We may not assume that students have a natural flair for this. We cannot assume that a common sense understanding of mathematics is inherent in our learners. It is something that must be cultivated. See [2] for suggestions along this line in a problem-solving context.

We must fight the tendency to succumb to a blind acceptance of abstraction and constantly fight for meaning. Abstraction is elegant, powerful and can wear the guise of simplicity. Abstraction is a seductive creature; it is all too easy to become its slave rather than its master.

What has this to do with flexibility? A great deal. Since a person's attitude to mathematics ought, in our view, to be based on their life experience, any teacher of mathematics must be flexible enough to incorporate other people's life experiences into their mathematical experiences. A teacher must be flexible enough to challenge students to make mathematics their own by bringing their everyday experiences into the learning arena. The results, in our experience, are well worth it.

Once real life has been brought into mathematics the teacher must then establish links between the concrete and the abstract. This requires enormous flexibility. The links must be tailored to the requirements and abilities of the given students. A link which will convince one person may not make sense to another.

The power of mathematics lies in its flexibility. By insisting on rigour and abstraction to the exclusion of the learner's own experience, we run the risk of destroying that flexibility. By avoiding any appeal to abstraction, we also run the risk of destroying that flexibility. The greatest gift we as teachers can give our learners is this flexibility.

We finish with a quote from one of our teachers: "Om eendag dit te kan verstaan was maar net wens." For those unfamiliar with this African language, "To understand it one day was just wishing."

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What Makes Word Problems Difficult?

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ABSTRACT. This paper investigates word problems encountered in calculus by first year university students. The study draws on work on representation (Lesh, 1987; Dufour-Janvier, 1987) and problem solving (Polya, 1957; Schoenfeld, 1987). Aspects of word problems that are investigated include language, representations and contexts. Aspects of problem solving that are investigated include transfer between situations and transfer between representations. The difference in problem solving strategies of experts and novices are compared and contrasted, drawing on work by Larkin (1980), Silver (1987) and Chi (1981).

The paper reports on preliminary data analysis of a survey, to be completed in the second term of 2001, by a large cohort of first year students and a sample of mathematics lecturers. The aim of the study is to identify problem solving strategies and heuristics used by students as well as those aspects of word problems which affect difficulty level, as perceived by the student, thus allowing a hierarchy of difficulty to be developed.

This study is intended to be of use to teachers of mathematics, to enable them to foresee difficulties for the student and to teach accordingly, so as to encourage flexible problem solving ability.

1. Introduction. This project was inspired by my experiences both as a science student and as an educator in mathematics. The University of Cape Town (UCT), at which I teach, offers several first year calculus courses for students in different faculties. I am currently one of the lecturers for the science students' calculus course, and have also taught the course designed for students in the commerce faculty. As a lecturer and tutor I regularly encounter students who refuse to attempt word problems, or who struggle to see how they should be approached.

The actual mathematical content of a word problem (i.e. the algebraic steps carried out to find the required solution) is usually simple, largely because it is difficult to describe in words a problem that is both complicated and still solvable. It is my hope that this research project will provide mathematics educators with suggestions on how to tackle word problems in a classroom situation, so as to make them accessible to the students.

2. Definitions. The characteristics of word problems can be categorized in different ways. The following schematic describes one way and includes all the terms considered pertinent to this report. They are defined below.

- **Word problem**: A set of statements describing a situation that can be modelled using mathematical symbols, but is described using nonmathematical language, or language involving a mathematical register. The problem instructs or invites the reader to solve for an unknown quantity using only the information included in the statements.
- **Concrete problem**: Also referred to as real life problem or real world problem, the concrete problem is one that describes a situation involving physical objects or real concepts that could possibly be encountered by the reader.
- **Abstract problem**: An abstract problem describes a situation that is not related to real objects of any sort. It can describe a geometric or algebraic situation.
- **Algorithmic requirement**: The majority of problems encountered in a first year university calculus course fall into this category. This sort of problem requires the student to make at least one calculation, involving formulae such as those for volumes of solids, and utilizing quantitative information supplied in the statement of the problem.
- **Interpretative requirement**: Some problems supply the problem solver with a lot of information, often in the form of a table or a detailed graph. The problem solver is then asked a series of questions that require little or no calculation, but that require an understanding of calculus, both differential and integral.

3. Problem characteristics.

3.1. Representations. Information can be represented in very different ways (several suggested below). Word problems by their nature represent information in written language, but sometimes include other forms of representation, such as sketches or graphs; quantitative tables are also sometimes used. Lesh *et al.* (1987), summarise 5 forms of representation:

- experience-based "scripts" / real world events
- manipulatable models
- pictures or diagrams
- spoken language
- written symbols.

I am concerned here with the first, third and fifth types as well as their relationships with one another.

Numerous researchers (Lesh *et al.*, 1987; Kaput, 1987; Craig & Winter, 1990) have found that translation between representations is problematic. Lesh *et al.* suggest that "the act of representation tends to be *plural*, *unstable*, and *evolving*" (emphasis in original).

Dufour-Janvier *et al.* (1987) consider the common practice of encouraging students to draw sketches to aid them in solving a problem (such as suggested by Polya, 1957). They discover that mere encouragement to draw has little effect and that it is not the act of drawing that is important, but rather the ability to "develop graphic codes and symbols". They suggest that the students be encouraged to use external representations of any kind as an aid, perhaps more as an indication to the teacher of where intervention and help are called for, if not as a particular aid to the students themselves.

3.2. Language. The language in which a word problem is necessarily couched can take on a variety of forms. Sometimes the problem is phrased such that someone with no knowledge of mathematics can understand it and sometimes a certain degree of familiarity with mathematical terms is needed. Pimm (1987) defines three important terms:

- Dialect: variant of a language according to the user
- Register: a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings (citing Halliday, 1975a)
- Semantic fields: a group of words closely related in meaning, often subsumed under a general term (citing Lehrer, 1974).

When one refers to a problem being phrased in "mathematical" as opposed to "normal" English one is referring to the use of a mathematical register. It is unfortunately not within the scope of this paper to investigate this to any depth.

3.3. Contexts. Many word problems are set in a context that involves real-life objects or events. This is not a necessary condition for a problem to be a word problem, yet it is ubiquitous. There is argument as to whether this is advantageous or disadvantageous.

The argument for word problems is that any situation requiring mathematical analysis in real life is in essence a word problem. Thus, by learning in school how to deal with word problems, one is learning valuable modelling and analysis tools that can be applied in engineering and business and, in fact, in ordinary things like grocery shopping.

The argument against word problems is that, in order for them to be useful, transfer has to occur (and it is doubtful whether it does: Carraher, 1985; Noss & Hoyles, 1996), and the problems students learn to deal with are fundamentally different from real problems in that they are puzzles not ill-structured problems (see sections 4 & 5 below).

4. Problem Solving.

4.1. Transfer between situations. It is impossible to teach students every single problem that they might ever encounter. Transferring a mathematical technique from one situation to another is an important skill and one that does not always come to students automatically. For instance Carraher *et al.* (1985) interviewed street children selling fruit in Brazil and found a distinct lack of transfer. Those who had had some schooling used traditional arithmetic techniques to add numbers on paper and yet used entirely different techniques to add verbally in working out prices.

4.2. Transfer across the school/real divide. For word problems to be useful in teaching people to deal with real world situations, it is important that transfer across the school/real divide be successful. Amongst others Noss and Hoyles (1996) express strong doubts on the possibility of transfer of school mathematics to real world situations, while studying banking practices.

Pozzi *et al.* (1998) looked at mathematical practices in nursing. They discover that formulae are memorized and are often not understood without reference to a particular drug.

The work practiced by a professional mathematician and the work practised in a traditional classroom differ enormously. Schoenfeld (1987:214) holds that "as long as the two cultures differ as radically as they do at present" transfer is unlikely to succeed. Students need to understand what it is to live in a culture of mathematics in order to transfer techniques. We need to "create classroom cultures in which students do mathematics naturally. When that happens, the 'transfer problem' will no longer be a problem."

4.3. Expert/Novice distinctions. There are distinct differences between the ways experts and novices solve problems. Three views of these differences are discussed below, namely level of representation, chunking and automation.

Chi *et al.* (1981) cite McDermott and Larkin (1978) as proposing the following taxonomy of representation:

- the literal representation of the problem statement
- (naïve) the literal objects and their spatial relationships; often including a diagram
- (scientific) idealised objects and appropriate physical concepts
- algebraic representation; including equations.

Chi *et al.* suggest that novices use the naïve representation to categorise problems whereas experts use the scientific representation. They conclude this after an in-depth study of how experts and novices categorize physics word problems.

Larkin *et al.* (1980:208) discuss the "chunking" of memory. Only a few concepts can be held in short term memory at one time ("four to six items, or chunks"). To an expert, however, entire sets of problem solving steps may be regarded as a single "chunk" due to familiarity; a novice would perhaps remember these as separate chunks, thereby leaving less short term memory for other concepts.

Silver (1987) discusses the automation of sets of actions. Someone who has carried out the same set of actions many times automates them and, as a result, they take up less time in short term (working) memory. Von Glaserfeld (1987) refers to the same idea when he considers athletes and how they need to watch themselves on video to realize how many of their actions are automatic.

5. Cognition. There is a vast body of literature on the subject of cognition, and, in fact, on the part that cognition plays in mathematics education. This paper, perforce, touches only briefly on it.

Kitchener (1983), citing Churchman (1971), distinguishes between puzzles and ill-structured problems. Puzzles have a single true answer, have at least one correct path to that solution, and will be solved if that path is chosen. Ill-structured problems, however, do not necessarily have any solution, might have several contradictory "right" solutions, or might not have enough information contained in them to allow for a solution. In real life problems that are encountered are more likely to be ill structured, whereas problems encountered in a school or university mathematics course are more likely to be puzzles. Kitchener postulates three levels of cognitive processing embodied in problem solving. These three levels are:

- Cognition - computing, reading, memorizing, perceiving
- Metacognition - knowledge about cognitive tasks, knowledge about particular strategies to solve the task, how and when the strategy should be applied, success or failure possibilities
- Epistemic Cognition - "the processes an individual invokes to monitor the epistemic nature of problems which demand specifiable solutions" (Craig & Winter, 1990). This level of cognition includes the knowledge about necessary and sufficient information for a solution as well as the ability to choose between different possible strategies.

Kitchener suggests that epistemic cognition does not apply to puzzle solving. When one is confronted with a puzzle in a mathematics course one is assured of there being a solution. Also, it is accepted that there is only one solution at which everyone who chooses a correct way of tackling the puzzle will arrive. In the case of ill-structured problems, there is the possibility of there being no solution, or of there being several different solutions depending on one's initial parameters or priorities. Epistemic cognition is what is being practiced when considering the solvability of a problem and thus plays a large role in problems encountered in real life, and has very little to do with problems presented to one in a mathematics course.

Schoenfeld (1987:190 -191) stresses the importance of metacognition for successful problem solving. One has to be able to monitor one's progress on a problem in terms of usefulness and probability of success. "Aspects of management include (a) making sure that you understand what a problem is all about before you hastily attempt a solution; (b) planning; (c) monitoring, or keeping track of how well things are going during a solution; and (d) allocating resources, or deciding what to do, and for how long, as you work on the problem."

6. Research question and design. The aim of this study is to isolate factors associated with word problems that affect the difficulty level as perceived by students. The data collection instruments will be questionnaires and interviews, with the questionnaires being the primary instrument. At the time of writing, only the data from the pilot study was available for analysis. The final analysis will involve a population of approximately 300 students and 15 lecturers and will involve a questionnaire as well as interviews. The pilot study involved 38 students and no interviews, only a questionnaire.

6.1. The Questionnaire. A full set of fourteen problems was drawn up according to the criteria listed in Tables 1 and 2 below. Not all of those could be used, however, as the questionnaire would be too long, therefore five problems having the characteristics described in Table 3 were chosen for the questionnaire. The students were required to arrange these problems in order of difficulty, ranging from 1 (easy) to 5 (difficult).

Table 1: Algorithmic problems

Option #	Context	Dimensions	Visual representation
1	Concrete	Two	No
2	Concrete	Two	Yes
3	Concrete	Three	No
4	Concrete	Three	Yes
5	Abstract	Two	No
6	Abstract	Two	Yes
7	Abstract	Three	No
8	Abstract	Three	Yes

Table 2: Interpretative problems

Option #	Type of calculus	Visual representation
9	Differentiation	No
10	Differentiation	Yes
11	Integration	No
12	Integration	Yes
13	Differentiation and Integration	No
14	Differentiation and Integration	Yes

Table 3: Characteristics of word problems in questionnaire (the terms are defined in section 2)

Problem #	Contextual Shell	Cognitive requirement	Visual representation included?
A	Concrete	Algorithmic	No
B	Concrete	Algorithmic	Yes
C	Abstract	Algorithmic	Yes
D	Abstract	Algorithmic	No
E	Concrete	Algorithmic	Yes

The five chosen problems are described briefly below.

A: A cylindrical container of known volume is to constructed. The dimensions of the can that minimise surface area are to be calculated. No diagram was included.

B: A container of known volume formed from a cylinder with hemispheres on either end is to be constructed. Given the costs of the various pieces, the dimensions of the container that minimise cost are to be calculated. A diagram is included.

C: A cylinder is to be constructed within a circular cone of known dimensions, where the top of the cylinder touches the surface of the cone and they have a common axis of symmetry. The dimensions of the cylinder that maximise the volume of the cylinder are to be calculated. A diagram is included.

D: A sphere is placed inside a cylindrical container of known base radius. Water is poured into the container until the sphere is just covered. The radius of the sphere that maximises the volume of water present is to be calculated.

E: A container of known volume in the shape of a rectangular prism is to be constructed. Given the costs of the various pieces, the dimensions of the container that minimise cost are to be calculated. A diagram is included.

It was decided to omit interpretative problems (see definitions in section 2) for reason of brevity. Although problems B and E appear to have the same characteristics, problem B involved circles and spheres, whereas problem E involved only rectangles and rectangular prisms.

6.2. Results. The numbers in the tables refer to numbers of students giving that response.

Total Population (N = 38)

	1	2	3	4	5
A	19	14	5	0	0
B	3	3	15	7	10
C	1	3	8	23	3
D	0	3	6	5	24
E	15	15	4	3	1

Language:

1st Language English (N = 20)

	1	2	3	4	5
A	10	10	0	0	0
B	0	2	10	3	5
C	0	1	5	14	0
D	0	1	3	2	14
E	10	6	2	1	1

1st Language not English (N=18)

	1	2	3	4	5
A	9	4	5	0	0
B	3	1	5	4	5
C	1	2	3	9	3
D	0	2	3	3	10
E	5	9	2	2	0

Degree:

BCom (N = 21)

	1	2	3	4	5
A	13	4	4	0	0
B	3	3	5	5	5
C	0	2	6	10	3
D	0	3	3	3	12
E	5	9	3	3	1

BSc (N = 16)

	1	2	3	4	5
A	5	10	1	0	0
B	0	0	9	2	5
C	1	1	2	12	0
D	0	0	3	2	11
E	10	5	1	0	0

6.3. Brief Analysis. The small numbers involved make a detailed statistical analysis unwise, so only large scale structures are commented on here. The final data will be analysed much more finely.

- Total Population: Problem A was considered the easiest. This is not surprising since the problem was a very familiar one. In the final survey the cost of the container will be the quantity to be minimised to bring it in line with problems B and E.
- An interesting point is the difference between problems B and E. The information given in each problem was similar (the volume is given, the diagram is shown, the quantity to be optimised is given). The only fundamental difference between the two problems is that B involves spheres and circles. Note that problem B is considered substantially more difficult than E.

- Problem D, despite involving the shortest calculation and the fewest variables (if you actually carry out the problem) was considered the most difficult. The most obvious aspects of the problem that suggest reasons are the lack of a diagram and the unusual nature of the problem (it is not common at all). This ties in with the work done on expert/novice distinctions i.e. memory chunking and automation.
- First Language: The same problems are regarded as difficult and easy by the different language groups, but the numerical spread is different. I suggest there are two levels of engagement within the problems i.e. verbal and mathematical. If the required level of verbal interaction is sufficient that it masks the difficulty of the mathematics, then the ratio between difficulties of problems will be small, since they all require a similar level of verbal engagement. First language English speakers will experience a very low level of verbal difficulty and thus the ratio of difficulty between two problems will be influenced almost entirely by the mathematical requirements of the problems.
- Course of Study: In the data for the total population problems A and E were regarded as the easiest questions, with almost the same weighting. When the data is split according to course of study it can be seen that the Science students, in general, find E a lot easier than A, and the Commerce students the exact opposite. There is no clear reason for this. If it occurs again in the main data collection exercise the reasons will be investigated in the interviews.
- The concrete/abstract nature of the problems did not seem to have much effect on the perceived difficulty level.

7. Conclusion. Word problems can be differentiated by many things e.g. whether a mathematical register is used or not, whether a visual representation is included or not, whether the context is concrete or abstract, etc. This brief analysis of the pilot data (a more in-depth analysis of a greater pool of data will take place in June 2001) would suggest that the primary influences on the perceived difficulty level of word problems are lack/presence of visual representations, and familiarity. This ties in with the concepts of translation between representations and automation of thought/action.

Unfortunately automation has little to do with the teacher and more to do with how much the student practises, but translation between representations can be taught if the teacher is aware that it is an area of difficulty for many students and therefore needs to be covered slowly and in detail.

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ATTITUDES OF ENGINEERING STUDENTS TOWARDS THE FLEXIBLE USE OF GRAPHING CALCULATORS IN A MATHEMATICS FOUNDATION PROGRAMME

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ABSTRACT. While literature abounds with enthusiastic reports and ambitious claims about active student involvement and qualitative learning in graphing calculator environments, this is not always the case. This paper explores the attitudes of a group of "educationally disadvantaged" students in an engineering foundation programme with regard to the use of graphing calculators in a mathematics support class. All thirteen students in the group were exposed to a set of TI-83 graphing calculators to which they had full access for two semesters during the year 2000 foundation programme.

Apart from frequent classroom demonstrations and easy accessibility to motivate the students to use the calculators, the programme and the format of its materials remained intact. Contrary to my expectations, the students were not very enthusiastic about the new technology. They showed very little interest in using the graphing calculators, preferring to use traditional pen-and-paper methods instead. Only two of the stronger students loaned calculators to use at home. The rest only used it in class under direct instruction.

Interviews were conducted with students at various stages during the programme to establish the reasons for their apparent apathy with regard to the available graphing technology. The reasons included: the fact that no calculators (including ordinary scientific ones) are allowed in the examinations; the fear of becoming too dependent on the calculator (forfeiting "the basics"); the accountability that goes with the loan of a calculator, and, the extra burden of having to learn how to operate the device. The interviews allow us to trace the students' apathy with regard to the use of graphing calculators to a primitive epistemology of mathematics.

1. Introduction. Over the past ten years, the mathematics education literature has shown a proliferation in enthusiastic reports and ambitious claims about the learning that takes place in graphing calculator environments (e.g., Borenson, 1990; Drijvers and Doorman, 1996; Gordon Hughes-Hallet, 1994; La Torre, 1994; Lauten, Graham, and Ferrini-Mundy, 1994; Demana and Waits, 1994; Sfard and Leron, 1996; Porzio, 1997; Schwartz, 1999; Torres-Skoumal, n.d./1999; Kutzler, 1999, June 13; Podlesni, 1999; Waits, 1999). Highly motivated by the overwhelming enthusiasm in the literature, I decided to start the millennium by introducing a set of graphing calculators from Texas Instruments (TI-83) into the foundation programme of the University of Stellenbosch. In doing so, I hoped to confirm some of the positive reports in the literature; and, to add a bit of technology to boost the programme. Much to my surprise, my classroom experiences did not confirm the positive reports in the literature. I needed to find out why.

First, a bit of background on the foundation programme: The *raison de 'être* of the programme is to render academic support to university students who are "educationally disadvantaged" by virtue of their inadequate schooling backgrounds. The programme is faced with a number of educational challenges. Two of the most obvious ones are: to bridge the gaps that are the result of the students' inadequate schooling backgrounds; and, to help the students at the same time to cope with the academic demands of a full mainstream course in mathematics. Experience has taught us that transmission-style lecturing is not the best approach to use under the given circumstances. We try to keep our approach flexible, i.e. to adapt our teaching style to the needs of the didactic situation; to experiment with new approaches; and, to stay student-centered. Students in the foundation programme must enroll for a full mainstream course in mathematics and in addition they have to attend non-accredited support classes in the subject.

It is in this spirit of flexibility and student-centredness that students were given the opportunity to decide whether or not they wanted to use the graphing calculators, and how they would prefer to use it, if they did. The calculators were freely accessible during the support classes and the students were frequently encouraged to use them, but never forced to do so. The format of the tutorials and assignments, for example, were not adapted to the graphing calculators because that would force the students to use it.

2. Methodology. This flexible, student-centered approach became our methodology with the graphing calculators, with the exception that the programme and the format of its materials were not adapted to the technology. The calculators were always freely accessible and the students had the opportunity to use them in the support classes, and, even to loan one for a full semester to use at home. In order to loan a calculator, a student had to sign a form that would keep him or her accountable for the safekeeping of the instrument.

It is important to remember that the students came from "educationally disadvantaged" backgrounds where access to even the most basic equipment is sometimes a problem. As was expected, none of them had any prior experience of graphing calculators, necessitating regular demonstrations of the keystrokes on the overhead unit and also to individuals. Meticulous attention was paid to the important issues of screen size and scale, which are notorious pitfalls that are part and parcel of graphing technology (cf.: Borenson, 1990; Dion, 1990; Guin and Trouche, 1999). The two students who loaned graphing calculators were given copies of the manual to assist them with the operation of the device.

The students were consistently encouraged to use the graphing calculators to solve their homework and tutorial problems; especially the useful graphing and numerical utilities (Porzio, 1997); and also to reduce the tedious calculations of the traditional pen-and-paper approach, which leaves more time for conceptual investigations (Borenson, 1990; Podlesni, 1999; Waits, 1999). Every available opportunity was used to demonstrate the usefulness of the graphics calculator, especially to explore typical tutorial problems or to check a solution. In doing so, I hoped that some of my enthusiasm would rub off on the students and that they would start to explore on their own. According to Torres-Skoumal (n.d./1999) intervention of this kind is necessary, because not all students are equally inclined towards the use of technology. This was confirmed in my own experience, but the interventions produced only limited success.

It has already been pointed out that the format of the worksheets and assignments was not adapted in any way to the graphing calculators. There are two reasons for this: Firstly, the graphing calculator is a cognitive tool, designed to assist with the job of solving mathematical problems. I believe that the job should not be adapted to the tool, but vice versa. This is important when learning when to use a tool, and when not to use it, is as important as learning how to use it (Kissane, 1996, March 1). Secondly, students are not allowed to use any calculators in the mainstream examinations. This presented a moral issue. Why use precious time to teach the students how to use a tool that they will not be allowed to use when they need it most? The easy way out was to let them decide for themselves.

With the exception of two very enthusiastic individuals, who also happened to be the top achievers, the general attitudes of students covered a "broad spectrum of willingness to use the calculator" (Torres-Skoumal, n.d./1999) that ranged from conservative skepticism to outright resistance. I decided to focus the study on the underlying reasons for the attitudes of the apathetic majority.

Clinical interviews were conducted with all thirteen engineering students in the group and it provided a rich source of research data. The core interview questions dealt with: the non-allowance of any calculators in examinations; perceptions about the possible influence of graphics calculators on a student's mathematical proficiency; why a student did or did not loan a calculator to use for self-study; how students preferred to use a graphics calculator; how students dealt with the novelty of the graphics calculator, etc. All the interviews were audio taped with the consent of the students. Field notes were also kept as the programme went along.

3. Results. Graphing calculators are still relatively unknown in South Africa, especially in the schools that produce "educationally disadvantaged" students. Only two of the thirteen students in the study had seen a graphing calculator before: one of whom had seen it in a catalogue. Before their encounter with the TI-83s in the support class, the rest of the students did not know that such hand-held graphing technology existed. Many were intimidated by the novelty of the device and its physical appearance. With its unfamiliar keys and impressive screen it looked difficult to operate. It was also difficult for the students to grasp how the effort that it would take to learn to operate a tool would be rewarded if it could not be used in the tests and examinations. They regarded it as an extra nuisance (Kissane, 1996, March 1) that would just add to the burden of the already crowded curriculum.

The non-allowance of calculators in tests and examinations was a major obstacle. Although the students all agreed that they would jump at an opportunity to learn how to operate a graphics calculator if they were allowed to use it in tests or examinations, not all were convinced that it would be such a good idea to do that.

Fears were expressed about themselves becoming too dependent on the calculator (too lazy to think); about "button pushing" leading to a loss of "the basics"; and, about examinations becoming "too easy" and hence worthless. Many felt that they do not need calculators anyway, because, by agreement with the lecturers, the test and examination problems are designed in such a way that the arithmetic can be solved without a calculator. All of these fears and comments were explored during the interviews; uncovering the rather primitive epistemology that mathematics is nothing more than arithmetic.

The fear that they would become dependent on the graphics calculator was one of the main reasons why so many students decided that they would be better of without it. In the following example, a student explains that it is better to minimize the use of a calculator in order to avoid becoming too dependent on the calculator and not being able to remember the calculations in the examinations.

I) Did the fact that you were not allowed to use a graphics calculator in the examinations influence your decision to take it home or not?

W) If one studies for a subject, then one tries to use the calculator as little as possible in order for one not to get too dependent on the calculator.... That is how I felt ... I don't even want to use my ordinary pocket calculator too much ... get too dependent on it ... One does not want to find oneself in the examination room and then one feels like working something out (on the calculator) and so on ... One wants to remember the stuff.

Students and lecturers use this argument frequently to explain why calculators should not be allowed in examinations, viz., that students will become too dependent on the calculator and lose the ability to think for themselves. The underlying assumption is that the calculator is able to "think" for the student, but what does that mean? I will suffice by asserting that calculators can only perform algorithmic procedures and that that can hardly be regarded as mathematical thinking!

Proficiency with pen-and-paper algorithms is often highly regarded by teachers, tested for by examiners, and well rewarded as a clear indication of the mastery of "the basics" of mathematical thinking. This stems from the primitive epistemological position that mathematics is nothing more than arithmetic. A calculator can perform the equivalent of a pen-and-paper algorithm with a higher degree of efficiency than a human being, but does that mean that calculators can think? Does it mean that the calculator has mastered "the basics" of mathematical thinking? My students seemed to think so, especially the weaker ones. That is why they felt that the calculator might deprive them of "the basics" by taking it out off their hands.

In the following example, a student expressed fear that the calculator might take too much of the "mathematics" out of a problem, leaving only the theory to be dealt with. He explained that the mathematics part is actually the arithmetic algorithms, while the theory part is the verbal, non-mathematical part.

H) If one is given a graphics calculator one will just type everything in... it will become too easy; it will not work ... If one has such a "powerful" calculator it will take so much of the mathematics part out of it, one will just have to apply the theory.

I) What do you mean by the "mathematics part"?

H) So much of the calculations. It will do all the calculations for you. You will give it the sum and it will you the answer. You will not necessarily know which method it used, or so.

I) What do you mean by the "theory" part?

H) The rules and so... It will tell you the turning points will be here and here, then you will have to say how many turning points there will be, and so... Whatever... It will be a maximum... Then you check on the graph to see what it looks like ...

I) So what do you mean by "theory"?

H) More the language ... Maybe I mustn't say the language... I don't know... The non-sum part of the thing... Many times they ask you: How do you do this sum, do this, and this, and that... and you have to infer stuff from your answers... and so

I) So which part is the "theory"?

H) The inferences that you make and so... Yes.

I) There are many people who argue that... that is the actual mathematics... that addition, multiplication and division is not really mathematics...

H) No, I think that is actually the mathematics... The calculations, for me, is the mathematics part□

I) And the other part?

H) That is more the applications and inferences that you make...

I) So that is not mathematics, or what?

H) That is also mathematics, but for me that is not so mathematical... It is more... I don't know... How shall I say ... It is more inferences that one makes, more... .

I) So the inferences are not mathematics?

H) It might be mathematics, but it is not mathematical (with emphasis), mathematical calculations as such.

This example clearly shows how lower-order thinking skills, such as the use of arithmetic algorithms, are regarded by the student as more important than the higher-order thinking skills, such as inductive and deductive reasoning. For the student (*H*) getting the arithmetic right is more important than drawing the right conclusions. According to the student (*H*) the higher-order reasoning should not even be regarded as real mathematics. This explains why weaker students often feel that they will be disadvantaged by the use of graphics calculators. They fear that it will deprive them of the opportunity to drill-and-practice their pen-and-paper arithmetic algorithms until they know them by heart. This perspective is often inherited from school teachers, many of whom believe the same.

Another concern is that "exams will become too easy". Kissane (1996, March 1) pointed out that a strong tradition of doing things "the right way" often leads the mathematics community to regard new ways of doing things as illicit, for example the notion of the calculator being a cheating device. Of course the examinations will become too easy if the focus remains on assessing routine algorithmic procedures that the graphics calculator had been programmed to perform. We will need to revisit our existing assessment devices to address that problem.

One of the reasons why the students were so reluctant to loan a graphics calculator had little to do with their epistemological position about mathematics. Many were afraid that something could happen to the calculator, in the event of which they would be liable to replace it. They had to sign a form to that effect before they could loan a calculator. I think the form was necessary, and they agreed with me on that, but it also reinforced their fears. The calculators are expensive and many students felt that the educational benefits of using it in their own time did not balance the financial risk of loosing it. The few students who used the calculators did so only in the classroom, but refused to loan it for use outside of the classroom. Even the two students who had loaned graphics calculators never carried it with them to campus, also for fear of losing it. On campus they used the calculators that were available in the classroom, preferring to keep the loaned ones safely at home.

4. Conclusions. Although this study did not confirm the positive reports in the literature about the teaching and learning of mathematics in graphing calculator environments, it does not provide conclusive evidence that it can never work. We have to look at the study against the background of the current situation in South Africa with regard to the availability and use of graphing technology in mathematics classrooms, the prevailing epistemological notions of mathematics, and the reluctance of departments to commit themselves towards the integration of the technology into their curricula. We are still trailing far behind the rest of the developed world where most of the available research was done. The results might have been different if the students had prior experience with graphing calculators; if the calculators were allowed in examinations; and if the programme was adapted to the use of calculators; and if the students had a better epistemological notion of mathematics. But these are topics for further research. This paper has just scratched the surface of some of the problems that might be encountered with a too flexible, unstructured approach to the introduction of graphing calculators into a mathematics foundation programme. Finally, I agree with James Kaput (1992) that the real question is not whether computational aids do better teaching job than static media, but rather what the optimal circumstances for each are.

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"GATE KEEPER" VS. "GATEWAY"

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ABSTRACT. The metaphor of mathematics as the "Gate Keeper" to university study and to various popular programmes is one that should be contested: mathematics should be an enabler, not a barrier. In the aptly named "Gateway" programme at the University of Cape Town, an attempt is being made to use mathematics in this way. As part of this programme our "Effective Numeracy" course serves both as a quantitative literacy course and a bridging course to introductory calculus. Students who enter this programme come from very diverse academic and cultural backgrounds. Our approach is that students who have had either bad experiences or insufficient mathematical exposure during their high school careers, should be engaging with mathematics in contexts that they find interesting, relevant and challenging. The integration of the core courses (Economics, Philosophy and Mathematics) should be well articulated and coherent. Students should be engaged (in small groups), with mathematics applied to social and economics contexts, allowing them to construct their own mathematical knowledge. The use of a problem-solving approach allows students to appreciate the utility of mathematics and creates opportunities for them to articulate their mathematical understanding verbally. Interactive Excel-based computer tutorials are used to support the learning of course content (and computer literacy) in the laboratory component, which is one third of the course. In this presentation we will describe our approach, show examples of our material and give quantitative and qualitative analyses of our successes.

1. Introduction. Achievement in mathematics is often made a prerequisite for entering certain courses of study, and as such mathematics often acts as a kind of "gate keeper" which denies people access to a large range of disciplines. Even when competence in advanced mathematics is not really essential to a discipline, it is still sometimes used as an indicator of the likelihood of success in a demanding course of study. There are many students who have the potential to succeed in a range of courses, who have not had the opportunities at school which would prepare them to meet these challenging entrance criteria. In the "Effective Numeracy" course at U.C.T., we aim to make studying mathematics for them an enabling experience that provides students with the tools and abilities that they really need to succeed in their course of study.

Students on a four-year extended curriculum in the Humanities faculty enter the "Gateway programme" in their first year, which allows them to continue a variety of economics-related programmes of study. The Mathematics course, Effective Numeracy, is one of the core courses in this year, the others being Economics (Microeconomics) and Philosophy (Quantitative reasoning). These are all designed to provide extra academic support and to prepare students to enter the normal academic stream in their second and subsequent years.

In order to be admitted to the Effective Numeracy course, students do not have to have achieved any minimum standard for mathematics in Matric, in fact a minority of students are admitted who have not studied mathematics at school since grade 9, usually because the school did not offer it (although, of course, they have all achieved sufficiently good marks overall to meet the University's entrance requirements). In some cases, students are placed in the Gateway programme not because of a poor mathematics background, but for other reasons, such as language deficiencies. This means that in the Effective Numeracy course the students have an enormous range of abilities and prior mathematical experience, which presents a very significant challenge in the classroom and for the curriculum design process. A positive aspect of this diversity is that the class contains a very representative cross-section of the student population in terms of race, language and place of origin.

2. Objectives and course design. The course has two main objectives, which are sometimes difficult to integrate with each other. The main goal is to provide the students with the confidence, skills and experience to make them "quantitatively literate" at an appropriate level for their course of study and their careers. This part of the course is based on the Effective Numeracy course that is discussed in Brink (1999). Then they must be prepared to enter an introductory calculus course in their second year, so the Effective Numeracy course must also act as a bridging (Precalculus) course. In addition, we feel that the ability to use computers appropriately (spreadsheets in particular) is an essential part of a student's quantitative skills, and so there is a heavy emphasis on computer competence in our course as well.

During the two and a half years that we have been developing and offering this course, we have been faithful to certain guiding principles, which have been adopted in response to the particular needs of our (diverse) group of students:

- It is essential to build the students' confidence and provide them with an unthreatening and supportive environment, and opportunities to succeed. Mathematics anxiety is prevalent amongst these (less successful) students and must be actively addressed. Interviews with some of our students confirmed that many of them were apprehensive about having to do a mathematics course when they entered university. Thus one of our main objectives is to change students' attitudes and perceptions about mathematics.
- The subject matter of the course is context-based. We aim to present the students with real-life contexts, from which the need for the mathematics concepts and numerical skills should arise naturally. Many of our students have remarked that they realised for the first time during our course, how mathematics is needed everywhere in their studies and in daily life. Previously, they saw mathematics as an isolated and irrelevant series of procedures that had to be learned in a rote fashion.
- Wherever possible, we have used materials of local interest, for example from Cape Town newspapers, and have attempted to focus on contexts that have relevance to the students' lives, such as borrowing money, microlending, the AIDS pandemic, and environmental issues. In 2000 we experimented with using a text book on which to base the quantitative literacy component of the course. Although the book was quite appropriate in terms of mathematical content, the fact that it was published in England, detracted from the sense we are trying to foster that quantitative literacy is vitally relevant to the student's everyday life. This year we are not using a textbook and have invested considerable time and energy into writing our own materials with local content. To maintain the sense of relevance, these materials will have to be constantly revised and updated.
- Computer literacy (at a basic level) is an essential part of quantitative literacy for students in these programmes of study, and is therefore an important part of our curriculum, as many of these students do not have these skills.
- Computers should be used to support the learning of mathematics concepts wherever appropriate. There are unique ways in which computers can be used to enhance the learning of the mathematics content of the course, and these are exploited where possible. Thus the computer component of our course has two aspects: computer-assisted-learning (of mathematical concepts) as well as developing computer literacy in itself.
- Co-operative learning is encouraged. Students work together and assist each other as much as possible in the classroom (and laboratory), and outside it.
- Assessment is continuous throughout the year, with evaluation sessions and assignments every three weeks, contributing towards a class record that comprises one half of the final result.
- Ongoing formative evaluation of the course is important for the process of continuous revision of the curriculum. This involves regular student evaluation questionnaires, interviews and tracking of our students as they progress through their programme of study. During 2000 we were also evaluated externally by representatives of the South African Institute for Distance Education (SAIDE), who reported positively on the course and on the laboratory component in particular.

Our greatest challenge is to make all aspects of the course flexible enough to meet the diverse needs of a class of students who range in ability from those who are functionally innumerate to those who have passed Matric mathematics on the Higher Grade. The Effective Numeracy class consists of 100 to 120 students divided into 3 groups, which meet separately for 6 periods every week all year, 2 of which take place in the computer laboratory.

The classroom sessions are run as "workshops" with limited presentation of course content at the blackboard. Students sit around tables in groups of 5 or 6 and engage with the course materials (provided as printed "worksheets"), while lecturers and tutors act as facilitators. Each class has one lecturer and 2 tutors. In this way, more capable students often help the weaker ones, and at the same time improve their own understanding by having to clarify their understanding sufficiently to make it comprehensible to other students. The high teacher to student ratio ensures that most students receive a reasonable amount of individual attention (or at least access to the teacher as part of a small group) during the classroom sessions. In addition, the students are encouraged to seek extra help individually or in groups at any time, which places great demands on the lecturers and tutors' time, but often relieves the frustration of both student and teacher that limited classroom exposure can generate.

The printed materials are also designed to provide a range of experiences, from very basic background material for the less-experienced students to quite challenging extension problems for the more advanced.

In the laboratories, each student works at his/her own computer, but the students are encouraged to assist one another (except during tests!) The laboratory tutorials are interactive Excel-based computer-assisted-learning materials; the student works on an Excel workbook that contains the content that supports the learning of selected mathematical concepts, as well as examples that exercise their understanding and their computer literacy skills. The advantage (in terms of a flexible response to different student needs) of presenting some of the course materials in computer tutorials is that learning from the computer materials can be self-paced, and students can repeat the lessons as often as they choose. Feedback to the students' answers to exercises is immediate so that they can judge for themselves the level of their progress. Students work through this material at very different speeds, and it has been necessary to specify the minimum material that must be completed in a session, as well as to provide plenty of practice examples to challenge the more capable students.

At the beginning of the year, there are always several totally inexperienced students who need a great deal of support with learning how to use a computer, but it is gratifying how rapidly most of them achieve the necessary level of competence. However, it is necessary, particularly for these students, to ensure that the environment in the laboratories is unthreatening and as free as possible of technical problems, which can be devastatingly discouraging to students who are already at a great disadvantage.

Assignments and projects (where students are required to express themselves in English on Numeracy topics) offer another area where it is possible to be flexible about accommodating students' differing levels of ability. Where possible, we offer the students the choice of doing a well-defined circumscribed task, or taking it further in an open-ended way.

3. Evaluation. To gauge the successfulness of the teaching of numeracy skills within real-life contexts, in 2000 we included in the mid-year class test repetitions of the most challenging examples from our Numeracy Competency Questionnaire, which we administer to all Humanities students entering first year at U.C.T. In this way we get pre- and post-test results for our students on exactly the kind of contextual problems that we are emphasising in the Effective Numeracy course. All the students on this course improved their results for the questions chosen, with the mean result increasing from 42% to 66% (a 57% increase). Linear regression analysis of the pre- and post-test results yielded $y = 42.92 + 0.55x$ (where x is the pre- and y is the post test result) with a correlation coefficient of 0.67.

In order to judge how successfully we are adapting our curriculum to the needs of the particular students in the course, we engage in a comprehensive course evaluation at the end of each semester. Students complete questionnaires (anonymously) that canvass their views about the relevance, pace, standard and effectiveness of the course generally, about their experiences in the classroom and laboratories, about the level of organisation, preparedness and approachability of their lecturers, and about their opinions of various aspects of the design and content of the computer tutorials in particular.

On the whole, these reveal that the course is well received, and provide us with information about how we should modify the content, pace or other aspects of the delivery. It is particularly useful to have regular information about how the students are responding to the computer tutorials, and to canvass their opinions about the details of tutorial design. One interesting observation is that students appear to find that the computer tutorials support their understanding of the course content better if they experience new concepts in the classroom, before they are introduced to them in the laboratory. They do not appear to transfer the knowledge they have gained about new concepts as easily from the laboratory to the classroom.

At the end of 2000 we conducted individual interviews with students selected at random, (with the assistance of independent evaluators SAIDE). As one of our principles is that the Effective Numeracy course should address students' confidence and attitudes towards mathematics, in these interviews we focussed more specifically on how students' attitudes towards the usefulness and relevance of mathematics (and computers) had been changed as a result of the course.

At least a third of the students interviewed had negative expectations of doing a mathematics course at university before they started the Effective Numeracy course, for example:

"I thought it would be tough, because I last did maths in Std 8 and people in school used to say that matric maths is difficult, so I thought, 'It's going to be a nightmare'. It wasn't like that...."

In fact about 80% reported a positive experience of studying mathematics on this course, for reasons like the following:

"Increased emphasis on understanding", "solution of real-world problems", "learning for self", "can be mastered with hard work and practice", "slower pace helps" and "maintained confidence".

"This course has helped, because there is not so much learning how to do and to calculate, but solving word problems and putting what you learnt into everyday problems and then working them out. Now I understand where the actual formulae etc. come from and how they are derived, whereas at school they just teach how to do things and get it right. This course has taught the origin of where things come from and why you're doing what you're doing"

In addition, about 60% of those interviewed said that they had applied what they learned in the Effective Numeracy course in their other courses of study:

"I didn't realise how this course has been designed so well to work synergistically: It's timed so well that as you start doing something in economics that requires a certain maths skill, we will have just learned that skill in maths."

Nearly 85% reported a positive change of attitude towards mathematics (and using computers):

"I was never 'into' maths before matric. In matric I just went through old exam papers (the teachers told us that there were only so many ways of asking the questions and only so many questions to ask, so if you did enough exam papers you would be able to do everything). Now I see that you actually need to understand what you are doing, it's been an adjustment. I never understood before where maths fitted in. Now we get word problems and we have to sift through them to get an equation or whatever. Now we actually have to understand the problem (especially in the homework assignments). I now see maths not just as a classroom subject, but something you use irrespective of what you do."

"Before, I was very anxious about using computers. Now my feelings have changed, now I know how to use a computer and I can do things on my own."

"Now I have no particular feeling about computers. I am not anxious. I know I can use it as a tool (I now have one at home). Before, I found the idea of using computers overwhelming. I didn't know what to do and I was sure I would lose all my work (as happened to me once at school). I was convinced they would be my downfall."

The Effective Numeracy course is an integral part of the "Gateway" to economics-linked programmes, and, as such, is intended to provide each student with an appropriate foundation of skills and knowledge necessary to succeed in their chosen programme of study. To collect information about the effectiveness of the course in this context, it would be necessary to track the progress of all the students during the "Gateway" year and in their subsequent years of study. This is a substantial undertaking, and given the short period over which this course has been running, we can at present only report verbal feedback, which we have received from past students. This has in general been most encouraging, such as this statement made by a member of our 2000 class, who did not do mathematics for matric and earlier this year achieved 83% for his first class test in his calculus course this year:

"The work that we did in MAM107H was related to my other courses ...... The Excel and Word skills that I acquired in MAM107H I still use in my second year Economics and my Statistics. The graphing, organising and solving problem skills are also helpful. I still use the study methods (of group study and

working on my own) in my 2nd year .... my attitude towards varsity, Maths and computers has totally changed. I can engage confidently with my fellow students in arguments and debates, I can sit confidently near the computer and do my work using my skills. This is all thanks to MAM107H, which took me from a public school student to a confident, proud, bona-fide student of the University of Cape Town."

4. Conclusion. There are several strategies we use in the course "Effective Numeracy" for Humanities students at U.C.T., in order to cater for the needs of very diverse students, while successfully improving their quantitative literacy and providing a bridging course for further mathematics. Students in this class range from functionally innumerate to having passed Matric mathematics on the Higher Grade. Some of the strategies we have employed are: workshop lectures, co-operative learning, team teaching, interactive computer-assisted-learning tutorials (one third of the course) and regular writing assignments.

Ongoing formative evaluation reveals that the course is well received, perceived to be relevant and useful, and is successful in improving students' attitudes towards studying mathematics and using computers.

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THE MATHEMATICAL PREPARATION OF PRIMARY SCHOOL TEACHERS

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ABSTRACT. Following TIMSS, there is great concern in New Zealand about the level of mathematical ability of our school students. It is felt that part of the reason for this is the way that mathematics is presented in primary schools and hence there is a spotlight on the standard of the mathematics of primary school teachers.

In 2000, a new university mathematics course was devised for pre-service primary teachers at the University of Otago. This paper will discuss the content of that course as well as its method of presentation. Briefly, the content covered arithmetic in bases other than 10; divisibility by 2, 3, 4, 5, 6, 8, 9, 10, 11; some "simple" algebra; Pascal's triangle and some "simple" counting; APs, GPs and compound interest; Pythagoras' Theorem; some circle geometry; and tessellations. A problem solving approach was taken with new material being introduced around problems as much as possible. Generally every effort was made to involve the students in the "lecture" class. In addition to the two-hour lecture each week, there was also a three-hour "tutorial". In these tutorials the students worked in groups of four to solve a series of problems based on the week's lecture material. This paper will also discuss the way in which the course was revised when it was given in 2001 and the purpose of those changes.

1. History and Philosophy. In 1997 it was decided that our University would initiate a degree that would train primary teachers. As part of this degree, the trainees were required to undertake courses from different subject areas. One of these areas was mathematics. At that time, the Department of Mathematics and Statistics had a number of entry points for students wishing to study mathematics in some form but none of these courses appeared appropriate for primary teachers. The "easiest" course available contained simple linear mathematics (equation of a straight line; solving linear equations; and related topics) leading to the solution of simple linear programming problems (but not using the simplex method); a little on standard functions such as polynomials, exponentials, logarithms and trigonometric functions; and the beginnings of calculus. Its aim is to bring the students taking the course to the level of high school graduates. However, this course did not appear to be appropriate for primary teachers for at least two reasons. First, the material seemed inappropriate. It was largely far removed from anything that they would ever need to teach. Second, it started at too high a level. There is strong evidence to suggest that most primary trainees leave school with an inadequate grasp of Year 10 mathematics. This means that they are likely to be able to do arithmetic but have trouble with algebra and can only apply what they know in limited situations.

The fears for the "difficulty" of this "easiest" course were quickly confirmed. While some students were able to cope with it, the majority had withdrawn by the end of the first two weeks. Hence it was decided to introduce a new mathematics course that would be more valuable for the primary trainees. This course was devised with the following as its underlying philosophy. We wanted the students to:

- have a positive experience of mathematics while still being challenged by it;
- experience a broad view of mathematics;
- gain a deeper understanding of the material that they themselves would have to teach; and
- experience a pedagogical approach to mathematics that might provide a model for their own teaching.

There were a number of reasons why it was felt important to base the course around the above points. From what is known of students entering similar courses elsewhere in the country (and in other countries too), it was highly likely that students in this course would have a poor view of mathematics, would never have succeeded in the subject before and may even have "mathematics phobia". Consequently we wanted them to see that there was some mathematics that they could do and even enjoy. At the end of the day there was an approximately 80% pass rate, though as it turned out, non-primary trainees also took the class and only 2 of about 40 teacher trainees actually failed. In many cases this represented a great deal of effort on the students' part. Many of them had indeed got over a "hump". Hopefully many of the teacher trainees had a positive experience of the subject that was reinforced by their success and that this positive experience will be manifested in the way they work with their own students in future.

It was also felt that teachers needed to have a "feel" for the subject as a whole - that it was not sufficient that they just knew a lot about the area that they would have to teach. Generally what they knew about mathematics was solely mechanical skills. They knew how to do arithmetic. They had some knowledge of the basics of geometry ñ they knew something of shapes and angles and maps but little or nothing about proofs. They had met the basics of algebra but had no understanding of how and why it worked. Generally too they saw all of this in isolation. So it was felt important to try to show how various parts of mathematics were connected. And it was felt important that they saw something of the process skills of mathematics, particularly with regard to the way that problems are solved. As an extension of this, and to see what mathematics is trying to do, they were shown something of the how and why of research mathematics. Hopefully then a wider vision of mathematics would be reflected in their teaching.

It is clearly important that primary teachers should know the material that they have to teach quite thoroughly but it might seem that the material that is taught in primary school should not present any challenge to pre-service primary teachers. However, this is not the case on at least two levels. First, there are many challenging puzzles that require only the mechanical skills of school mathematics and yet are relatively quite difficult. Such problems can be used to reinforce mathematical process skills. Second, the understanding of mechanical skills can require deep thought. For these students simple subtractions such as 342 - 254 have significant mathematical content underlying them (see Ma, 1999).

But there was not only a concern about primary teachers' content knowledge. There was also concern about the pedagogy that they use. Again, at least anecdotally, there is evidence that teachers' delivery is didactic and uninteresting, that worksheets are in common use, and that rote learning is also common. This may well be the result of traditional tertiary teaching. The lecture approach and the more traditional texts emphasise a "chalk and talk" pedagogy. This view of teaching may have been followed by secondary teachers because they saw university lecturers use it and because they themselves had learned their mathematics well enough through this approach. It then passed down the line to primary teachers who had enough trouble with content without thinking up a new pedagogy to use in their classes. Perhaps the spiral could be broken by this course using approaches other than the traditional one. But perhaps the answer lies with the content too. Cooney (to appear) suggests that "The development of a broader and more process-oriented view of mathematics is more likely to occur when students study the mathematics they will be teaching but from a more sophisticated and broader perspective."

2. MATH 100, 2000. In this section we give an overview of the new course MATH 100 that was presented for the first time in 2000 to about 60 students. It is worth noting that a little over half of these were pre-service teachers. The other students were majoring in Arts or Commerce.

The course ran for the 13 weeks of the first semester. Each week in addition to the two 50-minute lectures each student attended a 3-hour tutorial/workshop. The lectures were held in consecutive hours in the same flat lecture theatre, so there was some flexibility in the length of time we worked. However, we always took a break roughly halfway through the approximately 2-hour session.

The original intention was to have interactive sessions that would be more like school classes than lectures. In these sessions students were encouraged to do work that contributed to the development of the topic and to make regular contributions. From time to time the lecturer would go round and talk to small groups of students. This scheme worked to some extent though the lecturer forgot from time to time and delivered more traditional lectures. Occasionally there was a good reason for this as the topic being discussed appeared to be too hard to manage interactively. The individual lecture topics are listed below.

Lecture 1: Tests for division by 10, 2, 5and 3.

For what values of a and b are the two subtractions below equal?

400	4ab
<u>-ab4</u>	<u>-400</u>

Change 4 to 5, 6 ... Are there any patterns? Change 400 to 4000, 40000, ... Are there any patterns?

[Here we were trying to get them back into thinking about number. We also wanted them to start thinking about patterns and how they might be found. The idea of proof was also pushed from this early stage. Proof is an important part of mathematics and they needed to appreciate this.]

Lecture 2: Other bases. Addition, subtraction, multiplication, division in base 8.
[This was a way to remind them why the standard algorithms (in base 10) work. It was meant to give them a better appreciation of base 10.]

Lecture 3: 61 -16 = 45; 85 - 58 = 27; etc. Pattern? Can you justify it?
Test for divisibility by 9: a proof for a 3-digit number.

Linear equations in one and two unknowns.
[Number and patterns were used as an introduction to algebra. The 61 - 16 problem and divisibility by 9 are two things where the proof follows the same line both in specific cases and in general. Two specific cases were given and then the numbers were replaced by variables. This would have been enough for the lecture. Adding in linear equations pushed them too hard.]

Lecture 4: More in linear equations relating to word problems. The graphs of these situations.
[Here an attempt was made to show that "trial and error" could be replaced by a less chancy approach, i.e. algebra. The idea of this lecture and Lecture 3 was to show the power of algebra.]

Lecture 5: What is the unit's digit of 72 000? Prime numbers. How many factors does 30 have?
Can you generalise?
[Here the students were to recall prime numbers and use them in a novel (for them) context.]

Lecture 6: Pascal's Triangle in the context of $(x + 1)^n$.
[Given the general weakness in algebra, too much was covered in this lecture. Half of this lecture was given over to a guest lecturer. There were about 5 of these altogether that were meant to give students some idea about what mathematicians and statisticians actually do.]

Lecture 7: ^nC_r and the Binomial Theorem.
Σ notation; arithmetic and geometric progressions.
[Again this was too much for the two hours available. APs were introduced through the Gauss story.]

Lecture 8: Compound interest.
[The time here was again shared with a guest lecturer. As most of the students have loans and might one day have mortgages, it was felt they should know how to do calculations with compound interest.]

Lecture 9: Pythagoras' Theorem and Pythagorean triples.
[Most people only know one theorem and this is it. It also gives the opportunity to talk about Pythagoras as a historical figure. It was heavier going than expected even though the Theorem itself was not proved.]

Lecture 10: Circle geometry - equation and simple tangents.
[Again shared with a guest lecturer. Applications of Pythagoras in a co-ordinate geometry situation were shown. They should know something about circles. Maybe it would have been better to do angles in semi-circles and the like.]

Lecture 11: Tessellations by regular polygons.
[Seemed to work reasonably. This was an attempt to bring in some geometry from the curriculum at the grade 4 - 5 level. The lecture was again shared with a guest.]

Lecture 12: Skills test and revision.
[A one hour test plus a look at the mock exam.]

Lecture 13: Revision.

Some lecture time was given over to talks by members of staff who were not involved in the rest of the course. These talks were either on research that the staff members were involved in or on interesting mathematical topics. They were all pitched at a reasonable level. There was an examination question on these talks. ("Outline the material given in three of these lectures and indicate how the material might be useful in your future career.") Generally the students didn't appreciate the purpose of these talks. Despite the examination question, which they were given in advance, the talks were not taken very seriously.

The tutorials had a tutor in charge who was either a member of staff or a PhD student who was a member of staff of another university. The tutors had a demonstrator to help them. The demonstrators were all senior students. Each tutorial had an assigned number of problems that the students were to do in groups of 4. (In some groups the burden of the work was undertaken by one or two members. For two weeks towards the end of the course, students were asked to work in groups of 2. This was an attempt to make sure that everyone made a contribution.) This work was assessed with all students in the group being assigned the same mark. The work was supposed to reinforce and extend the work done in lectures, to provide problem-solving opportunities, to introduce them to new ideas that they would not have seen in class, and to revise material covered earlier in the course.

Below some typical problems from the tutorials are listed with comments. Many of the questions were of this type in that they had many steps to them. There were also more routine problems that gave the students a chance to practice skills that had been covered in lectures.

Example 1: What is the remainder when 1 327 is divided by 3? Is it possible to know the answer to this question without dividing 1 327 by 3?
Produce seven more 4-digit numbers that have the same remainder as 1 327 when divided by 3.
What is the pattern here?
Guess it. Check it. Prove it for special cases. Prove it in general.

[The students need to become fluent in arithmetic. One aim of such a question is to provide practice in arithmetic. But much of mathematics is about looking for patterns and this problem does that too. (It should be pointed out that they had looked at divisibility by 9 in the lecture preceding this tutorial so they weren't going into this problem "cold".) Finally students were encouraged to prove things where possible. Students did not lose many marks if they couldn't prove things in general. The tutors were also told to be generous with their help on the harder parts of the questions.]

Example 2: A tramping group walked 80 km over a long weekend. On Sunday they walked 7 km further than they had walked on Saturday. On Monday they walked 13 km further than they had walked on Saturday. How far did they go on Saturday?

[There was a problem-solving question for every tutorial. This was partly because it was felt that problem-solving skills are important. (It now is an essential part of the New Zealand Curriculum.) It was also partly to help students see connections if only when the same question was put into a completely different context.]

Example 3: Devious Dan employed Lara and Mara in part-time jobs. Lara negotiated a pay of \$10 for the first day and \$2 extra for each additional day that she worked. On the other hand, Mara said that she'd just take \$1.50 on the first day but each day after that she'd get 3/2 times the previous day's pay. Lara was quite happy with this arrangement until she realized what was going on. Why did she quit after 8 days? Mara said that Lara should have stayed on for at least 2 more days. Why?

[This is clearly an old chestnut problem relating to arithmetic and geometric progressions. But it was important for them to see the different rates of growth here and to be able to explain Lara's unhappiness. So, as with most problem-solving questions, there was an element of communication required.]

The examination was 3-hours long. It consisted of three parts. The first part consisted of questions that had short answers. This related to the basic skills of the course. The second section had some problem-solving questions along the lines of Example 2 above. And the third section had multi-step questions that required more thought but were built on the lecture material.

In addition to the examination, there were four types of assessment in the course. The tutorials counted for 20%; there was a skills test (10%) (which consisted largely of straightforward word problems containing applications of arithmetic and geometry (areas and perimeters); a mathematical investigation (10%) and an essay (10%). Apart from the examination, only the skills test was not undertaken in a group. The groups generally worked well, though there were some problems (alluded to above). They did make the running of tutorials very efficient as the tutors and demonstrators essentially only had to talk to six groups rather than covering 24 individuals. A student's final mark was the sum of all the constituent pieces or solely based on the exam mark, whichever gave them the better mark.

3. MATH 100, 2001. We felt that most of what had been done in 2000 worked well. There was no in-depth research undertaken to reinforce this but student feedback was positive and so was the feedback from the School of Education. However, it was felt that some of the topics were too hard. For instance, binomial coefficients were done because of the link with Pascal's Triangle. But it was really too hard for most of the students. At least, asking them to expand expressions like $(x + 2)^5$ was far too difficult. So in 2001 the binomial coefficients work was limited to some elementary counting, even though that is hard for many of them.

Another difficulty for them was the arithmetic and geometric progressions. Part of the problem was the algebra that was involved and part was the fact that it was done too quickly.

In 2001 an attempt was made to try to reduce the amount of lecturing and increase the amount of interactive work in lectures. There are a number of activities that have been developed over the years for use with bright junior secondary students and some of these worked well with the students in this course. For instance, the book by Hilton and Pedersen (1989) contains ways of folding strips of paper that can be weaved to make polygons and polyhedra. There is some good geometry in this activity and producing models is very satisfying for some students. So this was added to the course this time round.

Apart from a self-help program, Mathercise, no technology was used in the course. This year a web site was used to convey administrivia related to the course as well as the lecture outlines, tutorial exercises, etc. Graphics calculators were also introduced to the students.

4. Discussion. After working with the class for a few weeks it became clear that many of them had very poor skills, not just in mathematics as a whole but in arithmetic in particular. Some of them had no idea how to use brackets, for example. Consequently the expectations of them were far too high and, in places, the material presented was inappropriate. Given that some primary teachers go on to teach mathematics in the junior secondary school though, none of the content of MATH 100 should have been too hard, nor is it outside the range of what they should be required to know.

This does raise issues regarding exactly what mathematics we should expect students entering such a course as this to know, and what we should expect practicing primary teachers to know. There is probably nothing that can be done about the intake level of students. But it is clear that we need to find ways to extend primary teachers and increase their mathematical knowledge because courses such as this do not go far enough. This can only be achieved through in-service courses.

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Indispensable Manual Calculation Skills in a CAS Environment

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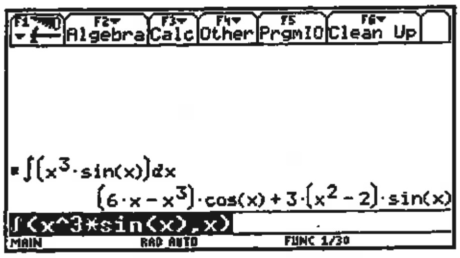
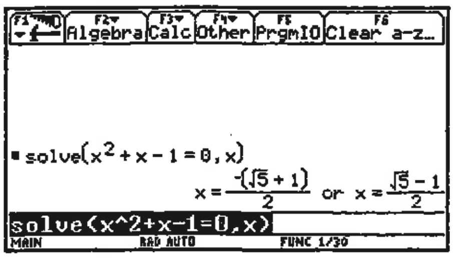
ABSTRACT. Which manual calculation skills are still needed when students use graphic/symbolic calculators or computers with computer algebra systems (CAS)? What should students be able to do manually, i.e. just using paper and pencil? This paper is the outcome of a two-day discussion on these questions, held by the four authors. Our answers and proposals are meant to be challenging, aiming at sparking off a broad discussion about what permanently available manual calculation skills we still need to teach and assess.

1. Computer algebra systems (CAS). Computer algebra systems (CAS) are tools that automate the execution of algebraic computations. CAS can simplify expressions, compute symbolic derivatives and integrals, plot graphs, solve equations and systems of equations, manipulate matrices, etc. In short: they automate most of the calculation skills we teach in school mathematics.





The CAS which are widely used in schools are the computer program Derive and the algebraic calculators TI-92 and TI-89. Introductions to using these tools are found in [Kutzler&Kokol-Voljc 2000] for Derive 5, [Kutzler 1997] for the TI-92 and [Kutzler 1998] for the TI-89. Soon such tools will be used as a matter of course, such as we use scientific (in some countries also graphic) calculators today. Using a calculator for differentiating $x^3 sin^2(4x+5)$ will be as common as using it to evaluate $cos(1.3786)$ or $\sqrt{5.67}$.

The above screen images give examples of what CAS can do.

2. Starting Point: Calculator-Free Exam. We assume an exam comprising two parts. In the first part no modern technical tool is permitted - not even a simple scientific calculator - whereas in the second part all kinds of technology, in particular powerful calculators or computers with CAS may be used. Some countries, such as Austria, are experimenting with two-tier exams. Other countries, such as England, use two-tier exams already. We believe that two-tier exams would be a well-balanced compromise meeting both the desires of technology supporters and the reservations of those who are concerned about the use of technology in the classroom. Some fundamental thoughts about two-tier exams are contained in [Kutzler 1999].

We assume a fictitious, written, technology-free exam. We look for questions and classes of questions that

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[1] It would be better to use the word "calculator" here. According to *The CASSELL Encyclopaedia* the word "technology" means "the science of the industrial arts; the practical application of science to industry and other fields; the total technical means and skills available to a particular human society; the terminology of an art or a science." However, the use of the word "technology" became ingrained in the academic literature about the use of calculators and computers for teaching. Therefore – and because we want to avoid confusion – we use it here as well.

we would include in such an exam. Drawing the borderline between questions to be asked in a technology-free exam and questions that would not be asked in such an exam is equivalent to listing the indispensable manual calculation skills. Therefore, the fictitious technology-free exam is a means to an end for us. Our discussion and its results are relevant far beyond the exam situation. They are fundamental for the development of mathematics education in the years to come.

After reconsidering the meaning and importance of calculation skills and restraining their role in teaching and learning, it is crucial to discuss the consequences for mathematics teaching. This will become the topic for our future discussions and work.

3.Three Pots: -T, ?T, +T. The border line between questions to be asked in a technology-free exam and questions not to be asked in such an exam clearly depends on many parameters - including the type of school. We try to give a universally applicable answer by creating three pots, which we name -T, ?T, and +T.

- The first pot, -T (= no technology), contains those questions which we would ask in a technology-free exam. Hence these are the questions we expect students can answer without the help of *any* calculator or computer.
- The calculation skills needed to answer the questions from pot -T should be mandatory from school year 8, or starting from the school year in which they are taught. The students are supposed to *maintain* these calcu-lation skills throughout the remaining school years (and, hopefully, beyond school) hence teachers may assess them *at any time*.
- The third pot, +T (= with technology), contains questions which we would not ask on the -T exam. Hence in situations in which such problems would occur, we would allow students to use powerful calculators or computers with CAS for their solution.
- The second pot, ?T, reflects our doubts, our different views, and partly also the inherent difficulties of this topic. We either were divided over the questions that ended up in this pot, or we agreed that we would not or could not put them into one of the other two pots. This pot shows how fuzzy the border line (still) is - at least for us.

Whenever feasible we outlined the spectrum and the border line of a class of questions by providing comparable examples for both -T and +T.

4. Higher Demands During Teaching and Exercises. The questions we put into -T are those which we would not ask in a technology-free exam - but we would not ask them in a technology-supported exam either. These questions appear sensible only in the context of appropriate problems, but not as isolated questions. Their best use could be to test how well a student can operate a calculator.

The questions we put into -T describe long-term manual skills. In order to reach this goal it certainly would make sense to let the students practice with more demanding examples at some stage.

To some extent it could make sense to let the students practice some of the examples from +T even *without technology*.

5. Other Important Skills and Abilities. It goes without saying that other important skills and abilities exist in addition to calculation skills. In a CAS teaching and learning environment many of those skills and abilities will keep their importance. Several will become more important. In any case, they are indispensable also (for details see [Heugl, 1999]). Examples of such abilities are:

- finding expressions
- recognizing structures
- testing conjectures
- visualizing
- using technology properly
- documenting calculations or problem solutions properly.

The ability to visualize allows a person to make a "proper sweep of the hand" to sketch a graph of, for example, x^2 or $sin(x)$.

Among all the skills and abilities teachers are supposed to teach in math classes, *calculation skills* have played and will play an important role. We teach them not only for their own sake, but to some extent because they are prerequisites for the attainment of "higher" abilities such as the above mentioned. Therefore the above mentioned and other abilities play a decisive role when judging the importance of calculation skills, hence they were part of our discussions. This is partly documented by some of the annotations we give.

6. Mathematics Education Will Not Become Simpler! We do not believe that mathematics education will become simpler - the contrary is true. The suggested lower level of manual skills reflects our belief that CAS will become standard tools for mathematics teaching and learning. It also reflects what we believe is our realistic approach as to what we want students to know through-out their school career and beyond. A consequence of the new tools is that mathematics becomes more useable and probably more demanding - but definitely not simpler. After the very unfortunate discussion about "7 years of teaching mathematics is enough" in the German and Austrian press some years ago we definitely do not want to create a similar debate about "trivial symbol manipulation is enough." Most important for us is the distinction between the goals "perform an operation" (to some extent this can be delegated to a calcula-tor) and "choose a strategy" (this cannot be done by the calculator.)

The following exposition has an impact on many aspects of teaching mathematics: the teaching methods, training methods, homework, curricula, the topics we teach, what teachers need to know, etc. We broached these issues but did not elaborate them. Therefore we do not mention them here.

7. Our Goal: Permanently Available Minimal Calculation Skills. We want to instigate a long overdue discussion about the mathematical, methodological, and administrative consequences of using CAS and other mathematics software for teaching and learning mathematics.

This paper is meant to be challenging, maybe even provocative. Let us face the challenges of the new tools and let us take the necessary steps! In particular this demands the willingness to say goodbye to familiar things if we see the necessity for it.

8. Questions and Classes of Questions. For this article we restrict ourselves to questions for which one could use powerful calculators or computers with CAS.

8.1. Arithmetic - long term minimal competence.

	−T (no technology)	*?T*	*+T (with technology)*
01	compute $3 \cdot 40$		compute $3.2987 \cdot 4.1298$
02	compute $\sqrt{81}$		approximate $\sqrt{80}$ to ... digits
03	estimate $\sqrt{80}$		simplify $\sqrt{80}$
04			calculate $\sqrt{11 \cdot \sqrt[3]{11}}$
05	factor 15		factor 30

The example $\sqrt{80}$ (and its variants -T03, +T02, and +T03) demonstrates how important and decisive the formulation of a question is for putting it into a certain pot. The less important the manual calculation skill becomes, the more important becomes the appropriate formulation in order to clarify the objective of the question. This becomes even clearer with some of the questions in the next sections. We agreed that the importance of the teaching goal "estimation" goes far beyond the given example (-T03). It is so important that we need to reach it without technology - although it may be worthwhile to use a calculator as a pedagogical tool, for example when testing the quality of an estimation, when computing the error, or when demonstrating the purpose of estimations.

To avoid a misunderstanding we repeat what we said earlier: The questions from pot +T are questions we would not ask in a technology-free exam. We would not ask these questions in a technology-supported exam either, because these questions appear useless as such, their best use might be to test how well a student can operate a calculator.

The questions from +T just require the skill of evaluating an expressions that typically comes from a more complicated problem. In the long term this should be delegated to a calculator. We need to make sure that the students *understand* what these expressions mean. But for testing such an *understanding*, we need different types of questions.

Nevertheless - this is another reminder - it certainly could make sense to use questions from +T in both technology-free and technology-supported "training units." This could be needed in order to make the questions which we put in pot -T a long-term manual skill.

Basically our proposals obey the following rule: elementary calculations (such as the factoring of an integer with only two factors, e.g. 15) are an indispensable skill (therefore these questions belong to -T), whereas calculations requiring a repeated application of elementary calculations (such as the factoring of an integer with three or more factors, e.g. 30) may be delegated to a calculator.

8.2.Fractions - long term minimal competence.

	–T (no technology)	*?T*	*+T (with technology)*
01	simplify $\frac{10^2}{5^2}$		simplify $7\cdot\frac{2}{5}:\frac{4}{6}$
02	simplify $\frac{10^2}{10^5}$		simplify $\frac{100x^3y^2}{10xy^5}$
03	simplify $2:\frac{1}{2}$		
04	simplify $\frac{2}{\frac{1}{2}}$		
05	simplify $\frac{5a}{5}$		
06	simplify $\frac{a}{5}\cdot 5$		
07	simplify $\frac{2}{x}\cdot\frac{x}{y}$		simplify $\frac{a}{b}\cdot\frac{b^2}{3ac}$
08			simplify $3x^2:\frac{2x}{5y^3}$
09	simplify $2a-\frac{a}{3}$		simplify $2a-\frac{a}{3}+\frac{a}{7}$
10	simplify $\frac{a}{3}+\frac{a}{7}$		
11	simplify $\frac{5}{x}-\frac{2}{x}$		
12	simplify $\frac{2}{x}-\frac{5}{y}$	simplify $\frac{2}{x}-\frac{x}{5}$	

-T01: Here we want students to see the obvious calculation 100/25 = 4. This is not trivial!

-T02: Expressions like this are needed in physics.

-T03: A corresponding alternative question (of higher value) would be: "*Why* is 2: 1/2 equal to 4?" This involves the ability to recognise structures.

We deliberately would not test if the rule $a/b + c/d = (ad + bc)/bd$ was learned by heart. We consider this a *background goal* - by which we mean a goal which does not need to be tested explicitly in a written exam. Learning such a rule by heart only leads to students who stolidly apply it for adding two fractions - instead of using the most often more appropriate approach of computing the least common multiple of the denominators. Equally, $a/b.c/d = ac/bd$ is only a background goal for us. Nevertheless, these rules (which also can be generated with a CAS) are an important teaching topic, also because they are good examples of the structuring of mathematical facts.

8.3.Expressions: With and Without Parentheses - long term minimal competence. We mentioned above that the formulation of a question is decisive for its value. In the following table we deliberately did without the usual request "expand" and instead requested "eliminate the parentheses." While the first formulation seems to suggest the application of the distributive rule, the second is non-sugges-tive, which hence increases the value of the question.

	−T (no technology)	*?T*	*+T (with technology)*
01	eliminate parentheses: $a-(b+3)$	eliminate parentheses: $(5+p)^2$	eliminate parentheses: $3a^2(5a-2b)$
02	eliminate parentheses: $2(a+b)$		eliminate parentheses: $(a^2-3b)(-3a+5b^2)$
03	eliminate parentheses: $2(ab)$		eliminate parentheses: $(2a+t)^2$
04	eliminate parentheses: $3(5a-2b)$		eliminate parentheses: $(5+p)^3$
05	eliminate parentheses: $(3+a)(b-7)$		
06	find equivalent forms of: $2a+2b$		
07	simplify $x^2y^2+(xy)^2$		
08	factor $3ab+6ac$		
09	factor x^2-4	factor x^2+4x+4	factor x^2-x-6

-T09: This question is important because it helps to develop the abilities deciding' and 'justifying.' Both abilities are needed for sensibly using a calculator's "factor" key or command.

The distributive rule $a(b + c) = ab + ac$ is a background goal here.

We had a long discussion about questions ?T01 and ?T09. Part of our group thought that the ability of recognizing structures needs this calculation skill. On the other hand, the Austrian CAS projects produced some evidence that using technology supports the ability to choose a strategy without requiring the development of the corresponding calculation skills.

8.4. Linear Equations - long term minimal competence.

	-T (no technology)	?T	+T (with technology)
01	solve w.r.t. x: $x-6=0$		
02	solve w.r.t. x: $5-x=2$		
03	solve w.r.t. x: $3x=12$		
04	solve w.r.t. x: $5x-6=15$		solve w.r.t. x: $5x-6=2x+15$
05	solve w.r.t. y: $\frac{y}{3}=5$		solve w.r.t. x: $2x+3=\frac{4}{3}$
06	solve w.r.t. x: $a\cdot x=5$	solve w.r.t. x: $a\cdot x-6=15$	
07	solve w.r.t. x: $x+1=x$	solve w.r.t. x: $2(x+1)=2x$	
08	solve w.r.t. x: $x+1=x+1$	solve w.r.t. x: $2(x+1)=2x+2$	
09	solve w.r.t. t: $s=v\cdot t$	solve w.r.t. x: $K=k\cdot x+F$	
10	solve w.r.t. r: $U=2r\pi$		
11	solve w.r.t. x: $\|x\|=1$		

-T06: This example is important, because currently available CAS do not make the necessary case distinction for values of a.

-T11: CAS often produces answers involving the absolute value function. Therefore students should know this function and handle simple applications technology-free.

8.5. Quadratic Equations - long term minimal competence.

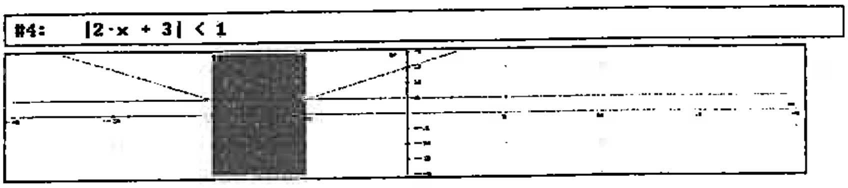
	-T (no technology)	?T	+T (with technology)
01	solve w.r.t. x: $x^2=4$		solve w.r.t. x: $9x^2=4$
02	solve w.r.t. x: $x^2-4=0$		solve w.r.t. x: $9x^2-4=0$
03	solve w.r.t. x: $x^2-x=0$		
04	solve w.r.t. x: $x^2-4x=0$	solve w.r.t. x: $x^2+4x+4=0$	solve w.r.t. x: $2x^2-5x+9=0$
05	solve w.r.t. x: $x^2=a$		
06	solve w.r.t. r: $A=4\pi r^2$		solve w.r.t. v_0: $x=\frac{1}{2a}\cdot v_0^2$

+T04 and ?T04 designate what some teachers may consider the most radical change: we eliminate the for-mula for the solution of a quadratic equation from the list of indispensable manual skills. However, we keep it as a background goal because of its important role in algebra and the inherent concept of case distinction. The traditional approach of solving quadratic equations with a procedure (by either applying the formula or performing the method of completion of a square) will become extinct (compare [Herget 1996].) For similar reasons the logarithm tables and slide rules disappeared "over night" after arithmetic computations could be delegated to scientific calculators.

8.6. Inequalities - long term minimal competence.

	–T (no technology)	?T	+T (with technology)
01	for which *x* is: $x-2<4$	for which *x* is: $x-2<x+3$	for which *x* is: $3x+1<2x-1$
02	for which *x* is: $-2x<4$		for which *x* is: $\frac{1}{x-1}\leq 2$
03	for which *x* is: $x<x+1$		for which *x* is: $ax<4$
04	for which *x* is: $x<x$		
05		for which *x* is: $\lvert x\rvert<1$	for which *x* is: $\lvert x-2\rvert<1$

The use of CAS means an obvious shift from calculation to visualization skills as is demonstrated by the following (Derive) screen images.



8.7. Differentiation - long term minimal competences.

	–T (no technology)	?T	+T (with technology)
01	differentiate w.r.t. *x* : $y=x^4$		
02	differentiate w.r.t. *x* : $y=7x^2+3x+1$		
03	differentiate w.r.t. *x* : $y=\frac{1}{x^2}$		
04	differentiate w.r.t. *x* : $y=3$		
05	differentiate w.r.t. *x* : $y=\sqrt{x}$		
06	differentiate w.r.t. *x* : $y=\sin x$	differentiate w.r.t. *x* : $y=x^2+\cos x$	differentiate w.r.t. *x* : $y=x\sin x$
07		differentiate w.r.t. *x* : $y=2\cos x$	differentiate w.r.t. *x* : $y=\sin^2 x$
08		differentiate w.r.t. *x* : $y=3\sin 2x$	differentiate w.r.t. *x* : $y=\frac{\sin x}{x}$
09	differentiate w.r.t. *x* : $y=e^x$	differentiate w.r.t. *x* : $y=e^{2x}$	differentiate w.r.t. *x* : $y=2^x$
10	differentiate w.r.t. *x* : $y=\ln x$		
11	differentiate w.r.t. *x* : $y=\lvert x\rvert$		

Traditional calculus courses are full of calculation skills. There is a particularly strong demand for change.

9. Concluding Remark and Request. As mentioned at the beginning, we like to see this paper as an impulse for a broad discussion about which manual calculation skills we should keep demanding and which we can let go. It was not our goal to provide a detailed and unimpeachable pedagogical analysis - we just wanted to give a pragmatic, brief presentation of our current judgement of the complicated issue of manual calculation skills. We deliberately wanted to be provocative and shake the mainstay of traditional mathematics teaching. Let us know what you think.

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DIVERSE MATHEMATICAL BACKGROUNDS IN SWEDEN

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1. Background. The Royal Institute of Technology (KTH), is one of the oldest and most well-known technical universities in Sweden. During its more than 100 years of existance, success in an examination from the natural science program in upper secondary school has been essential for entrance. The same rule applies for mathematics and science programs at all universities in Sweden.

But this year KTH has accepted 15 students (from a total of 3 000 new students) directly from the social science program in secondary school. Why did this happen? The answer is easy. Swedish technical and scientific universities can welcome 23 000 new students every year, but only 14 000 qualified students leave upper secondary school. Where are the missing 9 000 to be found?

In 1992 this problem was already obvious. A special preparatory year, containing physics, chemistry and mathematics, for students from the social science program, was started. After successfully completing this preparatory year these students have the same qualifications as students from the natural science program, and can therefore be accepted at technical universities.

In Sweden upper secondary school is organized into 16 programs, each one with its own compulsory subjects and courses. Mathematics at secondary level is divided into 5 modules, A to E. Only in one of the 16 programs, the natural science program, are all 5 modules taught. In the social science program course modules A to C are included in the syllabus, and in most of the remaining programs only module A is included.

Today all students coming out from non-natural science programs at secondary school are qualified for the preparatory year if they have studied module C in mathematics. If not they can take this course at a secondary school for adults. This year KTH has 400 students in the preparatory year, during which they study the same mathematics, physics and chemistry as in the natural science program at secondary school. The content is organised differently and adapted to the subsequent engineering studies, in order that the students learns how to study at university level. About 65% of the students who enter the preparatory year are successful and can start their engeneering studies. Of the remaining 35% some need more time to pass, but most of them find that natural science is not their passion and try something else.

The preparatory year is considered to be very demanding, and students completing it are highly coveted at all technical and natural science universities.

In the near future a preparatory semester for student missing just parts of mathematics, physics and chemistry, is planned to start.

Many social science students find physics and chemistry hard and not very interesting. Some think that one year is to much time to spend before starting university studies. The preparatory year is therefore not for them. At the same time there are some new engineering programs, for instance different computing and media programs, where the need for knowledge in physics and chemistry is not as great as in more traditional ones.

In Sweden, as in many other European countries, there are two kinds of engineering school. The traditional one is 4.5 years long culminating in a masters degree. The new school offers a 3-year bachelor degree course (after which it is possible to study for a masters degree). But the system does not allow for students to change schools. Students "lose" between 12 and 18 months if they want a masters degree from the traditional school. Therefore it is more prestigous to study at the traditional school. When there are not enough students for both schools, the problem of recruiting students is more complex for the shorter education.

2. Address in the challenge of diverse mathematical background. Accepting students from the social science program in upper secondary school directly into an engineering program is a new idea.

The 3-year long media engineering program which started in August last year accepted 45 students, of which 15 came from the social science program in upper secondary school and the rest from the natural science program. The student interest for this new program was great with 9 times as many applying as were accepted. This can be compared with the 3 year construction engineering program which only had sufficient applicants to fill 75% of its quota.

In all 3 year long programs 16 credits (of 120) are free "electives" of the student's own choice. The students from the social science program have to allocate 4 credits for a course in secondary mathematics to reach the same mathematical level as the natural science students. However it was not possible to give this course in the first semester. Also it was considered too late to give the other students their first mathematics course at the end of the first year. Therefore it was decided that a course in applied mathematics should be held for all students during the first semester.

Social science students do learn the trigonometry of right triangles, but that is in the very beginning of their upper secondary school, and they never use this knowledge after that. They have heard of derivatives, and also learned some rules, but they have not had any experience of applications. They have never heard of integrals and differential equations. They are not trained in solving problems the same way as the natural science students, and their mathematical skills, in general, are poor. Yet the same course was to be held for both kinds of students together!

Objective of the course, Applied Mathematics, 3cr

The course will give practice on, and applications of mathematical topics previously studied to generate motivation for further mathematical studies. The examples are to be chosen so that they are relevant for the program.

A traditional calculation program is to be used in handling mathematical functions.

The course will introduce a powerful and modern computer program in mathematics.

Syllabus

Algebra, geometry, trigonometry, equation solving, simple numerical methods, matrices, series, algorithms and differential calculus.

Use of a computer program for calculating and mathematics

Realization

Lectures, tutorials and computer laboratory workshops

Examination

Written and oral presentations, computer exercises.

There is no written examination. Assessment had to be done in other ways. The students were organised into groups of five in an earlier course, and these same groups were used in the applied mathematics course. It so happened that every group had at least one social science student and no group consisted of only social science students. This was essential for the trial.

The course started with recapitulation of geometry, right angle trigonometry, equation solving and derivatives. Some fractions and algebra was also revised. The students did not use a calculator except when solving trigonometry problems.

After this the students were introduced to Mathematica. They were given problems which led to equations considered hard to solve "by hand" even for a well trained natural science student, but easily solved with a computer program. The problems generated trigonometrical and other types of equations that social science students had never learned to solve, but with the help of Mathematica they were able to solve them.

Every group was given 5 to 7 problems, without answers, which they should solve together using Mathematica if needed, and write down the solutions in good order. Every group had a 10 to 15 minutes examination with the lecturer, where one of the problems (the lecturer's choice) was reported in written form and the solution of one other problem was reported orally by one of the students, again chosen by the lecturer. If the chosen student failed, the whole group failed.

Equations were followed by derivatives. Lectures were given on the rules and applications. The exercises on which the students worked led to functions of all kinds, many of which social science students had never used before. Trigonometrical functions is one example. This meant they had to use Mathematica. The same examination method as mentioned above was used twice for this part of the course.

As the course in Applied Mathematics can only use calculus taught previously in secondary school, integrals and differential equations had to be left out.

Matrices is not taught in Swedish secondary schools, but is part of this course. The reason for including matrices was that it would be possible to give examples related to the media world.

This part of the course had to start with lectures and exercises about what matrices are, when they are useful and how they can be applied. The applications chosen were cryptation, computer graphics and Markov chains. After some exercises by hand, Mathematica was used. The martrix part of Applied Mathematics became very popular and made the students eager to learn more. Assessment was done individually in the computerlab, where only Mathematica was open and no communication was possible, almost like an ordinary written examination. The students saved their files on special accounts which later were copied on a CD for marking.

The last components of Applied Mathematics were, two rather comprehensive exercises connected to economics which were done individually in Excel. The students were already familiar with the program. Many students found this part easy but not belonging to a course in mathematics.

3. Outcome. 69% of the social science students and 73% of the natural science students passed all parts of assessment. Of the remaining students only a few will have problems in passing the course.

Almost all social science students found the calculus part quite difficult. They thought that being part of a group had helped them very much, some of them even thought it would have been impossible to pass otherwise. Working in a group of 5 takes more time than studying individually, but the students thought they learned more this way. Some natural science students however thought they spent too much time helping social science students to learn. They would have prefered working individually. Almost all students were satisfied with what they have learned. The social science students had used more time for the course than natural science students. Both categories liked using Mathematica, social science students somewhat more than natural science students, which was expected as they had to rely on the computer program more than the natural science students.

So far there are only very small differences between the two categories of students in their results in mathematics or in any other subject studied this first year. This fact indicates that it is possible to accept students with less mathematics in their background than "normal students": at least if they are high performing students and the engineering program into which they go is not a traditional one.

DISTANCE LEARNING IN URUGUAY: TWO DIFFERENT EXPERIENCES

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ABSTRACT. Universidad de la República is a public Uruguayan university, where students do not pay for their courses. This university is organised in several faculties and each one has its own authorities, which make both the academic and the economic management of the institution.

One of these faculties is the Chemistry Faculty at Montevideo, where five different chemical careers are offered. In Uruguay, as in other countries of Latin America, when students enter at the university they have to choose their career in first year. For this reason, when we talk about certain subjects, it is implicit the career and vice versa, because courses and careers are strictly related.

At the beginning of 1999, authorities of the Chemistry Faculty offered for the first time first year courses of several chemical careers at Salto, a small city located approximately 500 km from Montevideo.

For this purpose, Maths teachers prepared web pages, video conferences, e-mail tutorials using electronic blackboard, and of course, presential classes (face-to-face) and assessment.

Results of 1999 courses were really bad: in the first test all the students at Salto failed! Several changes were made in the second part of the course, but it was too late, because they did not reach the minimum score in order to approve the courses. Fortunately, the situation has been changed dramatically in the present courses, which began in March of 2000. Several modifications were made to address the problems experienced with this first offering, which led to better learning outcomes.

In this paper we comment on the experiences in both years and changes introduced. The results of the assessment in both years are analysed statistically. We also present some conclusions based on the students' answers an evaluation questionnaire.

Finally, we suggest some recommendations for these kinds of courses.

1. Introduction. At the beginning of 1999, the authorities of the Chemistry Faculty decided to offer first year courses of several careers in Salto, a small city located at almost 500 km of Montevideo.

Courses at Salto were proposed in the modality semi-presential, that means using technology like web pages, electronic blackboard, video conferences, etc., complemented with practical face-to-face classes of exercises with teachers who came from Montevideo for this purpose.

In 1999, the Department of Mathematics proposed a teacher team consisting of two very enthusiastic professors about technology, who put the emphasis on web pages and electronic blackboard tutorials. They encouraged the students to use the e-mail for asking questions and the discussion group, called "the Forum". The last one was an electronic place were some topics about the courses were discussed. Moreover, the theoretical parts of them were taught using video-conferencing.

But, these efforts were not enough, because at the end of the year all the students at Salto failed.

The situation suggested the teaching approach should be changed. As a consequence, the teacher team was changed in order to put persons who can balance the technology-assisted classes with the traditional ones.

A new team began to work at Salto courses at the beginning of 2000. They continued with video-conference theoretical classes (twice a week) and tutorial classes using electronic blackboard (twice a week and several extra times near the exams). Not so often, they used forum and e-mail for mathematical questions. In fact, both were used in this new experience to organise administrative questions related with courses.

There were two real differences between both teaching approaches. The first one was about orientation of the 2000 theoretical and practical classes. For example, several exercises proposed in the last course, presented more applicability to other subjects of their career, than in 1999. This was the situation in one of the courses of the second part of the year, as it will be noted below.

The second important difference was the emphasis put in the presential face-to-face classes. In fact, every two weeks, one of the teachers went to Salto and expended the whole day in solve mathematical problems and especially difficult exercises. Several times two teachers went to Salto, because there were students of two different courses, and half a day for each group was not enough.

These presential classes were not a copy of Montevideo courses. Teachers assumed that students had already used technology media such as web pages, video-conference classes, etc., so they did not need to repeat the material on them. Only the most important concepts and procedures were taught again, before working with problems and special exercises. It is important to note that this hypothesis was not always true.

The results of this new experiment were very different from those obtained the year before. This point will be expanded later (in next section), and supported with statistical evidence.

2. Results of both courses: A statistical approach. In 1999, 34 students were enrolled at Salto mathematical courses, but in fact, only 14 continued with the experience. The other 20 students only went to initial class, and begun with the activities but they decided not to continue. As was said before, all the 14 students who continued the whole year of 1999 failed.

In 2000 Mathematical courses, 22 students were registered at Salto, but, as in the year before, only 14 of them continued, and 8 of these 14 students failed at the end of the first part of the year. The code of this first course is MAT 101 and the content is the typical Differential and Integral Calculus of real functions of one variable. As a consequence of these results, these 8 students were not able to continue in the second part of the year.

The other 6 students were divided (because of their different careers) into two different courses at the second part of the year. One of them registered for MAT 104, a typical Calculus course, like MAT 101, but with functions of several variables. The other students registered for MAT 103, a course of Probability and Statistics, with the emphasis put in Statistical Data Analysis. Only one of these 6 students failed in the second course. Combining the results of both parts of the year, we have [1]:

Course	Failed	Approved	No continued
1999	14	0	20
2000	9	11	8

These results can be used to obtain a contingency table in order to test the null hypothesis of independence, which means results are independent of the experience considered [2].

For this test, we have a chi square statistic of 16.81 and a critical value of 10.6, if we consider a significance level of 0,005. Note that the usual level is 0.05, so this is a very strong result! In fact, we can say that results were statistically different with a probability of being wrong, less then 5 in 1000 (0.5 %).

If we are strictly formal in the use of statistical results we can say that these results are a consequence of taking into account the students who decided not to continue with the experience. In order to solve this problem, we can construct a contingency table with only the first two columns of the first table. In this case, we have a 2x2 contingency table, so there exists a bias in the statistic, that can be avoid by the use of Yates correction [3].

In this case, Yates statistic is 9.008 and chi-square critical value is 7.88 if we decide to maintain the significance level at 0.005 as in the other test. So, we arrive at the same conclusion if we considerate the students who decided not to continue, or not.

It is possible to make the following objection: in 1999, all the students registered for MAT 101 and in 2000, we considered three different courses: MAT 101, MAT 103 and MAT 104.

To solve this new problem, we can consider this new contingency table, were only results of MAT 101 are taken into account:

MAT 101	Failed	Approved
1999	14	0
2000	8	6

We have again a 2x2 contingency table, and using one more time Yates bias correction we can compute a value of 5.30 against a critical value of 5.02, but now changing the significance level to 0.025. Therefore, in this case we can maintain our conclusion with a probability of being wrong less than 2.5 % (that means less than a half of the usual level of 5 %).

Obviously, this results confirms the intuitive conclusion: results of 1999 courses were really bad in comparison with 2000 ones. Another thing is that the percentage of students approved in Salto was practically the same than in Montevideo, in the case of the 2000 courses.

3. Other Results. Results mentioned before suggest analysing the details of both courses and if possible, trying to know more about the reasons of the differences noted in the results.

For this purpose, the instrument was a set of questions about the courses, proposed to the students. This set of questions was anonymous and it was given to students of both years (several students participate of both experiences).

The content of this set of questions was:

1.Do you think that use of technology as web pages, electronic blackboard, video - conferences, etc., is more useful in other subjects than in Mathematics? Why?

Almost all the students think that technology is more useful in other disciplines as General Chemistry, because they note that Mathematics is the most difficult and so, they need more help, and then presence of teachers is fundamental. Even more, they need teacher's help when they have doubts and so, get the answers later is not enough.

2.In the present conditions is it possible to teach at same time to Salto and Montevideo, or is better to separate them?

In the student's opinion, courses for Montevideo and courses for Salto must not be the same. As a consequence, we cannot recommend teaching simultaneously both classes. This implies a duplication of the work. Then, hours of teaching, and teachers themselves, must have an important increment and obviously, this means invest more money.

3.Which is the role of technology in mathematical courses? Do you think it can substitute the traditional class or it is only a complement of the other possibilities?

All the students said almost the same. Technology is an important complement, but it is impossible to substitute presential classes, at least at this level.

4.How much time do you need in an electronic blackboard mathematical class to get the same than in a traditional class of one hour?

Answers at this question propose different numbers. Anyway, all of them suggested that the same thing that can be taught in one hour in a traditional class, need significative much more time if it is taught in distance learning courses using electronic blackboard (between 50% and 100% of extra time).

5.Suppose you have to learn a long mathematical proof or you are asking about a long exercise: is it possible to use the electronic blackboard for this purpose?

For this question is possible to say almost the same than in the last one. For a long mathematical construction, electronic blackboard is not very useful. Likely, it is a problem of mathematical maturity, but at this level they are not able to understand for example a demonstration if it is presented sectioned in different pages of an electronic blackboard. Answers of the students depend of the subject they are coursing. Students of MAT 103 said that is better, but not fundamental, to see the complete construction of a mathematical proof. Likely, this answer comes from the fact that demonstrations of MAT 103 are not so long. That is why the student of MAT 104 did not think the same.

6.In your experience, did you find troubles derived from use of technology? Did you need any extra time to adapt yourself to this kind of courses?

In this question, answers are very different one from each other. Students mentioned there were some technical problems with the equipment employed. Nevertheless their success or not depended fundamentally of their previous experiences. Some students got that experience because they were repeating the subject and other ones had personal experience as PC home users.

7.Is there any difference between theory and practice (in mathematical classes) in distance courses? Why?
All the students noted important differences between theory and practice for this kind of courses. Sixty percent claimed for presential practical classes and the other forty percent asked for presential classes for the theory. One thing is to understand a mathematical construction, and a very different thing is to make oneself this construction, in both theory and practice. This special task needs a very different kind of help from the teacher.

8.Is it possible for you to study Mathematics from a web page without any help?
Results suggest that is not possible at this level. Even more, all of them asked for presential help from teachers. Although they are university students, they are the same persons of six months ago. Maturity (and mathematical maturity, in particular) cannot be the consequence of three months of holidays. As was noted by Alsina [4], this transition from secondary schools to universities needs special attention.

9.What do you think about the forum? Was it useful or not? Why?
Only a few part of the group gave an answer for the first part of this question. This few students said that forum would be a good idea in ideal conditions, but they observed some operative problems. In any case, they do not use forum, or they used it in very few opportunities. Of course, answer depends of their interest and predisposition to use of technology

10.Only for students who know both experiences (1999 and 2000 mathematical courses at Salto): was there any difference between them? If you answered "yes", which were the most important differences in your opinion?
All of them noted the differences in the "style" of both courses. In MAT 103 all the students observed that 2000 courses adjusted in a better way to their careers. Nevertheless, they noted other differences, but the answers were very different, making impossible from a statistical point of view to inform against those answers.

11.Do you want to add any comment not included in other answers?
It seems that questionnaire was very complete because nobody suggested any other thing. Therefore, it was a good instrument at least for knowing about both experiences and differences between them.

4. Conclusions. Almost all the conclusions were mentioned before when we commented on the results of assessment and the answers of the questionnaire. Anyway, we make some important remarks:

a) Distance learning courses demand more effort from students and teachers and it is not recommendable to teach simultaneously with other courses. They have special needs: more time, more attention, other kind of difficulties, etc., and so, they must be planned and taught independently.
b) Video-conference and electronic blackboard are more useful for theoretical classes or other kind of situations with a low degree of interaction. Anyway, not all can be transmitted successfully by this way. For example, large mathematical constructions are not easy to understand for this kind of students when they are not able to look at the whole building.
c) There exists an important problem of mathematical maturity of the students. They need help and the presential activities play a fundamental role in this sense. These activities cannot be substituted at this level.

As a final comment, these courses need good planning, technological equipment, good teachers, specialists in distance learning, etc. Of course, all things above-mentioned need to invest money. Without an adequate investment in education, results are not possible.

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NEW TECHNOLOGY'S PROMISE FOR MATHEMATICS EDUCATION

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ABSTRACT. This paper offers insights into how technology is meeting the needs of teaching and learning of mathematics and economics. Recent advances in mathematics education include didactic features for geometry, statistics, symbolic algebra, and electronic measurements. Integrating computer technology into the curriculum (1) provides easy data gathering in a hands-on environment; (2) allows data analyses while performing experiments in real time; (3) expands student opportunities to explore and solve real-world problems, and (4) enhances pedagogy.

1. Introduction. Institutions of higher education have always emphasized learning activities such as critical thinking quantitative reasoning and communicative skills. The information and knowledge explosion is forcing educators to reexamine their curricula and delivery systems. The traditional lecture-based teaching places more emphasis on knowledge and comprehension. However, by incorporating technology into the curriculum, teachers can incorporate higher-order taxonomies: applications, analysis, synthesis, and evaluation into teaching and learning. Interactive learning allows students to become more active participants in the learning process. This interchange from passive to active exchange of information allows students to develop their critical-thinking and problem-solving skills and expand their knowledge (Freberg; Shaw and Jakubowski; Lang; Levine; Pierce).

As technology innovations expand, instructors are adopting the Internet, computer networking, and hypermedia instructional materials to keep students on the cutting edge of the information revolution. Learner-centered environments support independent work as well as collaboration among learners. These classrooms provide students opportunities to connect prior learning with current experience since learners have access to a variety of tools and resources. Computer technology can also support the ways learners construct their own understanding of problems. Technological innovations are affecting economic, social, political, and educational institutions worldwide. Consequently, teaching and learning is being affected by technology's capabilities (Dias). As students become more exposed to information in courses and classrooms where the Internet, computer networking, and hypermedia-instructional materials are standard, low-tech classrooms will become obsolete. The new technologies present tremendous opportunities for teaching and learning, because they shift the focus from the teacher and the physical classroom to the learner. Therefore, the instructor functions less as a source of knowledge and more as a consultant in the learning activity.

Research on the effectiveness of computer-aided instruction (CAI) in teaching and learning of selected undergraduate courses suggest (1) test scores increase, (2) use of graphics, pictures, geometric shapes, and real-world statistical analyses help students to understand complex course materials, (3) instructors have more flexibility and easier access to previously covered materials, and (4) students have greater in-class exposure to sophisticated tools such as spread sheet programs, applications of mathematical concepts, and how to find and download data from electronic sources (Debertin and Jones; McLean-Meyinsse; Monson).

This paper describes a hands-on teaching and learning tool that integrates computer technology into the classroom. The discussion focuses on instructors' abilities to use technology in the classroom. The enhanced instructional tools (1) give easier access to data sources via the Internet, (2) allow collection and analysis of data while experiments are being performed in real time; and (3) provide real-world, exploratory, hands-on learning opportunities. Specifically, the paper reports on the experience of integrating technology into selected courses in the Departments of Mathematics, Agricultural Economics, and Economics at Southern University, Baton Rouge, Louisiana.

2. Technology Integration in Selected Mathematics and Economics Courses. In Spring 1999, faculty members in Departments of Mathematics, Agricultural Economics, and Economics implemented CAI in selected mathematics (geometry and statistics), agricultural economics (price analysis and special problems), and economics (microeconomics) courses. The technology integration extended the course objectives while engaging students in meaningful learning, and fostering interdisciplinary project-based instruction, team teaching, and individually-paced learning. CAI allowed the teaching and learning experience to include active learning, experiential learning, content design, tutorial development, on-line testing and grading, and workshops for the faculty.

Traditionally, mathematics, and abstract economic theories are difficult to grasp. Research suggests that experiential learning, through participation in interactive demonstrations, allows undergraduate students to get a deeper, more lasting understanding than is possible in the usual lecture format or lecture discussion format. Consequently, the instructors incorporated more opportunities for cooperative learning, critical thinking, and problem solving in the selected courses. We made extensive use of computer software packages (Geometers Sketchpad, SPSS, Eviews) and laboratory and Internet projects. The real-world applications clarified many of the abstract concepts and aided understanding.

3. Findings. Analyses of test scores in all courses show that statistically significant differences existed between the mean scores on final examinations for courses currently using CAI and those from previous years. Scores also were statistically significantly influenced by use of the computer laboratories, graphing calculators, and other experiential learning opportunities. The instructors also executed surveys to determine students' reactions to the new technology. Some of their comments are summarized below.

- The use of the computer and LCD projector requires less time compared to switching overhead transparencies or writing the information on the chalkboard. It is more efficient and benefits the professor and students.
- Easier viewing, better and more pleasant backgrounds
- Lectures are more organized; there is less eyestrain because of the interesting and eye-catching backgrounds.
- Made the lectures more organized and interesting because of the graphics and eye-catching colors.
- Lecture notes are easier to read; lectures my attention.
- The overhead projector is bland and unattractive.
- Internet and laboratory exercises help my understanding of course materials.

4. Concluding Remarks. Based on the study's results, CAI improves the faculty's teaching techniques and instructional effectiveness. It enhances access to the most advanced computer technology for multimedia presentations, and provides the faculty with the resources to prepare and graduate quality students who can compete in the global economy. CAI places technology in instructors' hands at the point where it can be most effective - the classroom, and give them the needed resources to stay on the frontier of innovative instructional technology. Additionally, CAI encourages faculty members to learn concepts and theory so that they can give students the skills, knowledge, and intellectual training required for a future that will continue to undergo rapid and fundamental changes. Integrating technology into mathematics and social sciences is useful in introducing and teaching geometric models, shapes, designs, and patterns, a variety of class and laboratory activities, and simplifying abstract economic concepts.

The technology is ensuring that students and faculty stay on the cutting edge in regards to teaching, learning, and research. CAI enhanced curricular offerings and improved students' performances in core courses. Instructors are no longer limited by the availability of computer laboratories. They are now able to teach students with the latest advances in instructional technology from the classroom. Growing up in an era of rapid technological changes, students are anxious for an enriching educational experience. CAI in the classroom enables students and faculty to stay abreast of current technology related to their fields and become totally computer literate. When students and faculty learn these state-of-the-art capabilities, student retention can be impacted positively. Students often have difficulties grasping abstract concepts presented by the lecture-only method of instruction. Thus, CAI may be an attractive recruiting and retention technique for colleges. Finally, and perhaps the most important impact is the broadening of the students' intellectual horizons and career opportunities.

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A Course for Student Tutors

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ABSTRACT. Most large university Departments of Mathematics face the annual challenge of finding and training a cohort of effective tutors for first year classes. In this paper, we describe the experience of our department in developing and delivering a course to meet this challenge. For the past nine years we have offered for academic credit to senior undergraduate students a course called Tutoring in Mathematics. The course provides tutors to a first year foundation calculus course at the University. The study guide states that students who take the Tutoring in Mathematics course should:

consolidate their own understanding of basic mathematical concepts and techniques;
gain theoretical and practical knowledge about both the learning and teaching of mathematics;
gain personal satisfaction from using their own knowledge to help other students;
gain sufficient experience to assess their own attitudes to, and suitability for, teaching as a career (without having to make a major commitment in that direction).

We discuss such things as the history, the curriculum, the practicum, some of the outcomes, and the future of this course.

1. History. The Tutoring in Mathematics course MATHS 202 has been taught in the Department of Mathematics of the University of Auckland since 1991. It was designed by the second author based on his observations of two similar courses; one in Mathematics at the University of California, Davis; and the other in Science at The University of Auckland. Students enrolled in each of the latter courses undertook their practicum component in neighbouring high schools. The logistical problems associated with this aspect of these courses led directly to the demise of (at least) one of them. From the outset we were determined to retain control by having our practicum component located in our Department (to date in MATHS 102). This has the additional benefit of providing peer-support for students in one of our own courses. Our course was initially designed as a yearlong course, and was taught in that format for its first four years. However, in 1995 the University changed to a two-semester academic year, and we have since developed MATHS 202 into a semester-long course, usually taught in the first semester. This allows students who have completed MATHS 202 in the first semester to be employed by the Department as student tutors for courses in the second semester, an opportunity for which many more tutors wish to avail themselves than is generally available in courses at an appropriate mathematical level. Since the inception of MATHS 202 course, the members of the class have been paid the usual hourly rate for undergraduate student markers for their assignment marking work, and if they continue as tutors in the second semester, they also receive payment for the tutoring they do. Successful completion of the course brings academic credit for a two-point course, usually as part of a 42-point degree programme. A similar and more extensive programme, established recently at the University of Arizona, is described in Chapter 7 of [1]. Entry to MATHS 202 is described in the regulations as being by permission of the Head of Department, and to ensure that they have the appropriate skills and interest in tutoring, all students applying to enrol in MATHS 202 are interviewed prior to selection. The interview examines such issues as the tutors' motivation for wanting to enrol in the course (e.g. some students in the past have been attracted to the course merely because of the lure of payment for marking!), and also assesses their communication skills. There is also a minimum pre-requisite of reasonable grades in at least two other mathematics courses, although in reality, most tutors selected for the MATHS202 course have greater mathematical experience than this minimum requirement. While it is clearly desirable that the tutors have as much experience and as high a level of qualification in mathematics as possible, raising the pre-requisite level too high has the effect of both reducing the pool of suitable students, and eliminating potential tutors who have the most recent experience in first year mathematics.

2. Curriculum. The Study Guide for MATHS 202 suggests that students who take the course should:

- consolidate their own understanding of basic mathematical concepts and techniques
- gain theoretical and practical knowledge about both the learning and teaching of mathematics
- gain personal satisfaction from using their own knowledge to help other students
- gain sufficient experience to assess their own attitudes to, and suitability for, teaching as a career (without having to make a major commitment in that direction).

Students in MATHS 202 attend three one-hour lecture/discussion sessions each week. These sessions have a focus of:

- One session dealing with practical aspects and administration of the tutoring and marking for MATHS 102
- One session looking at aspects of mathematics education, journals, seminars etc.
- One session to consider mathematical matters to assist in the marking of MATHS 102 assignments.

The MATHS 102 course (in which they do their practicum) is a bridging or foundation course consisting of an introductory module, plus five other modules. The major topics are the following.

- Mathematical modelling and the idea of a function.
- Polynomial functions.
- Exponential, logarithmic, rational, and piecewise functions.
- Gradient functions - differential calculus.
- Trigonometric functions.
- Area functions - integral calculus.

One important feature of MATHS 102 is that half of the assignments are designed as collaborative assignments. This means that the tutors (students in MATHS 202) must be able to assist and assess work in this context. Part of the study guide [2] information to all students in MATHS 102 states:

"Discussion is important in the process of mathematics learning. Being able to communicate your understanding is an important aspect of mathematical knowledge. In this course you will be given an opportunity to develop these skills.

A collaborative assignment task in this course is an activity in which a group of students (usually three) attempts to solve a mathematics problem together. The solution is submitted as a joint effort and is assessed as a joint effort. That is, all three students will gain the same mark.

Collaborative work is highly valued and is, therefore, a compulsory part of your course work. It is done in tutorials every second Friday. You do not have to work with the same students every time but all members of group must be from the same tutorial group. The task will be handed out at the beginning of the tutorial and the tutors will have completed part of the assessment before you leave. Some tasks will be done in the computer laboratory.

The assessment is in two parts:

- **Oral** in which tutors will assess the extent of the collaboration of the group members in the problem solving process and the understanding which the group 'as a whole' has of the problem and its solution.
- **Written** in which the written solution of the group will be handed in and marked in the usual way.

The oral and written components will be equally weighted."

The use of technology, especially graphics calculators, is also an integral feature of the teaching of MATHS 102. MATHS 102 may be best described as a bridging mathematics course with an applications focus that places a high priority on problem solving and understanding of concepts. Less emphasis than is traditional in many such courses is placed on basic skills.

Assessment in the Tutoring in Mathematics course is 100% course work, made up as follows:

35%	practical - tutoring and assignment marking
25%	journal/log documenting your analysis of tutoring and marking experiences
25%	seminar presentation
15%	written assignment.

There is no formal final examination for MATHS 202.

3. Practicum. The MATHS 202 practicum is a one-hour tutorial each week of the semester with a group of about fifteen MATHS 102 students for each member of the MATHS 202 class. These tutorial sessions alternate between tutorials where assistance will be given with the five written assignments, and collaborative tutorials as described earlier, which emphasise the importance of interaction and collaborative work in problem solving. The student-tutors from MATHS 202 mark the MATHS 102 students' assignment work according to marking schemes developed and discussed in one of the MATHS 202 class sessions. These sessions consider such important assessment issues as *marking correct on error* (i.e. following through a student's work after making a mistake, and awarding marks for work subsequently done correctly), or how to mark a student's work, which varies greatly from the answers given in the marking scheme. The tutors often develop strong bonds with the MATHS 102 students under their responsibility, as they will be marking the assignments for much the same group of students throughout the semester. Each tutorial class also has in attendance one of the lecturers or post-graduate students involved in the teaching of the MATHS 102 course, who are there to both support the MATHS 202 tutors, and provide an additional source of assistance for the MATHS102 students. This high degree of resourcing is felt to be critical to the success of the programme, but it has been subject to criticism from some staff also, with questions being asked about the relative effectiveness of directing such a high level of resourcing into one area of the Department's operations.

Each student-tutor is required to keep a journal (or log) recording the learning achievements and/or difficulties of their group of MATHS 102 students. The journal also gives tutors the opportunity to reflect on their own experiences in the role of student-tutor, an important component of personal development and an essential element of teaching practice. The seminar presentations introduce the students to a wide range of issues in mathematics education. This can be very illuminating to students with backgrounds predominantly in empirical science, as is frequently documented in their journals. The seminars are presented in pairs to their classmates on an aspect of Mathematics Education drawn from class discussions, or issues they may have observed in the MATHS 102 tutorials. The need to present a seminar in front of the class is also a new experience for many science students, but it does give them a small taste for another necessary teacher skill. Certainly, in this respect the way in which MATHS102 is delivered provides a model of some aspects of good teaching/learning practice, for example in the use of collaborative work and technology [3], which serves as useful experience for any of the tutors who decide to continue on into the field of mathematics teaching.

The written assignment varies in nature, but recent examples have included designing their own marking scheme for one of the MATHS 102 written assignments, describing the mathematical issues they considered in assigning marks to each question, and then comparing their scheme to the one provided by the course coordinator.

4. Outcomes. This course provides ideal training for student-tutors. So it is no surprise that a significant number of the tutors employed in the Department's many student support programmes are graduates of this course. They are highly regarded as well-trained tutors by our colleagues. MATHS 202 is also a fertile recruitment ground for future high school mathematics teachers, with a high proportion of MATHS 202 students continuing on to a further course in teacher training. Indeed, many of the students enrol in MATHS 202 as the first step in a degree structure, which is deliberately directed towards a career in teaching. On the other hand, several members of this class over the past five years have indicated that it convinced them that teaching was not a career option for them. Another apparent effect is that in recent years the Mathematics Department at the University of Auckland has become more willing to use undergraduate students as tutors. It surely seems that is no coincidence that this shift in policy and practice has occurred concurrently with the establishment and evolution of the MATHS 202 course.

5. The future. In its ten years of existence, MATHS 202 has certainly met its objectives. However, there is some pressure from our Mathematics colleagues for us to expand MATHS 202 and enlarge the class. We have resisted these overtures to date. Evidence from reading journals and examining the marking of the student-tutors suggests that, even at the relatively undemanding mathematics level of MATHS 102, many of them find their own mathematical understanding being tested, and at times stretched. This suggests that more problems may be experienced in for example interpretation of students' solutions when marking, if this programme were to be extended to higher-level mathematics courses. The need for more experienced and confident tutors for higher level courses means that the pool of prospective candidates would also be correspondingly smaller. Nevertheless, we are considering modifications to MATHS 202 so that it might produce a group of student-tutors for other first year mathematics classes. This is already happening to a degree, with many of the student-tutors in more advanced first year courses having previously successfully completed MATHS 202.

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Throwing Out the Bath Water?
Adapting Curricula to Reflect Changes in Technology

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ABSTRACT. The use of graphics calculators, and lately super calculators, has been increasing dramatically in school and tertiary mathematics teaching in recent years. However, in most cases the evidence is that the calculators have been merely incorporated into existing curricula as a tool to facilitate current teaching and learning schemes. Technology in general has not often been regarded as a major influence on the mathematics curriculum, with little attention being paid to how technology may affect both what is taught, and the sequence in which it is taught. This paper will profile the early stages of the implementation of computer algebra systems (CAS) into a first year mathematics course at The University of Auckland. The ability of these calculators to trivialise many of the tasks previously examined in this course has meant that a comprehensive curriculum revision is now needed. This paper will present progress to date in the curriculum review process. Questions such as "Which topics will need to be taught/examined differently; what material can/should we discard; what new topics may be taught?" will be considered.

1. Background. The principal entry-level course, Mathematics 151, at The University of Auckland has undergone a rapid transformation in technology use. Up until semester two 1999, any calculator or computer use by students at all was discouraged, and indeed prohibited in any of the course assessment. In 2000, graphics calculators were introduced to the course. Initially, the use of these was mostly confined to teacher demonstration. Although students were allowed to use them for course-work, there was no significant encouragement of this, and their use in formative assessment was still not allowed. Calculator use was also restricted to models like the TI-83, with those such as the TI-89 and the TI-92 with algebraic capabilities being excluded. This has changed in 2001 with the active promotion of the TI-89. The calculators are being extensively used in teaching by most of the teaching staff in the course, and students are strongly encouraged to buy the TI-89 with algebraic capability. The Casio FX2 was also considered, and the final decision to standardise on a single model was made after considering such factors as price structure and model capabilities. The use of the calculators has also been actively promoted in the small group tutorials that were introduced simultaneously as an integral part of the course in 2001. This paper considers some of the issues that have arisen during the implementation, and gives some examples with justification for some of the curriculum choices.

It is interesting to look at the reasons behind the technology development, although an extensive discussion of the arguments surrounding the appropriateness of technology use in mathematics education lies outside the scope of this paper. There has long been a strong divide between opponents and disciples of technology in mathematics education. Indeed, technology issues are central to what has been labelled the "maths wars" in the United States. The often highly emotive nature of this debate is clearly illustrated in two opposing articles that appeared initially in 1996 in *The Mathematical Intelligencer*, and subsequently came to this authoris attention in a list-serv debate in 2001. In the first of these articles, Koblitz argues that although there are some appropriate uses for computers in mathematics education, in his opinion "Computers should not be a major component in math education reform"(1). Koblitz describes the movement towards technology as an example of *gimmickry*, labelling it as *computermania*. He lists four broad areas that he sees as being the *down-side* of computers; ìthe drain on resources; bad pedagogy; anti-intellectual appeal; and corruption of educators" (1). For example, Koblitz condemns much of the effort to introduce technology in the classroom as being profit-driven, stating that "The intrinsic value of a pedagogical idea is not considered as important as its saleability." The technology-in-education movement has some of the characteristics of a religious evangelical campaign, fuelled by corporate and foundation money" (1). Dubinsky and Noss (2) counter much of Koblitz's arguments as being "implistic", noting that there is a growing body of literature describing evidence of (computer) effectiveness. For example in referring to Koblitz's claims of "bad pedagogy", Dubinsky and Noss (2) agree that whilst "teaching by demo" alone may be bad pedagogy, this does not rule out a multitude of alternative methods for implementing computers in mathematics education. There are numerous other writers such as Kissane (3), who are also strongly critical of traditional viewpoints such that expounded by Koblitz. Kissane perceives many of the arguments against technology as somewhat spurious, stating: "Rather than insist that the traditional skills are necessarily of permanent importance to all pupils for all time, we might use such observations to help us decide which things really are important, and be able to argue why they are important. We will need a better argument than the one making vague reference to needing to know how to do something at some later time" or not really understanding until you can complete the procedure the traditional way. Frankly, I find none of these arguments particularly convincing, and suspect that many pupils find them not so persuasive either (3).

Whilst they do agree with some of Koblitz's observations, Dubinsky and Noss generally regard Koblitz's arguments as unhelpful to the technology debate, stating that what is really needed now that "the initial flush of enthusiasm over 'new technologies' is beginning to pass" is a dispassionate, well-informed examination of costs, benefits, and difficulties (2). This view is strongly supported by Penglase & Arnold (4), who after reviewing an extensive body of current literature into the use and effects of graphics calculators noted that: "Sadly, the answers offered by research to (important) questions at the end of the first decade remain elusive and conflicting. As assumptions concerning the role and effectiveness of graphics calculators assume almost the status of dogma amongst educators and policy-makers alike, it is vitally important that the research basis for these assumptions be examined critically" (4:59).

Since the meta-analysis by Penglase and Arnold, there has been an expanding number of studies into the effects of graphics calculators on student learning that lend support to proponents of graphics calculator use. These include for example a recent study in 2000 by Hollar and Norwood (5), which found that students employing a graphics calculator approach to intermediate algebra in a class at university level demonstrated significantly better understanding of functions than did students in classes using a more traditional approach. It is in the environment of controversy and vigorous debate over the use of graphics calculators described above that the developments to the Maths 151 course need to be considered. Until 1999, the members of the teaching team of Maths 151 were mostly unfamiliar with the power of graphics calculators, and certainly there were some members of the team who sympathised with Koblitz's (1) arguments. Some on the team were ignorant of the educational possibilities of the technology available, and others believed that students need a basic grounding in skills and concepts before being introduced to the power of technology, meaning that the "no calculators" viewpoint held sway. In 2000, an experienced secondary school teacher and another member of the mathematics education unit, both of whom had much experience using graphics calculators in their teaching, joined the Maths 151 teaching team. They made use of the TI-83 in their teaching, and continued to demonstrate the possibilities of the technology to the then coordinator of Maths 151. This exposure ultimately led to a growing enthusiasm for the potential and power of graphics calculators, which has subsequently seen them introduced fully into the Maths 151 course in 2001. The intention is that the use of such technology will be fully integrated into the teaching, learning, and assessment of Maths 151. Some of the implications for this implementation will be discussed in the next section.

2. Implementation Issues. It seems that in any discussion on the use of technology in mathematics education, two key questions emerge: should we use technology in mathematics education; and if so, why and how do we best use it? The first question has been briefly addressed in describing the background to this study, and will continue to be a hotly contested source of debate in mathematics education for some years to come. The second of these questions is very complex, and it may be further broken down into several key areas.

- What access to the technology is necessary for effective use? Here we may consider subsidiary issues such as whether to make it compulsory for the students to purchase graphics calculators, or whether their use should be primarily as demonstration tools, with student use being optional. Should their use be allowed in assessment?
- What material should we teach the students? Should we make any changes to the existing curriculum in terms of content, or the order in which we teach subjects? What should we retain, what new mathematics is now accessible to students using the technology?

There has been much recent discussion surrounding the *integrated* use of technology in mathematics education. For example Kissane *et al.* (6) argue strongly that to be effective, graphics calculators should be fully integrated into all aspects of the teaching and learning of mathematics, from curriculum through to assessment. Leigh-Lancaster (7) uses the concept of *congruency* between curriculum, pedagogy, assessment and values to describe the integration, arguing that *congruency* between these components is critical to the effective use of computer algebra systems (CAS) in mathematics education. He gives as an example the overall use of graphics calculators in the State of Victoria in 2000, where there is a high degree of congruency between the above components. In Victoria, teachers use graphics calculators in instruction, students use them in learning, and the calculators are also used in both coursework assessment, and examinations. In support of this, Dugdale *et al.* note with regard to assessment that "It makes little sense to use calculators to express algebraic concepts graphically, then examine the students with tests that do not include graphical representations"(8:351). Many writers note the need to examine existing curricula from a technological perspective. Both Ruthven (9) and Kissane (3) observe that exciting possibilities exist for the introduction of new topics into the curriculum, and further that existing concepts may be treated differently, either in approach or in the order that they are taught. Ruthven (9) for example perceives that there has already been an important curricular shift with an increasing emphasis on the visualisation of relationships through graphic images. Hong and Thomas (10) note that students often lack conceptual understanding, being locked into a process-oriented style of thinking which is an obstacle to their understanding of important concepts. They give several examples where the facility of graphics calculators to provide students with superior visualisation of functions has greatly aided students' conceptual understanding. Harskamp *et al.* see such examples as further justification for curriculum review saying iit would be especially interesting to design a curriculum in which the role of the graphics calculator is to bridge the gap between a procedural and a structural concept of functions (11:51). An integrated approach to the use of technology as described here suggests that students should have universal and easy access to the technology, a position that introduces concerns of equity. Zevenbergen presents a strong warning in this regard when she notes that: "There is also a need to consider who has access to these tools for learning. Just as literacy was the key to power in the pre-industrial age, and mathematics and science in the industrial age; many authors argue that technology is the key to power in the post-industrial age. In earlier historical periods, women and the working classes were excluded from literacy and then mathematics. It can be argued that in these new times, the same group of students (and people) are excluded from power through their lack of access to technology." (12: 23)

As is described in the background to this study, the decision to implement graphics calculators into Maths 151 was based more on enthusiasm for their potential benefits, than directly on research findings. However, initial discussions between members of the teaching team, the course coordinator, and members of the department familiar with both research into, and the use of, graphics calculators did many of the issues previously outlined. Two initiatives were implemented in an attempt to minimise equity issues. Firstly an arrangement was made with Texas Instruments to supply reconditioned TI-89 (CAS) through the department's Student Resource Centre (SRC) at a greatly discounted price. Secondly, it was agreed that although the use of the calculators would be permitted and encouraged in all facets of the course including final examinations, the cost of the calculators prohibited their being made a compulsory part of the course's equipment list. This meant that students had to be given reassurances that all formative assessment measures would be designed to be as *calculator-neutral* (7) as possible. It was recognised from the outset by the course coordinator that this was not an optimum arrangement (c/f. *congruency* between pedagogy and assessment noted earlier (7)). However, as with several other decisions made in the initial stages of implementation in the Maths 151 course, the course coordinator felt that it was more important to get the calculators being used by both teaching staff and students, than it was immediately to perfect all aspects of their implementation. This was seen as especially crucial in the Maths 151 situation, given the degree of negativity, and hence difficulty, experienced in initiating their implementation in the first place! Even at meetings held after the decision to implement calculator use had already been made, there was still much debate over the appropriateness of this move. Warnings were given about weaker students using them merely as a crutch, and cautions raised over the potential of them being a *black-box* style phenomenon whereby students merely feed in numbers and pull out solutions with little knowledge of the processes or concepts involved. An interesting warning was sounded in regard to assessment issues however, when one of the teaching team members observed that often the procedurally based questions, which the calculators perform so easily, make up exactly those questions in tests and examinations where students can obtain marks more easily. Removing such questions from assessment could have the effect of making the examination more conceptual, and correspondingly harder! The intention to make the final implementation as integrated as possible was clearly present in the initial planning meetings. In considering curriculum issues, although it was recognised that at least some curriculum revision would be necessary, it was not clear at this initial stage what this might be. Given the inexperience with the use of graphics calculators by many of the teaching team, it seemed obvious that at the very least a semester of teaching the course using the calculators would be necessary before some informed decisions could be made regarding curriculum issues. Thus the main impetus was to implement the use of the calculators, with students being given as much encouragement as possible at all stages to purchase a calculator. Despite the TI-89is not being available for sale until the 4th week of semester and not all members of the teaching team using them, initial observations of the take-up rate for calculators by the students are encouraging. Nearly half of the students have purchased calculators from the SRC, which the coordinator has observed is almost equivalent to the proportion of textbook sales in past courses. It will be interesting to investigate why the remaining students have not invested in a calculator, although one may suspect that cost is a significant factor! The inherent danger in this implementation approach is that the whole process takes on the nature of an experiment, with both staff and students in the role of guinea pigs. There is an element of risk that some of the less than optimal facets of such an experiment may lead to the ultimate sabotaging of the entire project. Certainly a comprehensive review surveying both staff and students will need to be carried out at the end of the first semester in all facets of the use of graphics calculators in Maths 151. This may enable some recommendations for curriculum review to be suggested, and perhaps encourage a move towards greater integration of the calculators into such areas as course assessment, e.g. *calculator-active* as opposed to *calculator-neutral* examinations (7).

3. Some Examples of Possible Curricular Revision From Maths 151. In discussing the relationship between technology and curriculum, it is necessary to define what we mean by *curriculum*. Kilpatrick and Davis (13) describe several ways in which the term *curriculum* is used or referred to. For example is the curriculum located as a national document listing a broad description of topics to be studied, or localised at the classroom level as is commonly dictated by school schemes or even more commonly, textbooks? Are we talking about the *intended* or the *implemented* curriculum? In this regard universities are somewhat different from schools in that they enjoy a certain autonomy over the design of their curricula, and we should therefore expect a correspondingly stronger link between the *intended* and *implemented* curriculum. However this is not always the case, and Hoyles *et al.* note that "however much professional mathematics educators might wish it, curriculum change is not simply a matter of proposing desirable innovations or of engaging in rational debate about the form and content of the curriculum" (14: 172). It is clear that at the micro-level of a local university mathematics course such as in Maths 151, both these conditions are necessary if effective curriculum changes are to be effected. It does not necessarily follow that there is also a commitment to implementing changes to the curriculum. Tucker and Leitzel (15) for example observe that where graphics calculators are used merely in problem sessions to complement traditional lectures, there is little change in the content of the lectures. Almost all of the institutions in their survey which had instigated calculus reform using graphics calculators continued to teach all the material that they had taught their students before the change, reporting that their course syllabi had changed little as a result. Many of these institutions expressed concern that the result of this was that the courses tend to become *packed* with extra material (15).

Initial discussions on curriculum reform in Maths 151 suggested several areas that will require examination.

- Content areas that may need to be, or could be taught in a different order.
- Content areas that may be trivialised or made redundant by the use of the graphics calculator. This could include for example some routine algebraic skills.
- New content areas or richer conceptual understanding accessible using the graphics calculator.
- The possibility that some of the new content areas opened up by the graphics calculator may be trivial in nature and of limited educational value.

Some of these have already arisen, and it is expected that more will manifest themselves as the use of graphics calculators in Maths 151 progresses. One obvious example comes from looking at the (typical) 1999 semester one test, where students were asked to graph a simple cubic function, showing the intervals where *f(x)* is increasing, decreasing, concave up, concave down, and also indicate all local maxima and minima. Much of this is clearly trivialised by even simple non-CAS graphics calculators, and given that a certain amount of teaching time has been devoted to questions like this, it is clear that some revision of the curriculum is needed. Such examples of a purely graphical nature are widely documented in the literature. A more complex example has arisen in linear algebra, where in the past considerable teaching time has been devoted to finding the inverse of a matrix in order to solve systems of linear equations. The TI-89 will find the inverse of a matrix directly, which has generated much discussion already as to what steps students need to be able to perform in order to understand fully the concepts involved, and how much may be left to the calculator. This has significant implications for the curriculum, and the jury is still out at this time. It is clear already however that reaching a consensus over such issues is not easily achieved, as there are important considerations of lecturers' values and views on the nature of mathematics attached to every consideration. The review at the end of semester one promises to be every bit as animated as the earlier implementation meetings.

4. Summary. This paper has outlined some of the issues involved in the implementation of (CAS) graphics calculators into the teaching of Maths 151, a first year mathematics course at The University of Auckland. A need for congruency between curriculum, pedagogy, and assessment has been described as critical to the successful and effective use of technology in mathematics education. Some examples where a revision of the curriculum and/or the assessment of the course will be necessary to achieve any semblance of congruency have been given, and the implementation process will continue to be monitored, with many more such examples expected to arise. Penglase and Arnold (4) reflect that there is a particular need for studies such as this one, and a similar larger scale study in Melbourne (16) into technology and the curriculum, when they note that: "(The majority of) those studies which purported to investigate curriculum issues" have been seen to be instead studies of pedagogy, as the use of the tool remained intertwined with the effect of the learning context. Of more benefit, then, may be those studies, which directly attempt to address the issues of graphics calculator use within particular learning environments. (16:79)

The danger is that if mathematicians and mathematics educators do not engage themselves in such debates, then the criticism levelled earlier by Koblitz (1) about curriculum design being driven by corporate profits may indeed come to pass. Kissane (3) provides a suitable conclusion to this discussion when, citing Podlesni (1999:67), he warns that: "Are we getting to the point where technology companies are making *de facto* curriculum decisions for us? Are we doing our job as teachers or relinquishing part of it to the electronics industry?"

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TEACHING LARGE CLASSES USING INTEGRATED WEB-BASED MATERIALS

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ABSTRACT. Ever-increasing class sizes and decreasing contact times with students have meant that there is no time for extensive discussion of problem areas. I have tried to solve this by having all lecture material and computer tutorials on the web. Students are expected to read the relevant section before each lecture and the time is now spent in discussion of the material with students being able to participate and their questions leading in any direction that might be advantageous. Examples from computer tutorials are also integrated into the lecture/discussion times so that students can see immediate applications. This paper tries to describe and evaluate this approach.

1. Introduction. Twenty years ago I taught classes of some two hundred students with help from five permanent tutors and post-graduates and, of course, there was no computing component. Now I have classes of over seven hundred with no permanent tutors and help from part-time tutors, many of whom are second or third year students and there is a large computing component in the form of Maple and Matlab.

We always hear about "flexible delivery" with its usual meaning. Actually I think this method as it is often presented is, in fact, very inflexible, especially for mathematics and physics. If a student reads a theorem or definition on the computer screen and cannot understand it, as can often happen, they are helpless until help can be given from a tutor, if they should be present, or from an e-mail from their lecturer. So they just come to a halt - no amount of help options will cope with all the possible questions that need to be answered. Only a direct exchange of questions and answers with an experienced teacher will solve the student's problems (unless we all have access to totally interactive sessions on the web).

I worked out that if I had consultation hours from 8am to 6pm every working day of the week each of my students could see me for only **two minutes** each week and have **1 2/3 minutes** of tutorial help, so they were missing out on the very important opportunity of discussing difficulties face-to-face and having their problems solved. What to do about this deteriorating situation? This was my solution.

2. Method.

2.1. Importance of defining material for each course. Each course that I teach has three components - lectures, tutorials and computer lab sessions - but I have always found it was very difficult to define each of these in terms of references to text-books. Now I have the material for all my courses on the web [1] I have a definitive statement of the material. I believe this is very important for several reasons. We find, as many of you will, no doubt, have discovered, that text-books never fit the students' backgrounds - they are designed for American or British students who have different syllabi. So I always write my own lecture notes so that they fit exactly the level required. My own son has just finished Grade 12 so I know the school maths syllabus very well - and having taught first and second year I know exactly the background of our first, second and third year students. Most of the textbooks cover half of each of the courses I teach so that students often require two textbooks for one of our years.

I think it is important to have course material publicly displayed so that it sets a standard - I can remember when I did O and A levels in the U.K. that all students knew what was expected by what had been on the exams of the last few years. It didn't really matter if your own maths teacher left out some part of the material because you could easily verify that it was required by looking at the previous exams.

On the other hand in our Queensland State exam system we have moved to having exams for the final two years as internal school-based assessment so there is no body of state exams available for students to consult. Each year we have the Raybould Fellowship which allows a specially selected maths teacher from the State school system to come to our department for six months and I look after them during their stay. Through conversations with these teachers I have been amazed at the number of topics which have been either completely left out or only partially covered on the basis that the teacher could not really see why they were important. If there had been an equivalent body of past exams students themselves would have realized the omissions and demanded they were covered. Certainly there is a set syllabus but students never read them! - but they do practice questions from past exams.

2.2. Importance of discussing difficult concepts. On the positive side, topics that are covered at school allow considerable opportunities for discussion and sufficient questions and answers to satisfy the student.

Having my lectures notes on the web acting as a definitive statement of the required material for the course solves the problems mentioned above, but how to have the same discussion in a university course? Every time I mark exam scripts I can see just how much I have failed in really getting to the heart of some topics. The ones that always come up are: Taylor series, limits, infinite series, integration methods, vector spaces, eigenvectors, second order D E's with forcing terms, Laplace transform methods, Fourier series, multiple integrals - not quite the whole courses!

As I explained there is hardly any time for students to discuss their problems with me or their tutors. The solution seemed to be to create a discussion forum within the lecture. For this to work well it is important for the students to know exactly what will be discussed in the next lecture period and to read it and try to understand as much as possible of the material and to be prepared to ask and answer questions on it. The lecture notes on the web are all divided up into small sections with headings and all have references to the prescribed textbooks. In the lecture theatre I put up the relevant lecture material taken from the web on one overhead and keep a second overhead for comments. The difference between a purely web-based approach where the student is in front of a computer screen and mine is dramatic. Suppose we discussing a theorem, then as we go through each component of the theorem I ask students to explain exactly what it means. Problems always occur with "if and only if" situations. Many students confuse *pq* with *qp* and think they are interchangeable. Stopping and discussing examples about these logic situations when they first occur (writing them on the second overhead) saves time and misunderstanding later on when they re-occur.

2.3. Typical problems that need to be discussed. Students always have problems with what constitutes a proof. They do not realize that to prove that a result is true they must show that it is true for **all** values, but, to show it is false, they only need **one** counter-example. I am sure we have all seen proofs like this: to show $10n - 1 + 3n(n+1)(n+2)$ is divisible by 9 for all positive integers, the student tries $n=1$, giving $10-1+18 = 27$ so OK; for $n=2$, $100-1+72 = 171$, so OK. After some number of tries they give up and claim the result is true for **all** n! So it is very important to have time to discuss these concepts.

An excellent discussion opportunity occurs with considering one-one and onto (injective and surjective) situations. We can think about these in terms of geometrical, algebraic or calculus approaches. Let's suppose we want to show the function $f(x) = x^3$ is one-one, but $g(x) = x^2$ is not. First for the geometrical approach we draw the graphs of $f(x)$ and $g(x)$. Ask students what they can "see" from these graphs. Students will say that if you use the "horizontal line" rule that $f(x)$ is 1-1, but $g(x)$ is not. Is this a proof? For $g(x)$, if we verify that say $y = 1$ meets the graph where $x = 1$ **and** $x = -1$ we have found one counter-example - sufficient! For $f(x)$ it is a demonstration but not a proof since we can "see" but have not "proved" it is true for all points on the graph (opportunity for discussion on the third overhead of what constitutes a point on a graph).

Now for the algebraic approach: If $f(x_1) = f(x_2)$ then $x_1^3 = x_2^3$. Does this imply $x_1 = x_2$? Discussion of cube roots usually produces the outcome that students believe cube roots of negative numbers do not exist. So again they suggest trying various positive numbers (for safety!) to show it is true. When you finally persuade them it is as easy as $f(x_1) = f(x_2)$ implies $x_1^3 = x_2^3$ implies $x_1 = x_2$ **END**! it takes a while to sink in. For $g(x)$ our original method will work easily, e.g. $g(1) = 1$ and $g(-1) = 1$, but 1 is not equal to -1, so here we have a counter-example.

Lastly for the calculus approach. We need to show monotonicity or lack thereof. Now $f'(x) = 3x^2$. What can we say about this? Many students cannot deduce that this implies that $f'(x)$ is greater than or equal to 0 for all x. A good time to digress and talk about "positive definite", a concept which seems to have been neglected recently - use a third overhead or board (if available) to record these tangential results.

The next discussion point is: What does $f'(x) > 0$ imply? Again students have problems. More discussion on the connections between monotonic and the sign of the derivative. More examples on the third overhead. Finally we agree that $f(x)$ is monotonic for all x and therefore 1-1 and indeed we have a proof. For $g(x)$, $g'(x) = 2x$. What can we say about this? Again, students want to say that $g'(1) = 2$, $g'(2) = 4$, etc., and miss the only important fact that $g'(x)$ **changes sign** and so $g(x)$ is **not** monotonic and therefore not 1-1. Does the calculus approach constitute a proof? - yes, indeed, as the result for $f(x)$ is true for all x.

Get students to come up with many examples as possible of functions that are 1-1 and those that are not.

2.4. Different levels of information on different overheads. By now it should be clear to the reader that three levels of information are being developed, each on its appropriate overhead (or board) - firstly, reproduction of the web-based lectures and discussion of these; secondly, examples to illustrate that lecture material; thirdly, a mixed collection of examples and results that lead off tangentially from the main material. The student already has the web-based material in hand and only has to write down extra examples and results from the 2nd and 3rd sections corresponding to their particular needs and levels of knowledge.

2.5. Computing in lectures. As a department we have been using Maple for the symbolic parts (Mathematica would do as well) and Matlab for the numerical parts. I have developed a series of web-based tutorials for Maple which introduce the student to what Maple is and then to its use for arithmetic, algebra, calculus, graph sketching, Taylor series and their uses in approximations; linear algebra, including solutions of linear equations, Gaussian elimination, linear independence, matrices, eigenvalues and eigenvectors; ODE's, including exact solutions of separable, linear, exact and Bernoulli equations, second order linear with constant coefficients, including forcing terms and resonance; series solutions, Laplace transform solutions and numerical solutions using dsolve(numeric).

Matlab [2] introduces students to arithmetic, complex numbers, graphs, matrices and vectors, including solving systems of equations, polynomials, M-files and their uses, maxima and minima of functions, zero finding, integration using quad8 and ODE's using ODE23 and ODE45.

Again, it is important to have this body of material publicly displayed so the student knows exactly what is required for the computing part of their course and can use it later as a reference for further applications.

Each week the student does a computer lab tutorial where they learn some new aspect of Maple (first years) or Matlab (second years). Then, in the lecture, I demonstrate their newly learned techniques as applied to a relevant problem. An example might be: use Maple to solve the differential equation $mx'' + kx = F\cos(pt)$, $x(0) = a$, $x'(0) = b$, exactly and plot the solutions for suitable choices of the parameters and initial conditions as the frequency of the forcing term approaches the natural frequency of the system and then discuss and plot the resonant solution when they are equal.

For Matlab we might use ODE23 or ODE45 to demonstrate numerical methods for non-linear ODE's. and solve $mx'' + cx'|x'| + kx = mg$ for various representative initial conditions and values of the parameters and then plot the solutions.

Again on the main overhead I display the theory and then on the second we write in the appropriate steps as the students come up with the right commands. It is a good idea to try out the effect of "wrong" suggestions - the best way for students to learn is seeing the effect of their suggestions.

One method that doesn't work well is getting students to learn computer methods in the lab and then never see them used in the lectures. This just confirms beliefs held by many students that computer methods are a waste of time. It is often better to have no computer methods in the course - let the computer scientists or engineers do it all - rather than have them as a mere add-on. Best, of course, is to have them as an integrated part of the course and introduce something like this easy but insightful example to persuade those students who are still convinced that one can always solve problems **exactly** so who needs computers? There are three depots, one at $(0, a)$, one at $(0,-b)$ and one at $(c,0)$. Where should the central depot $(x,0)$ be located so that the total distance travelled from each depot to the main is least? The student sets up the total distance

$$L(x) = (a^2 + x^2)^{1/2} + (b^2 + x^2)^{1/2} + c - x$$

and finds its derivative

$$L(x) = x / (a^2 + x^2)^{1/2} + x / (b^2 + x^2)^{1/2} - 1$$

and tries to solve it for x. Maple can't do it but tells you it would be a root of the octic equation

$3x^8 + (4a^2 + 4b^2)x^6 + 6a^2 b^2 x^4 - a^4 b^4 = 0$, which the student can't solve. So you have to resort to some sort of numerical methods - get the student to admit they just can't do it exactly!

2.6. Continuity of discussion. Whichever form of discussion you are having - on lecture material, further examples or computer methods - you can continue it from one period to the next by asking your students to be ready to carry on the discussion or finish off the problem or have better ways of doing the present material. A good example of this is: spend some time finding the Maclaurin series for e^x and $\sin(x)$ and ask how to find the series for their product $e^x\sin(x)$ by the same brutal way. Stop at the end of the period and ask students to come back with faster ways - give a prize to those who consider $e^x . e^{ix}$.

2.7. The importance of different approaches. The great advantage of the discussion approach is the flexibility that it offers. I always like to present "bull in the china shop" approaches and "smart" ways of doing things. Good examples are: we are taking about determinants and we consider the Van der Monde determinant. First we take say the 3X3 case and evaluate it using the traditional expansion method. Then we talk about the factor theorem and see how we could do it almost immediately using this method and then use it to evaluate the general nXn case. For those students (and there seem to be many!) who haven't seen the result that if $f(x)$ is a polynomial and $f(a) = 0$ then $x - a$ is a factor of $f(x)$, we can follow this further with factorisation of other polynomials.

Second example: find the Taylor series for $f(x) = (x+2)/(2x+3)$ about $x = 2$. The "bull in the china shop" approach sees us writing down the general Taylor series, finding lots of derivatives of $f(x)$ and trying to guess the general form of the nth derivative and finally trying to put it all together (exhausted!). The "smart" way sees us setting $x = 2 + t$ and merely using the known Maclaurin series to write down the required series with its general term immediately. This leads to considering how useful it is to transform co-ordinate systems to make life easier (again, often left out) - like finding the partial fractions for say $(x^2+3x+11)/(x+2)$ and how to integrate for example
$(x + 1)/(x^2 + 4x + 13)$ by first completing the square. Once students see there are so many ways of doing problems, many of them so much faster and more elegant than their first lengthy approaches, they will hopefully get used to thinking first before rushing in like the proverbial bull.

3. Discussion. This new approach of having lectures as discussion sessions and the inclusion of computing as an integrated part of the presentation has proved very successful. Students come up after every session and say how much they have enjoyed the experience. You may well ask how large can the groups be so that it is successful. Well, I have used it with groups of four hundred students and see no reason why it could not work with any number of students provided they can see the material displayed and hear the lecturer.

Do students take part? The answer is, at first, no, but as soon as students realize they can only get their questions answered by actually asking questions themselves, however embarrassing it might be, they will soon join in. A good idea is to present a proposition and then get a straw vote on its validity or not and try to get each side to defend their viewpoint. If you can make this debate lively all students will benefit and enjoy it.

Apart from my own observations of the increased participation by students and the expression of interest from most students I can report that students did very well in their exams and computer assignments. Gone were many of the errors that students usually make and improved was the understanding shown in attempting harder questions.

4. Conclusion. With greater pressure on lecturing staff and the tutorial system something had to be changed. This method of transforming the traditional lecture into a discussion session with integrated computer methods has filled the gaps left by the declining availability of tutorials and has allowed students a real opportunity to participate in a true learning process. The other important factor is having the material on the web defining the course content and the computer tutorials. Having these available allows the flexibility required to conduct the course in this way and leads to students being able to expand their horizons and see many of the beauties of mathematics that they would have otherwise missed.

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THE DEVELOPMENT OF ON-LINE COURSES FOR BUSINESS MATHEMATICS AT THE HIGHER COLLEGES OF TECHNOLOGY, UAE.

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ABSTRACT. The United Arab Emirates (UAE) is a relatively young country and its educational development is making steady progress in bringing a high standard of learning to it nationals. The Higher Colleges of Technology (HCT) is a system of post-secondary colleges for nationals of the UAE and was established to provide students with vocational and technical education in Engineering, Business, Information Technology, Health Science and Communication Technology.

The rapid technological changes in education being brought about by the Internet are here to stay and we are informed that, in this 21st century, online learning will constitute fifty percent of all learning and education. The need for all educational institutions, including the HCT, to keep up with these ever-expanding changes is essential for their survival as legitimate centres of learning. The HCT is thus striving to become more innovative and flexible in meeting the learning needs of its students and to reach out to the UAE community. The development of on-line courses goes towards meeting these objectives and is being strongly encouraged and supported.

The authors are currently working on the development of two on-line courses in college-level business mathematics, which are to be offered system-wide on the HCT Intranet. They report on the design and objectives in particular relation to the needs of students working in an environment where they are making the transition from previous traditionally-based Arabic classroom experiences to the more student-centred HCT learning approach which uses the medium of English. The web design tools being used are WebEq, MathML, WebTex, Toolbook Instructor, MSExcel, MSWord and Frontpage.

1. Introduction. Students at the HCT are currently able to enter into four types of programs: Certificate, Diploma, Higher Diploma, and the Bachelor in Applied Science. The Certificate programs are two years in length. They introduce students to general and specific occupational skills and develop basic proficiency in English, computing, and mathematics. The Diploma requires a further year of study in which English proficiency is further developed and occupation-specific skills at the technician level are emphasised. Students following a Higher Diploma (HD) program are required to successfully complete a one-year Foundations course before being permitted to enrol in HD. The HD programs are three years in length and involve a combination of theoretical knowledge and practical applications at the technologist level.

All classes are delivered in English, and entry to programs is dependent both on high school performance and ability in English. All students are native Arabic speakers and are required to successfully complete all their courses in English. As entry standards to the HD programs are higher, students entering this program generally have a better command of English. For many students, being taught in English, working in a relatively new type of educational environment, and having to become accustomed to both male and female expatriate teachers, are additional barriers to be overcome. Teachers therefore have to be constantly looking for ways in which to assist students in making the transition from students' previous traditionally-based classroom experiences, where Arabic is main mode of instruction, to the more student-centred HCT learning approach using the medium of English.

Many of the learning difficulties in mathematics that we have identified in our students at HCT are closely related to limitations in the English language. This however manifests itself more at the Foundations and Certificate/Diploma levels [1]. While students at the HD level have less trouble in coping with English, teachers, nonetheless, have to be alert with their diction and phraseology. The language and terminology associated with a particular mathematical topic can be quite considerable and great care has to be taken to ensure that each of the related mathematical concepts is treated with a meaningful approach to students [2]. Sensitivity to the social and cultural background of students would bring further relevance and meaning to the students' learning.

To further enhance student success, the HCT revised its educational model in 1998 to reflect current trends in learning-centred education with it new learning paradigm. This has brought about a shift from providing instruction to producing learning, with the creation of independent learning centres (ILCs), custom designed labs for integrated learning and increasing use of technology and the internet. Laptops are now becoming quite commonplace in classrooms around the system, and laptop technology is being integrated into learning goals. Students are now able to link to the College Intranet and access transcripts, timetables, examination results and useful student information. Students also have mail accounts and are able to communicate with their peers and teachers.

At the Abu Dhabi Men's College, a Laptop Implementation Committee has been established to familiarise and assist teaching faculty with integrating laptops into student learning. The development of on-line modes of learning are being encouraged in order to provide students with more flexibility in their learning as well as in being in reach of the wider community. Under a Professional Development scheme, funds are available for staff who wish to explore ways of implementing technology in their teaching. The authors, while not experts in web design and on-line learning, have some experience in teaching and learning mathematics using multimedia [3], and have produced interactive CD-ROMS for students in the UAE [4][5]. The project reported here is on-going and in its early stages.

2. Aims and objectives. The project's aim is to produce two on-line courses for the Year 1 Higher Diploma Business Mathematics courses currently being taught at all of the HCT Colleges. The basic approach, from the standpoint of the authors, being relative newcomers to web and on-line course design, has been to try something fairly small, but effective, manageable and applicable to the "computer classroom". "On-line" is therefore interpreted here as "aspects of a course on the college intranet" which would

- provide development of concepts
- give students an opportunity to use interactive learning models
- allow classroom use where students and the course instructor would be able to interact
- allow students to review and reinforce their learning independently in self-paced study.

Our students in the UAE are surrounded by computers and technology. A growing number already own laptops and are using them with little or no anxiety. They are familiar with the internet and, in general, are quite comfortable with computers. They are eager to incorporate technology into their learning. While this may be so, students have yet to develop effective techniques for learning independently and on-line. Much teacher guidance and direction is still necessary. The intent of the authors, therefore, in developing these on-line courses, is to structure them such that they can be used a effective learning tools primarily in the classroom, making it possible for students to participate in a synchronous communication learning environment. The teacher's role of a provider of information, becomes more as one of a guide and facilitator of information [6]. The classroom interaction with the on-line components would further stimulate work in small groups and discussion [7].

There were several reasons for selecting a course at the Year 1 level in the HD Program. Students enter Year 1 on successful completion of the Foundation year and enter their chosen major. They would therefore be both good achievers and motivated. Additionally, in the first year of their HD studies, students start with the development of essential early concepts related to their major. This would form a good basis for an on-line course that could, at a later stage, be extended to treat the Year 2 material.

MATH 100 and MATH 150 are currently the two main Year 1 Business Mathematics courses, and these are offered right across the College system. With a large and wide student target, these two courses were selected for the on-line project. The site would also prove useful to students following other courses related to business mathematics, as well as provide a review those students in Year 2 Business students. These on-line courses in basic business mathematics, offered system-wide on the Intranet, would go towards meeting one of the stated objectives of the HCT in becoming more innovative and flexible in meeting the learning needs of it students. As both authors are involved in teaching these courses, and would be able to observe students, first hand, through integration of the on-line components in the classroom. This would provide evaluation as well as valuable feedback for further modification and improvement.

3. Structure and Design. The development of the MATH100 course is currently underway. In looking at the design of this course, the learning goals and terminal performance objectives have carefully been considered.

The goals, while contributing to the overall objectives of the course, are not necessarily hierarchical in the learning sequence, and can be approached fairly independently by the student. With the course textbook, in which the goals are linearly structured from beginning to end, students and teacher tend to work progressively from cover to cover. The on-line course will provide a less hierarchical and flexible approach, which may better suit the learning needs of the student [6][8]. The learning will be partitioned into modular "learning blocks" each consisting of a specific learning goal. This modular structure would enable both students and teachers to integrate all or a selection of learning blocks into their learning/ teaching as required.

Five overall goals have been identified for MATH100 to form the basis for the structure format. (See Figure 1, Appendix A). Their corresponding learning strands have been given the following simple names: Basic Concepts, Financial Mathematics, Linear Models, Break-even Analysis and Statistics. Students will start from an introductory title page with links to each Course Home Page; from here they can select the particular learning strand they wish to study. Each strand will allow the student to sequence through, ("through-sequencing") a selection of learning blocks according to his/her needs. While these blocks will have a hierarchical learning structure, the student is not confined to following this path and is able to control learning according to his/her needs and interests.

Particular attention will need to be given to those pages, which inevitably will require students to read several lines of text. Such pages would generally be introducing some new topic or concept and would involve simple definitions, explanations and examples. Not wishing to emulate a textbook on-line, the presentation of these pages will need to be "friendly" and "alive". They would need to use language that is both simple and meaningful in the local UAE context, with carefully chosen images and the use of interactive elements. They could further have hyperlinks to interactive and simulation models allowing students to investigate a given concept. The authors see these interactive models as being central to the learning process. Students will be able to investigate problems by varying parameters to explore different possibilities and interpret outcomes. It is hoped that these models will assist students in developing appropriate learning techniques and help to improve investigative thinking, stimulate the students' interest in the given topic and further motivate learning, and provide students with an opportunity to apply some of their classroom learning to simple real-world type modelling. Some of these models could be adapted for integration with other business and mathematics courses.

The course design will also enable students to communicate with the authors via email for any clarification or matters related to their learning. Students will also have access to a glossary at all stages of their study trail, for definitions of terms and meanings of difficult words. The home page also provides links to the course outline, a learning pacing schedule guide, a "How to Study" guide, a description of the course assessment, and a news page with any announcements.

4. Interactivity. As stated above, interactivity with the learning is central to the design. In addition to the interactive models, which are described further on, the authors have decided to use WebEQ [9] to create dynamic interactive equations to bring meaning to some of the symbolic forms used, and to develop step-by-step stages in working with some equations. WebEQ is a suite of software tools for putting mathematical expressions in Web pages, and enable the user to create interactive math applets. While the authors accept that in most situations, interactive equations are overkill and can be slow to execute, there are nonetheless instances where they can be of some value to the learning process. For example, the cumbersome-looking formula in Figure 2 (Appendix A) can be made less formidable by providing, at a glance, a brief description of each of its components. WebEQ allows the student to obtain this information by moving the mouse pointer over the appropriate component. When this is done, the component is highlighted and a description appears in the status line simultaneously. The WebEQ Equation Editor is quite similar to that found in MSWord and most equations and symbols can be managed.

The layout of equations, using the WebEQ Equation Editor, is based on MathML, a markup language, and can be imported directly into an HTML file. A Wizard program enables the processing of the MathML source. It is also possible to write equations by hand using the WebTex markup language which is a TeX-like input language for WebEQ. The interactive equation applets can be controlled by a wide variety of parameters.

The interactive learning models are being developed mostly with the use of Toolbook Instructor [10], a very powerful authoring package, which the authors have used extensively and is highly suitable for interactive learning. Toolbook uses a plug-in called Neuron which enables it to be used in a Web browser. Each Toolbook model is designed with a small number of interactive pages to minimise its download time. Figure 2 (Appendix A) shows an example of an interactive model to investigate the linear regression line. Students are able to add up to a maximum of twenty points, each with a mouseclick, anywhere on the grid. The line of best fit is shown, and varies with each additional point. At the same time, the slope, the y-intercept, and the equation of the line can be observed.

Further on-line student activities will involve links to Excel spreadsheets, MSWord documents and useful Internet sites.

5. Conclusion. The on-line MATH100 course is in now in its development stage and is expected be in a final working draft format for it operation and testing in the first semester of the 2001/2 academic year. In its initial implementation it will be integrated with the normal classroom sessions, with the teacher giving students direction and guidance. It will be possible to monitor the effectiveness of the on-line modules and observe student reaction and interaction. At Abu Dhabi Men's College, a good number of classrooms are equipped with computers and/or Smartboards that are on-line. Students are also able to connect their laptops to the Intranet. With further development and modifications, the next major step will be to have a fully-fledged on-line course, which will be open to the wider community of the United Arab Emirates. The follow-on course MATH150 will be created along the same lines and is to be put to trial in the second semester. On-line courses are relatively new to the United Arab Emirates, and as such, the onus for their development is being left to those educators who not only see it as desirable, but who also wish to keep pace with the technological changes which are influencing new pedagogical approaches. In the development of the two courses described here, the authors have taken an opportunity to further their own skills whilst helping the HCT to achieve its goals for technology in education.

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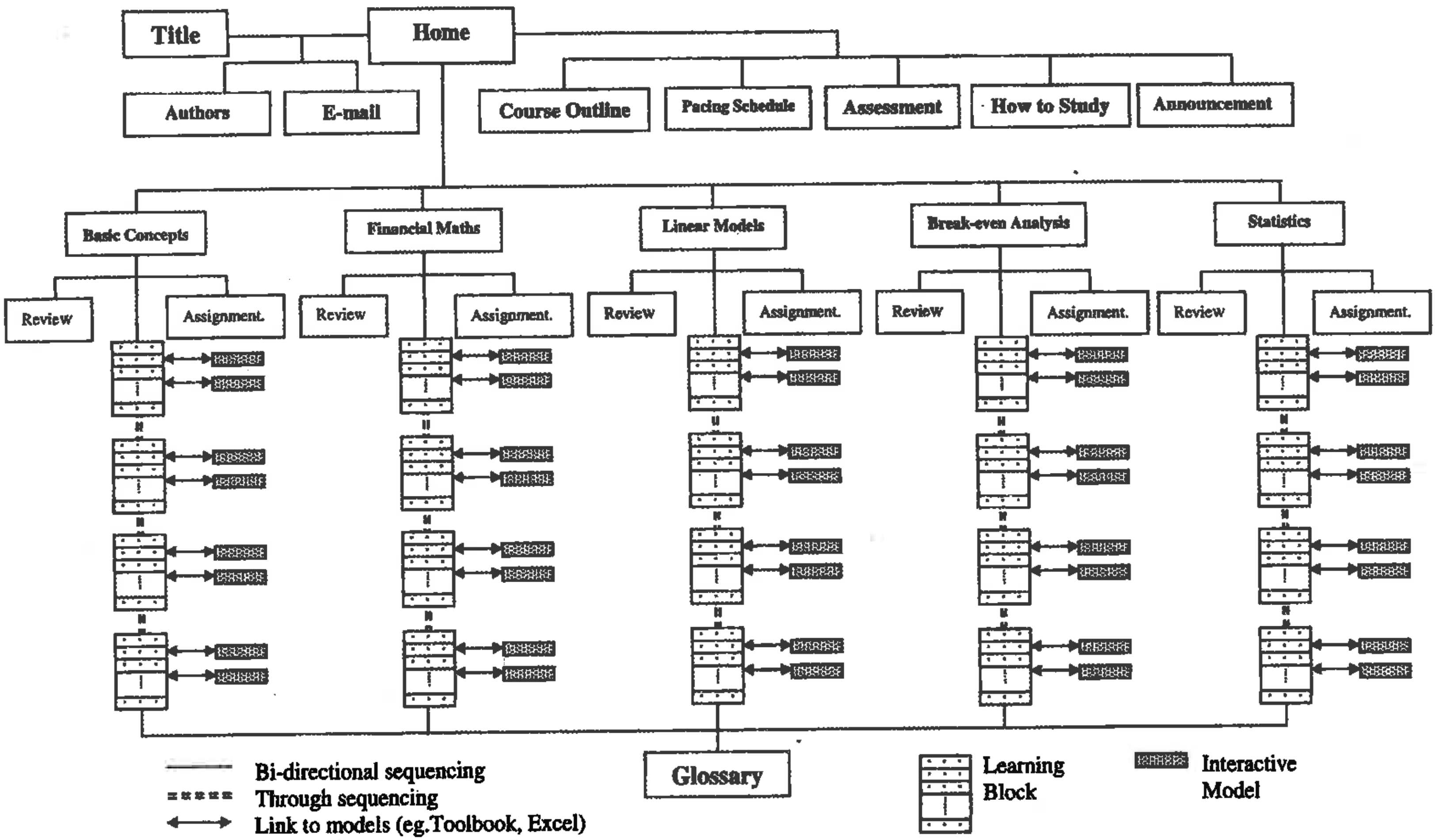

Fig. 1: On-line Study Sequencing Format

$$b = \frac{\left(\sum x^2\right)\left(\sum y\right) - \left(\sum xy\right)\left(\sum x\right)}{n\left(\sum x^2\right) - \left(\sum x\right)^2}$$

This is the sum of the 'squared values of x'

Fig. 2: Interactive equation

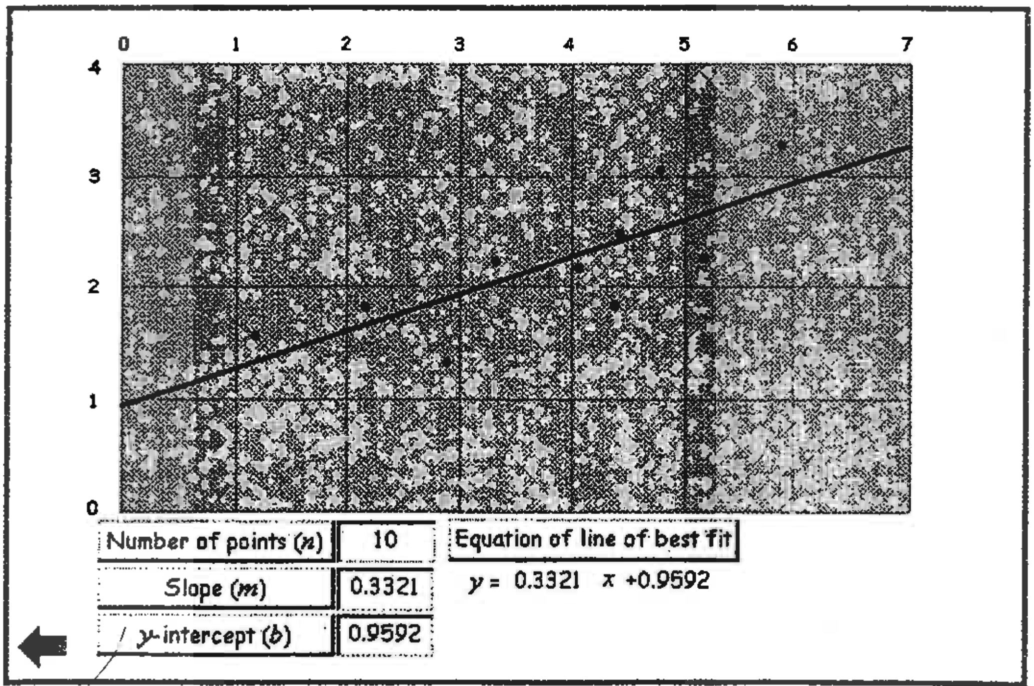

Fig. 3: Interactive Model

USING WEB-BASED RESOURCES TO TEACH TAYLOR SERIES

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ABSTRACT. An experiment was conducted to investigate the use of Web-based multimedia representations in the teaching of concepts of Taylor Series to undergraduate mathematics students. An extensive search uncovered two applets, one revealing step-by-step the formula for calculating Taylor polynomials, the other providing a graphical illustration of successive approximations of a function by polynomials.

The topic was introduced to the students in two ways: a traditional classroom explanation, and Web-based resources with a worksheet guide. Half of the students received the traditional teaching first, followed by the Web-based resources, while the remainder experienced the treatments in the reverse order. A short test was administered after each teaching session, and then again one week later.

While sample sizes (total n=10) were far too small to obtain statistically significant results, some useful trends could be seen. Both groups showed marked improvement on the second presentation of the concepts, no matter in which order they were presented. The best results however were shown when students first had their interest engaged by the applets.

A comparison was also made of the novice students' reactions to the applets with the way expert teachers viewed them. The paper concludes with some suggestions as to how mathematics educators might improve the instructional design of these multimedia representations and integrate them into traditional teaching to enhance both student motivation and understanding.

1. Introduction. In recent years there has been an increased availability of multimedia material for the teaching and learning of mathematics, as indeed for all subjects. The use of graphics, sound and animation has enabled new representations of concepts. Originally such programs were delivered on disk. Teachers had to plan for purchase and distribution before they could incorporate multimedia into classroom activities. Now the increased sophistication of Web browsers and the development of Java Applets (small programs which download from the Internet in a browser and run on the users' computer) have made the trial of such material much easier.

This ready availability of a new type of teaching resource brings challenges for mathematics educators. How should it be incorporated into existing methods? Do novice students have the same perception of the material as do expert teachers? What is the best way to present mathematical concepts using screen animations?

Research into the effects of multimedia technology on learning can be found across the broad disciplines of psychology, education, and computer science. Cognitive load theory (see for example Sweller [11], Mayer and Simms [4]) investigates the effect of combining presentation modes such as text, graphics, animation, and audio. Use of multimedia instruction for undergraduate students is the subject of numerous studies (Laurillard [3], Solomon [10], Watkins [13] are a few of many). The variety and contradictions in these results indicate a need to be more specific in the type of instruction, the subject matter and the age of the subjects when trying to generalise learning effects. Many investigations of the use of multimedia for mathematics are conducted on school-aged subjects (Goldman *et al.* [2], McCoy [5], Moreno and Mayer [6]). There are fewer specific investigations of the use of multimedia and Internet resources in undergraduate mathematics education. Szabo and Poohkay [12] have shown that use of animation can improve scores on a mathematics test in a study conducted on undergraduate education majors. Abrams and Haefner [1] argue for the inclusion of traditional classroom elements along with graphics, animation and text in an undergraduate mathematics course delivered over the Internet.

The study described in this paper is a pilot investigation of the use of Internet resources to complement classroom teaching. Instruction was limited to a one-hour session on one particular topic, with a small group of students. Observations are made on the design of available material as well as the effects on student learning. Some preliminary recommendations are made on how simple but well-designed animations could be incorporated into the traditional undergraduate mathematics classroom.

2. Methodology. The study was conducted with a class of ten undergraduate students studying introductory calculus. Most were prospective teachers, and did not have strong mathematics backgrounds. Taylor series was chosen as the topic for this experiment, as it was new to all of them. The researchers searched the Web for possible material, and selected two applets. Worksheets were prepared, to guide the students through the activity and prompt them to write answers to relevant questions at each step. One researcher was in the computer laboratory with the students to observe their use of the material. Assistance was limited to the mechanics of the programs, with no help being given on the mathematical concepts. The traditional classroom explanation had the lecturer introduce the motivation for the use of Taylor approximations, talk about matching successive derivatives at a point, and then give an example of the use of the formulae for a simple function.

The students were randomly allocated to two groups. One group received the traditional instruction first, followed by the Web exercise, while the other group experienced the treatments in the reverse order. Each period of instruction lasted half an hour. A short test was administered after each teaching session, then again one week later. On every occasion the test asked for written responses to two questions: "What is the purpose of finding a Taylor polynomial?", and "Describe how a Taylor polynomial might be constructed, for example for the function $f(x)=\cos(x)$ at $x=2$". Responses were scored out of a total of nine and converted to percentages on the basis of use of key words and concepts. The worksheets completed during the Web exercise were also scored out of ten on the basis of correct responses to the questions asking for interpretation of what was seen on the screen at each stage.

3. The applets. The two applets used were the best we could find in an extensive search of the Web. There appears to be less available on this topic than for other areas such as statistics (see for example Roberts and Pierce [8]). The applets chosen for this experiment displayed complementary aspects of the topic, one algebraic and the other graphical. The first encouraged the student to follow the steps of the theory by clicking on a "continue" button, which caused successive lines of algebra to fly onto the screen. Students could then step through examples in a similar way, and the worksheet guided them to investigate the polynomial approximation to $f(x)=\sin(x)$ around $x=0$.

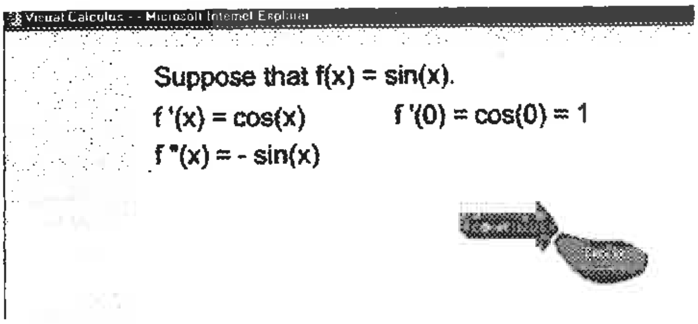

Figure 1: Screen dump of algebraic applet

The second applet showed a graph of $\sin(x)$ with the linear approximation at $x=0$. The user could change the order of the approximation, and thus visualise the effect of using higher order polynomials. The applet itself provides neither theory nor explanation so is obviously intended to be used in conjunction with other instruction. The worksheet we provided asked questions such as: "How far do you need to go to get a good approximation over the domain shown in the graph?" and "Do you think this approximation will be as good over a wider domain? Give reasons."

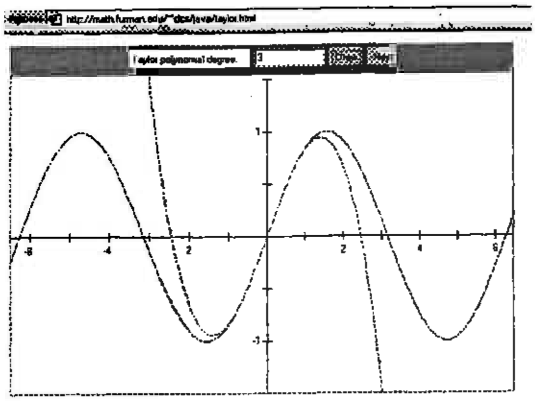

Figure 2: Screen dump of graphical applet

The URLs for these applets (both accessed 30 January 2001) are:

- http://archives.math.utk.edu/visual.calculus/6/power.3/index.html
- http://math.furman.edu/~dcs/java/taylor.html

Another extensive source of mathematics applets is

- http://www.ies.co.jp/math/java/

4. The results. Test results showed that very few of the students gained a reasonable understanding of the topic after this brief instruction period. Only 25% of subjects scored more than half marks on the final test. However some interesting trends were evident. All students improved their score from the first to the second presentation of the material. Most then showed a slight drop in the week to the final test. The group that had the Web-based instruction first, followed by the traditional classroom instruction, showed the greatest improvement from first to second test.

Table 1: Mean test scores by treatment group

	Test 1	Test 2	Test 3 (1 week later)
Web instruction first	10% (n=5)	43% (n=5)	38% (n=4)
Traditional first	13% (n=5)	31% (n=5)	26% (n=4)
Overall	12% (n=10)	37% (n=10)	32% (n=8)

Investigation of responses to the questions on the worksheet showed that students in both treatment groups had a greater understanding of the graphical applet than of the algebraic representation. Many students did not understand the algebraic steps that flew across the screen, and simply clicked on the "continue" icon repeatedly in order to get to the end of the exercise. On the other hand, most could see what was happening with the increasing order of approximation of the graphs.

Table 2: Mean Web worksheet scores by treatment group

	Algebraic applet	Graphical applet
Web instruction first	36% (n=5)	65% (n=5)
Traditional first	30% (n=5)	65% (n=5)
Overall	33% (n=10)	65% (n=10)

5. Discussion. The subjects in this study were keen to explore Web-based resources to assist their learning .As future mathematics teachers they felt they would need to incorporate new technologies into their own lesson plans. Their enthusiasm for learning from screen animations was not however matched by understanding. Particularly with the first applet, most did not have the algebraic insight to interpret what was on the screen in front of them. This applet essentially reproduced a textbook treatment. In a textbook, the many steps of a general derivation of the theory or of a particular example are set out sequentially on the page. Students must be taught to read mathematics by following the algebra one step at a time. Many students take one look at a page of symbols and give up. The value added by the animation of this applet is that the information will only appear on the screen in the correct sequence. The worksheet that accompanied the exercise in this study posed questions in an attempt to ensure the students thought about what they were seeing at each step. Our experience shows that more is needed. Perhaps some brief explanation, such as "differentiate the function again", "substitute for x", and "calculate the coefficient" could appear on the screen with the algebra.

The students found the graphical applet much easier to understand. They enjoyed being able to watch the approximation by increasing order polynomials become closer and closer to the sine function. The illustration was however limited by not showing the actual equation of the approximating polynomial. There was no way for students to appreciate how the box heading "Taylor polynomial order" at the top of the screen related to the formula and its derivation. An applet that incorporated aspects of both algebraic and graphical representations could be of great benefit to student understanding.

Our observations of the students as they worked at the screens gave cause to reflect on the different perceptions of novices as compared to experts. When selecting the applets, the researchers were happy with the algebraic treatment, and felt that it offered much more information that did the graphical applet. We did not foresee how difficult the students would find the interpretation of the symbolic representation. Many wrote an incorrect response to an early worksheet question, and then continued with further misconceptions as each subsequent step appeared. There is a need for some instructor intervention, either live or programmed, to correct misconceptions as they occur. The simpler graphical representation was much more popular with the novice students, as well as being better understood. Students were seated at individual computers, but in an environment where they could discuss their work. The algebraic applet was attempted in near silence, with students resolutely clicking through the steps. The graphical representation on the other hand brought comments of "hey, look at that!"

It is not surprising that results of the first test, after only half an hour's instruction, were so low. All students demonstrated greater understanding after the second presentation of the topic. Two lessons are better than one, no matter in which order the instructional methodologies are delivered. The most interesting indication from this study is the better performance by those who experienced the Web-based instruction first. These students had their interest aroused by the novel introduction. While their average score on the first test was slightly lower than that of the group that first experienced traditional teaching, they showed a greater improvement when they did receive the classroom explanation. We may speculate that this is because having been actively involved in "seeing" the concept illustrated on screen, they had a concrete experience with which to associate the theory. This finding is consistent with some of our earlier research (Roberts *et al.* [7]) on providing motivational examples before introducing the theory in the teaching of undergraduate statistics. Rogers [9] showed how self-directed learning activities for adults are best aimed at the solution of specific and immediate concrete problems.

6. Conclusions. This study was a preliminary investigation into the incorporation of Web-based materials into the teaching of undergraduate mathematics. It was restricted to a small sample of students and a to particular topic, Taylor series, taught in a single session. The aim was to discover the range of existing Internet resources and to trial their use in conjunction with traditional classroom instruction.

The range and quality of multimedia material available for the teaching of mathematics will continue to grow. Currently the resources freely available on the Internet for undergraduate calculus are somewhat limited. The applets chosen for this study appeared to the instructors to give interesting and adequate explanations. The students, however, did not find the animated screen presentations on their own to be sufficient. Future instructional designers should consider a mixture of graphical and algebraic representations. There should be some means of ensuring the student engages with the material on the screen, by either written responses to supplementary questions, or the inclusion of a greater level of interactivity than merely clicking a button to move to the next display.

The Internet on its own is unlikely to ever be an adequate replacement for the classroom teacher of undergraduate mathematics. The concepts of calculus are complex, and it would take a very sophisticated puter program to duplicate the benefits of immediate guidance and diagnostic feedback from a live tutor. wever there is certainly a place for the use of multimedia to extend and enrich the learning experience. The ults of this study suggest that the use of multimedia is best done before the lecture, to introduce concepts l motivate the students to discover more. The traditional explanation can then place the motivational ltimedia into context, and consolidate student learning.

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FACILITATED COOPERATIVE LEARNING IN FIRST-YEAR MATHEMATICS

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ABSTRACT. Cooperative Learning is an effective method of instruction. In approximately a decade and a half of classroom experimentation, the author has found that students, especially less prepared students, are more likely to succeed and to learn in the environment provided by group work and allowing active participation. However, many students have no experience in working in small groups. This can be addressed by providing a modest training in the skills required to function effectively in these groups.

1. Background. There have been periodic statements of dissatisfaction with the teaching of mathematics, as well as science and engineering, in the United States for some time now. "America's undergraduates - ***all*** of them - must attain a higher level of competence in science, mathematics, engineering and technology [1]". Much of the failure of students to perform satisfactorily has been assigned to the methods of instruction. As a consequence, research on alternative teaching strategies, such as active learning, cooperative learning, etc, has become of increasing interest. For the beginner, the sheer volume of material available is daunting. In this matter, the "teaching centers", or "centers of academic excellence" are useful sources of information and assistance.

Cooperative learning, see for example [2,3,4], is a structured form of group learning. It offers a variety of options. Group members can be self-selecting, can be assigned, can work together briefly, or for an entire term. Importantly however, cooperative learning presumes a level of commitment to the success of the group on the part of its members. Experience has shown that it cannot be assumed that all students, certainly not first-year students, possess the skills needed for successful group work.

2. Structure of the class. Each 50-minute class is divided into a presentation of new material (15 -25 minutes, depending in the material), a period for teamwork (20 minutes, or so), and a 5- minute daily quiz. This is discussed elsewhere [5,6]. The purpose of this organization is to transfer some of the responsibility of the investigation of new material from the instructor to the student, and to emphasis learning.

Part of the learning that occurs is that of the instructor about the class process. This type of "organizational learning" is an effective class improvement technique. [7]

3. The teams. Crucial to the operation of the class is the organization of the work groups that we call teams. Each student takes an "assessment" examination on the first day of class. This is a short test to sort the students by the ability to solve simple equations, etc. The teams are then organized heterogeneously with respect to these results. One high scorer, one low scorer plus one or two middle range scorers and placed into a team. This team, absent any difficulties, will stay together for the term. Each team elects a team leader, who attends a session outside of class for training. The leadership rotates within each team as the team is assigned a new project.

4. Facilitating cooperative learning. It is the participation of the team leaders in these out-of-class sessions that is the primary "facilitation" that is provided by the instructor. [8] There is additional "facilitation" that takes place within the classroom. This is the role that is played by the instructor as a roaming resource during the teamwork section of each class. During this time, the instructor responds to questions from individual team members whose teammates cannot answer the question and/or they were reluctant to ask during the opening session. This opportunity to guide students in an essentially one-to-one situation is quite important and useful to both the student and the instructor, as a check on the effectiveness of his remarks in conveying the information he intended.

5. Computer based projects. Projects intended to explain and/or elaborate the material presented in the class are assigned to each team on a weekly or bi-weekly basis. The team members collaborated to produce the

...t which was due one week after it was assigned. The projects used the Maple software program ...ped by the University of Waterloo, Canada. Although are a number of alternatives available, certainly ...*matica* is one, however convenience of use, and simplicity of programming favored Maple. The ...ted result from each project was outlined for the team leaders at the extra-class meeting.

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...PPENDIX A
...XHIBITS
...re are examples from two Maple projects to suggest how they can add to the classwork.

...ROJECT 1

...NTRODUCTION TO MAPLE
...OCAL LINEARITY

...OJECT DUE DATE: Feb 7
...RPOSE: The purpose of this project is to introduce you to the Maple software package, to the Computer ...b and to some of the ways that you can learn from plotting functions.
...NTS:

- You must ALWAYS indicate multication with a '*'. That is, *xy* is NOT understood as multiplication, you MUST type *x*y*.
- Powers are indicated by the caret '^'(or shift-6 key). So, x^2 is typed as 'x^2'.
- To indicate functions (with simple domains), we use the assignment (colon and the equal signs with no space between) ':=' and the minus sign and the 'greater than' sign '->'.
- To indicate the function $f(x) = x^2$, we type *f* := *x* -> *x*^2; Note the ; it signals the END of a statement and MUST be there.
- **NOTHING that you type is understood by the program until you hit the ENTER key! Remember this. If you change anything, you must RE-ENTER the new line! Otherwise the change has NO effect.**

The ENTER key will be indicated by '<>'.

COMMAND	COMMENT
>f := x -> x^2; <>	Type in the function $f(x) = x^2$.
> plot(f,-2..2); <>	DRAW the graph of $f(x)$ from $x = -2$ to $x = 2$. DO NOT PRINT

TO CHANGE THE DOMAIN OF THE GRAPH, USE THE UP-ARROW '' TO RETURN TO THE PLOT COMMAND AND THE LEFT AND RIGHT ARROWS '<-' '->' TO MOVE THE CURSOR (the blinking dash) TO THE NUMBERS, THEN BACKSPACE OVER THEM TO DELETE THEM, AND TYPE IN THE NEW NUMBERS. **(The arrows are located at the lower-right corner of the keyboard.)**

QUESTION 1. What happens to the shape of the function $f(x) = x^2$ if we change the plot domain to (0..2)? to (0.5..1.5)? to (0.9..1.1)? to (0.99..1.01)?

QUESTION 2. What is the slope of the line $y = 2x - 1$? (Remember: $y = mx + b$)

> line1 := x -> 2*x - 1; <> Define a function called line1.
(Caution: do NOT mistake the letter 'l' and the number '1'. They look the same!)

> Plot(line1,0..2); <> Draw the graph of line1. DO NOT PRINT

> Plot({f,line1},0..2); <>Draw TWO graphs together. Note the use of the BRACES { and }. This is necessary when plotting more than one function.

REPEAT THE LAST COMMAND CHANGING THE DOMAIN AS DONE IN QUESTION 1. **PRINT THESE PLOTS.**

A second project:
PROJECT 6

DEFINITE INTEGRAL AND THE AREA UNDER A CURVE.

PROJECT DUE DATE: May 5, 2000
PURPOSE: We want to see if it is possible to obtain an accurate estimate of the area **under a curve.** We **KNOW** how to find the area of some common geometric figures, so we shall use that knowledge. In particular, **we know how to find the area of a RECTANGLE. So we shall use different size rectangles and attempt to APPROXIMATE the area under a curve *(between the curve and the x-axis)*.**

COMMANDS:

>with(student):	This is the basic calculus subroutines library.
>n := 2;	A convenient parameter = no. of rectangles.
>f := x^2 + 1;	The first function we shall use is $f(x) = x^2 + 1$.
>leftbox(f,x=0..2,n);	This command will construct rectangles whose LEFT corner touches the curve $f(x)$. It will construct TWO rectangles from 0 to 2. PRINT THIS PLOT.
>left := leftsum($f(x)$,x=0..2,n);	The command calculates the AREAS of the rectangles. NOTICE the DIFFERENCE in this command, we need $f(x)$ in this command. Output is expressed as an algebraic sum. This is of NO immediate help to us, but we need it for the next command.

>evalf(left); Calculates the NUMERICAL answer.

QUESTION 1. Verify the calculation made by the "leftsum" command by hand. What value do you calculate? Does it match the leftsum command? (If not, you have a problem!)

>rightbox(*f*,*x*=0..2,*n*); Like the leftbox command, only here the RIGHT corner of the rectangle touches the curve. PRINT.

>right := rightsum(*f*(*x*),*x*=0..2,*n*); Calculate area of rectangles.

>evalf(right); Find the NUMERICAL value of the area.

QUESTION 2. Verify the calculation made by the "rightsum" command. What value do you calculate? Make sure it matches! Discuss the difference between the "leftsum" and the "rightsum". Why do they differ? Does one give better information than the other? Under what conditions might one prefer one to the other? (When might you want to over-estimate? When under-estimate?)

Now we are going to change the **NUMBER** of rectangles we are using.to do this simply "arrow" up to the *n* := 2; command and change the number 2 to 4 AND hit ENTER in each command below it, to change them.

QUESTION 3. Do the NUMERICAL VALUES of the leftsum and rightsum change? How?

REPEAT for 10 rectangles, also for 25 and 100 rectangles.

PRINT THE PLOT for 10 rectangles.

QUESTION 4. Do you believe that it is possible to use rectangles to obtain an accurate measure of the area between a curve *f*(x) and the *x*-axis? Compare your answer with the answer you get from the command below.

>int(*f*,*x*=0..2);

Designing Effective Learning Environments for Web-Delivery

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ABSTRACT. Throughout the world, people are creating both instruction and learning environments on the World Wide Web. These range from short episodes that are Web-based versions of traditional computer-based instruction or training (CBI or CBT) to months-long courses that offer university credit. Both types are proliferating at a great pace, and both have good and bad exemplars. The issue that this paper addresses is how to make Web-based learning effective, no matter what format it is.

1. Introduction. The single most important thing to remember is that if you are planning to create learning environments for the Web, you need to start the process at the beginning, even if you have lots of material from existing traditional courses. This does not mean that you will have to discard what you already have, only that you may have to discard any preconceived notions of how your existing assets will fit into the Web-based course. In this paper, I will describe a process that works for all types of Web environment and illustrate it with a case study from Capella University.

2. Learner goals and audience analysis. To begin at the beginning means that you have to define what you want your learners to learn, as well as understanding who your learners are and what environment they operate within. This latter part is a traditional audience analysis. (Many of the details of the process described in this paper are given in greater detail in Alessi and Trollip (2001), particularly in Part 3.)

In the case of Capella University, each course has its own content goals, which have to be established by the faculty teams responsible for curricular development. Currently, there are about 450 courses, so the range of content goals is very diverse. Although we would like to be in a position in which being on the Web does not define our content goals, the current constraints of the Web does impact what can be covered. For example, bandwidth considerations preclude many of our learners from being able to view streaming video or run Web-based simulations and other similar applications. This means we have to be very careful about selecting content that depends on such media. Fortunately, there are ways around these constraints in most situations.

Aside from issues surrounding content, there are other very important aspects of our audience that have to be taken into account. First, the average age of our graduate learners is nearly 45 years old. By definition, they also hold at least a Bachelor's degree. Second, most of them are working professionals. Third, most have families. Fourth, they live in all 50 states in the United States and in about 40 other countries. Fifth, about half of them access the Web through America OnLine (AOL) and a large percentage (perhaps 30%) have 28.8 kbs modems. Only 10% have high-speed connections greater than 56.6 kbs. Sixth, about 15% of the learners do not have a computer capable of running multimedia applications (most notably sound). Finally, all are taking courses for university graduate credit.

One other important item at the beginning of the process is determining the financial aspects of course delivery. Someone has to decide how much can be spent on each course. As with the audience, the range of course costs can be very broad. Some higher education institutions say that they are spending about a million US dollars per course. Others spend only a few thousand. How much can be invested in each course depends on how much money and other resources are available, such as personnel time, access to media production, availability of programmers, and so on.

2. Design. Once you have determined the audience attributes and goals and you know what your budget is, you have to decide how they impact your design process to build and deliver your Web-based courses. Each analysis will yield a different set of designs, so there is no one correct way. However, if you follow the *process* correctly, your outcomes are likely to be successful.

2.1. Audience attributes. Each of the audience attributes influences the decisions that have to be made in

...ing the learning environment to build to deliver the courses. For example, if your audience analysis ...l that most potential learners were Spanish speaking and only spoke English as a second language, the ... level of the materials you developed would likely have to be lower than for native English speakers. ...any in your target audience were poor typists, you may have to leave out exercises that required ...ed input. Or if your learners did not have ready access to computers, you may have to decide not to use ...livery in favour of some other means to which all had access. Let us now look at the implications of ...f Capella's audience attributes defined above.

...**Average age is 45; holds a Bachelor's degree**. These two attributes have profound implications, the ...mportant one of which is that the learners are adults and will want to be treated as such. They are not ...ers who are happy to be told what to do and learn. Typically, adult learners know what they want and ...ot tolerate being treated as children.

...om an instructional perspective, this likely means that they will not be happy being lectured to. That is, ...ttempt to transform traditional classroom lectures into a Web format is likely to be unsuccessful. For ...e faculty members, this is a difficult transition because they are used to being the "fount of knowledge" or ...ge on the stage." In this learning environment with these learners, the instructor role has to be more of a ...litator (or "guide on the side"), guiding the learners through the course. For some, this is a loss of ...tige, and they feel their importance is diminished.

Adult learners also place a great deal of importance on relevance. They are less tolerant of having to spend ...e on activities or courses that do not help them directly. From a design perspective, this means that most ...vities in a course must result in knowledge or skills that the learners can take back to their workplace as ...ckly as possible.

One other item from adult learning theory that has to be considered in the course design is convenience. ...dults are typically busy and greatly appreciate every effort to make their lives less complicated. Web-based ...arning environments are intrinsically convenient, so this is almost a built-in benefit.

...1.2. **Working professionals**. The fact that Capella learners are working professionals means that the course design has to ensure content relevance, as mentioned above. In addition, because the learners have a great deal of experience it is important for them to feel valued. That is, they want to contribute their experiences and share what they have learned on the job with the other learners. This means that the course design has to accommodate this sharing.

One other implications of the learners being working professionals is that they do not want, if possible, to have to drive through heavy traffic at the end of the day in order to attend classes. Not only are they tired, but they prefer to go home. Once again, the issue of convenience plays a big role.

2.1.3. Learners have families. The implication of this is simple. Learners would prefer to go home after work so they can spend time with their families. They also do not want to spend much time on weekends away from home.

2.1.4. Learners are scattered all over the world. There are several implications of having learners all over the world. First, because of the difference in time zones, any synchronous activities are going to pose difficulties for some learners. Therefore, it is better from a design perspective to have largely asynchronous courses.

Second, one of the most important design issues in this situation is to build courses that take the distance out of distance education. That is, the course design must encourage community, collaboration, cooperation, and closeness. As soon as learners who are at a distance feel that they are isolated, they are more likely to drop out of a course. So every effort must be made to make learners feel wanted. (In reality, good faculty practices also play a very important role in minimizing distance.)

2.1.5. AOL access and slow modems. Both of these issues mean that many learners have little bandwidth with which to play streaming media, download large files, and so on. From a design perspective, this means that courses need to be largely text-based, except that media-rich pieces may have to be sent to learners via CD-ROM or videotape - both of which add to the logistical complexities of running a course.

2.1.6. 15% of learners cannot play sound. If you accept this state of affairs, your design will have to leave out any multimedia. At Capella, we decided that this was too great a constraint and required all learners to have multimedia computers.

2.1.7. Graduate credit. Since all courses are being taken for graduate credit implies that learners have to both put in the amount of time typically expected of a graduate course, and be able to demonstrate that they can perform at a graduate level.

2.2. Finalizing the design. As can be seen from the brief discussion above of the major attributes of the target population, very specific design decisions have to be made in order to accommodate the preferences and styles of the learners. Sometimes, specific attributes may demand contradictory design features. For example, synchronous activities, such as chat rooms, certainly help break down the barriers of distance. However, they are also very inconvenient. As a designer, you have to make a choice between these opposites by determining which offers the greater benefit for learners.

It is also at the design phase that you need to start factoring in the budgetary issues. The type of course you would develop if you had $500 000 for it would be very different from the one for which you had $50 000, and different again from the one developed for $5 000. (It is important to note that the whole budget and project management processes are extremely important, but will not be covered in any detail here. Alessi and Trollip (2001) has extensive coverage of both.)

In the case of Capella University graduate courses, the following design was decided upon and implemented.

First, for convenience, courses are all Web-based and built upon an asynchronous model. That is, within reasonable constraints, such as weekly units and a 12-week course, learners can make their contributions at their own convenience.

Second, to accommodate slow bandwidth, courses are largely text-based and rely minimally on media other than graphics. Although streaming audio is acceptable, learners are adamant that they are not interested in listening unless the content contributes substantively to the course. They do not want audio for audio's sake.

Third, in order to draw upon the rich experience brought to the course by the working professionals who are the learners, the core of the course is an asynchronous threaded discussion group that encourages learners to both share what they have learned over the years and to help each other. This has been extremely successful, and some learners form virtual networks that carry on long after the course is finished.

Fourth, to build community and to reduce distance, every learner is *required* to post a substantial response to two or three questions posed by the instructor each week. In addition, each learner must respond to the postings of at least two other learners. This creates the beginnings of a dialog that often blossoms into a set of interactions that exceed those found in most traditional classrooms. With a dozen to fifteen learners in a class, I have often had 150 - 175 postings per week in the discussion group. Although some are short, such as "Good idea" or "That is interesting," most contributions have substance. We are currently experimenting with a slight change to this process, whereby instead of just responding to two postings, learners must now also ask a question about the content. We think this will improve learning by requiring deeper processing, as well as improve the dialog and interaction.

One other design feature that we are beginning to build into courses is mandatory collaborative projects. Not only does this help build community, but also provides invaluable practice for learners for their workplaces. More and more, we are told by employers that virtual teambuilding skills are very valuable.

Fifth, to provide relevance, we encourage learners to focus most projects on issues at work. This both helps them articulate issues that are important to them, and often results in them getting feedback and advice from other learners who have experienced similar situations.

Sixth, Capella courses typically require learners to spend about 8 to 12 hours per week, reading, responding, or working on projects. These projects form the second major basis for evaluating learners (the first being participation in the discussion group).

Seventh, and this is not a design issue but rather an implementation item, the success of even an well-designed course will depend to a large extent on the quality of the instructor. Online learners demand that their instructors are present - that is, they must be participating in the course at all times. It is usually not good enough for an instructor to make a few comments here and there once or twice a week. Adult learners will generally not tolerate this and may well drop out (or contact the university administration to complain).

At Capella University, all potential instructors are required to be learners in a faculty development course (online, of course) so that they may learn the techniques of being an online instructor and experience being a learner in the same environment. Such faculty development is essential for successful online courses, as we have found to our dismay when untrained instructors have tried to go online, often with poor results.

3. **Conclusion.** The discussion above is intended to highlight the process of designing an effective Web-based learning environment. The Capella example serves only to show how the process is applied. The Capella model may not work if any of the underlying audience attributes are different, if the instructional goals vary, or if the budget is not the same.

I am confident, however, that the process - a typical instructional design process - would work if the goal were an instructorless, self-standing piece of instruction that was to last an hour or so. It would also work if the Web-based environment were an adjunct to a traditional classroom. What is important is that a clear process is followed that is built around a clear understanding of the target audience and the intended instructional goals.

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The book below contains many references pertinent to the discussion of this paper.

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INTERACTIVE MATHEMATICS IN THE LAPTOP UNIVERSITY

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ABSTRACT. This paper describes a paradigm shift from the rote-learning environment of high school to a computer assisted approach in a Precalculus course using real world situations for groups of women students in a non-English speaking country.

Students in the UAE are taught mathematics in schools in a strictly traditional way and nearly always in Arabic. When they enter Zayed University the only thing they expect to be different is the language in which the subject is conducted. However, their differing mathematical backgrounds, varied exit goals, the constraints of a condensed schedule and pedagogical considerations make traditional methods unfeasible and undesirable.

On entry to the course there is an initial period when students review the rules of algebraic manipulation and graphing functions on paper. Nearly all work after that is computer-based using Maple on the students' own laptops. Problems are mainly word-based and are taken from the areas of Business, Ecology, Finance, Information Science and the Sciences. The students' work is written on Maple worksheets, saved in their own folders and copies sent to the tutor for assessment when requested.

1. Introduction. High school mathematics teaching in the UAE is largely instrumentalist in nature. The instrumentalist teacher views mathematics as "an accumulation of facts, rules and skills to be used in pursuance of some external end. Thus mathematics is a set of unrelated but utilitarian rules and facts" (Ernest 1989). Students are expected to achieve mastery of these mathematical rules and techniques. The teacher is the instructor and judges success or failure of the students by their ability to demonstrate skill in manipulation and rote learning. No relevance or reference to the 'real world' or the nature of mathematics is expected. The aim of the author's Precalculus course at the Abu Dhabi Campus of Zayed University is to move from the traditional rote-learning model to the teacher-as-facilitator model with the students becoming problem-solvers and engaging in the process of mathematical modelling of 'real world' situations using a Computer Algebra System to investigate the models. In this context it is important that students attempt to interpret results and communicate ideas in written and oral form. improving critical thinking skills. Since, for the majority of students, this courses will be their last mathematics course it is felt important that students acquire some appreciation of the role mathematics plays in the modern world and that the course should also have some relevance to their further studies, whether Business, Finance, Information Systems or the Sciences. Last but not least it is hoped that students will enjoy the mathematics they encounter.

2. Background. Zayed is a new university for UAE National women and is the first of its kind in the Middle East. One of the University's missions is to ensure that all our students are capable of using the power of the computer to maximum effect so every classroom is connected to the University network and the Web. Every student has a laptop and during the first two years students become acquainted with Microsoft Word, Excel, Access, Web design packages and a variety of software useful in their English Language studies. In addition, several mathematics tutors are using Maple to enhance the mathematical experience of students.

This paper describes a Precalculus course taken in the Fall 2000 Semester by two classes (38 students). The aims of the course are:

- to increase the students mathematical knowledge at precalculus level
- to introduce students to mathematical modelling and the role of mathematics in the real world
- to improve students problem-solving skills
- to introduce students to a Computer Algebra System
- to engender critical thinking skills through interpretation and appraisal of results of investigations
- to improve students' attitude towards mathematics.

Maple has been chosen because of:

- its ability to allow students with limited mastery of the basic rules of algebra and techniques to still solve problems in mathematics;

- its power to display 'live' graphical results and permit 'what-if' exploration;
- its command line structure which benefits students (the majority) who wish to study Information Sciences for their majors;
- its ability to incorporate text and spreadsheets into the mathematics worksheets thereby making it possible for students to produce reports and communicate results of their investigations in English.

3. Content and Teaching Method. At the beginning of the course some time is allocated to reviewing basic algebra and graphs using pen and paper. It is considered necessary for students to appreciate the rules and techniques of algebra and the concept of a function but it does not follow that students need to be experts at complicated algebraic manipulations. Similarly, students should have an appreciation of graphical representations used in the study of functions but again a broad understanding of the graphs of standard functions is possible without the more intricate techniques involved in hand-sketching these curves. This is followed by an introduction to Maple showing how a computer algebra system deals with the same algebra and graphs. After this period most of the work is computer based. The students study elementary mathematical models arising from a variety of applications, which take the student from the idea of a function to the beginnings of The Calculus. Periodically, there will be sessions concerned with synthesis of mathematical knowledge acquired so far.

The students copy a set of folders and sub-folders from the shared drive on the University's network onto their local drive and it is then their responsibility to maintain, update and use the folders as a resource. Course work, assignments and additional notes and information are sent to students either by e-mail or by placing files on the shared drive. Communication between tutor and student is via email, shared drive and by personal contact.

The course has the following five main themes.

- Introduction to Maple
- The Function Concept
- Mathematical Functions in the Real World
- Rate of Change
- The Idea of a Limit

The approach to this course is almost entirely by way of modelling and the usual topics such as functions, graphs, trigonometry (periodic functions), etc., are introduced and developed through models taken from a range of applications. A case study is presented to the class, usually posted electronically, and discussed in class. The students then use Maple worksheets to answer the questions posed and also include interpretation of findings within the worksheet. This is saved in the relevant folder on the students' local drive and e-mailed to the tutor for assessment if requested. Collaboration and discussion between students is encouraged.

The cases start from an elementary standpoint, introducing students to the function which models the situation and the cases then develop through time towards a meaningful interpretation of rates of change and an informal introduction to limits. Problems are revisited periodically, each time adding some new idea to existing ones. For example, the logistic function is introduced to model an ecological problem and over time revisited to involve students in solving inverse problems, the rates of change and introducing the idea of a limiting value.

Assignments are sent electronically to the students and returned to the tutor by the same methods for assessment when requested. Feedback is provided through class discussion, e-mail correspondence, individual consultations and marked assignments. At the moment the final assessment is a combination of assignment marks, short tests, midterm exam and final exam but this may change for the next semester's classes.

6. Summary. The approach to this course is similar to many courses being introduced under the calculus reform initiative and the author is grateful to all those involved in the initiative for the amount of literature which exists to help teachers who wish to adopt the new paradigm (For example, Gordon, 1998).

The challenge in this particular situation is that of breaking with a deeply embedded tradition of rote learning. Students on the course initially want to know what they will have to memorise for the exam and it takes time for them to realise that this is a very different course from the ones they have been used to.

A second feature of this course is that the students are working on modelling problems in, what is for them, a second language. In fact the students are tackling three language areas in one course: English, Mathematics and Maple, as well as learning new concepts and a new learning approach. The problems that students experience are often linked with the languages rather than the mathematical concepts.

7. Future Plans. It is planned to continue the work started this semester, concentrating on defining a suitable research method to investigate the effectiveness of this approach. The model suggested by Bansenaur *et al.* (Balas, Bansenaur, Clay, *int. al.*, 1998) after Wright *et al.*, 1997, would seem an appropriate one at this point in time; students on the 'new' Precalculus course and also students on a more traditional one would be interviewed by faculty from other departments and students ranked according to the interviewer's perception of the student's competence.

The next semester will see an expansion of the number of students taking this course.

8. Concluding Remarks. The results of informal interviews conducted during and at the end of the semester have been encouraging. Despite the tradition of rote learning and the mechanistic nature of earlier examination formats the students on the course have appreciated the benefits of a course which has some relevance to their future studies and which broadens their idea of what mathematics is. The use of Maple has sharpened the computer skills of those going on to study Information Science by illustrating the need for strict adherence to syntax. And finally, and very satisfyingly, many students have said that they have enjoyed the course and now like mathematics.

This is work in progress but it is hoped that this opportunity of working with students with unlimited access to computer time will provide useful information about the effect of the computer on teaching and learning.

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Targeted Intervention: An Applied Mathematics course for Geology and Chemistry majors

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ABSTRACT. Because of the need to better analyse and interpret experimental data and the increased use of mathematical tools in geology and chemistry textbooks, it is becoming necessary to teach students quantitative skills beyond the scope of first year mathematics courses. This paper describes a one semester second year applied mathematics course, designed mainly for chemistry and geology students with three specific objectives: to develop students' ability to quantitatively analyse problems arising in their own field, to illustrate the great utility of mathematical models to provide answers to key chemistry and geology problems, to develop students' appreciation of the diversity of mathematical approaches potentially useful in the chemical and geological sciences. The course was divided roughly into two parts; statistical theory and methods, and applied mathematics tools and modelling. Three projects were given to illustrate the use of computer packages in solving a given problem, to develop a web page and to write an open-ended essay and publish it on their web page. It was found that it is essential to integrate the course with students' existing major courses and also to involve faculty from the respective departments. Another goal was to motivate students to continue with more statistics and mathematics courses by inviting people to demonstrate further applications of quantitative skills in industry. The paper analyses the successes and shortcomings of the new course.

1. Introduction. Despite the fact that quantitative methods and theoretical developments have become central to modern science, the quantitative training of undergraduate chemistry and geology students is generally considered insufficient to support the content found in some standard geology and chemistry textbooks (Boggs, 1995; Atkins, 1998) and used in laboratory sessions (Macdonald and Bailey, 2000). In the struggle to act on this insight, one typical reflex from chemistry and geology departments has been to add number-crunching examples and exercises as adjuncts to existing exercises. This approach has certainly not been successful and departments interested in building the quantitative skills of their students have recently taken various courses of action; some departments have developed a sequence of courses in which quantitative skills are developed (Paola *et al.* 1995), some departments offer a specific course that focuses on mathematical approaches to problem solving in geology and chemistry (for example see Vacher, 2000), some departments teach a shadow course taken concurrently with a calculus course (for example see Lutz and Srogi, 2000), and some departments focus on student projects throughout the curriculum (for example see Keller *et al*, 2000).

The chemistry and geology departments at the University of Cape Town requested that a second year one semester applied mathematics course be introduced in 2000 to improve the mathematical skills of their students. Their request was extended by including both statistical methods and theory and mathematics applied directly to solving problems in the chemical and geological sciences. Chemistry major students at the University of Cape Town are required to take two semesters of mathematics, either as a single course or as two one semester courses (with the first one semester course taken in their first year and a second one semester course taken sometime in subsequent years of their degree). Geology students only require a single semester first year mathematics course but neither chemistry nor geology students require any statistics. This paper has two objectives: 1) to describe the content included in the new course that integrates both statistical and mathematical skills and 2) to report some observations of the use of this approach.

2. Need for a new course. There is a mismatch between previous quantitative skills learned and the quantitative skills required, mainly because of the lack of both statistical methods (and theory) and mathematical tools applied directly to solving problems in the geological and chemical sciences. For example, error/uncertainty analysis and basic data analysis is usually "taught" by handing out a couple of pages of notes containing a few formulae, and the student is then expected to use the formulae as part of their laboratory sessions. Students do not get to learn the material and it becomes a meaningless ritual, in which students add a few lines of calculation at the end of each laboratory report. However, students are also expected to analyse and interpret the results of their experiments. Also, several multivariable techniques are used in the standard textbooks, which students have not encountered.

There is empirical support that shows that students do not naturally generalise what they have learnt to new situations (for example, out-of-class situations where students may not even consider applying their relevant quantitative tools to chemistry or geology problems). A simple example may be students' inability to identify the difference between a variable and a constant in an equation from their chemistry or geology courses.

There is also a need to reduce the perception of science students that mathematics is only needed for mathematics courses, to enhance students' appreciation for the importance of theory in the scientific process and to encourage students to take more mathematics and statistics courses. Encouraging students to take more quantitative courses and the attempt to reduce the perception that mathematics is only needed for mathematics courses is essential because of the increased use of quantitative tools in geology and chemistry journal articles and the increased establishment of new interdisciplinary graduate programmes in, for example, computational chemistry and geostatistics. A recent report by the National Science Foundation of the U.S.A. (2000) highlighted this general need:

"... it is obvious... that students of mathematics should be able to understand problems in science, and that students of science should understand the power and roles of mathematics. Each areas of science has its own unique features, but the different areas share common features that are often of a mathematical nature".

The design of the course was motivated by the specifications of four fundamental course objectives: 1) to develop students' ability to quantitatively analyse problems arising in their own field, 2) to illustrate the great utility of mathematical models to provide answers to key chemistry and geological problems and 3) to develop students' appreciation of the diversity of mathematical approaches potentially useful in the chemical and geological sciences.

3. Mathematics, statistics and computing included. There were three immediate implications for the design of the course: 1) to know what quantitative skills were being expected of students in textbooks and lectures, 2) to have a readily available set of "real-world" data and applied examples and 3) to learn or know about subjects outside of mathematics. To gather examples and to know what quantitative skills were required, several meetings with faculty members from both chemistry and geology departments were arranged. To gather relevant data, various faculty were consulted but most data and examples were taken from journal articles.

Though mathematicians often rely on applied context as a source of problems, the ultimate focus in mathematical thinking is on abstract patterns, where context obscures structure. Data analysis also involves the search for patterns, but whether those patterns have meaning and value depends on how they are linked to the context (for examples see Cobb, 1997). This implies that to teach the statistics component of the course well, it was not enough to understand the mathematical or non-mathematical theory of statistics but to have a ready supply of "real-world" data and examples available and to know how to use them to involve the students' critical judgement. The typical stages in statistical analysis (Figure 1) suggest a strict left-to-right progression, but in reality, the data analysis process is neither linear nor unidirectional.

Figure 1: **A scheme representing the phases of data production and analysis** (Cobb, 1997).

I adopted the suggestion by Cobb (1997) that statistical analysis should begin with exploratory data analysis (EDA), that is, methods for exploring and describing the data. This is because the basic methods of EDA are conceptually and algorithmically simple and because it is not required to distinguish between population and sample, or to discuss the features of randomisation that protect against bias. Students enjoyed exploratory data analysis and it set a good foundation for the design and analysis of experiments. Given an exercise, students were required to plot the data using STATISTICA and then to describe the shape of the distribution of the data, specifically looking at variations in modality, skewness and kurtosis, and then to examine the spread and symmetries using a boxplot.

The mathematics taught in the course cannot emphasise the search for abstract patterns and must be applied directly to solving problems in the geological and chemical sciences. The approach adopted in the course was to encourage students to learn the mathematical theory and then to apply the theory to a variety of problems in the sciences. One intervention that has worked is giving students multiple problems (from different areas in the sciences) that have related solution structure but that appear different. Integrating new knowledge with existing knowledge means helping students create appropriate links between "pieces" of knowledge. This applies both to integrating what students will learn in the course with what they have been taught in chemistry and geology (see example 1 below), and structuring instructional methods (lectures, tutorials etc.) appropriately (discussed in the next section).

Example 1: Van der Waal's gas equation. The assumptions of the ideal gas equation (PV = constant) are, under certain conditions, incorrect. It is not difficult for students to realise that at high pressures and low volumes it can no longer be assumed that molecules have zero volume or that no interactive forces exist between them. Figure 2, plot (a), shows what the ideal gas equation predicts, and indeed what occurs at high temperatures for many gases.

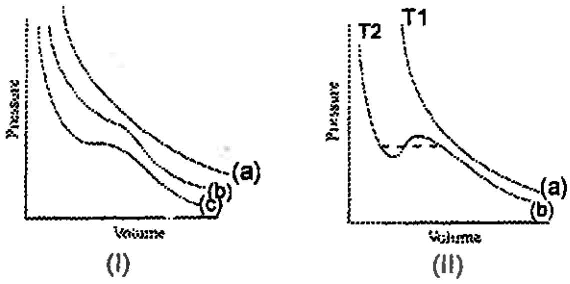

Figure 2

(I) Pressure-volume isotherms for real gases (a) above the critical temperature, (b) at the critical temperature and (c) below the critical temperature. (II) Isotherms predicted by the van der Waal's equation (a) above the critical temperature and (b) below the critical temperature. At the critical temperature an inflection is predicted.

At low temperatures, the isotherms show a short horizontal portion and students generally explain this behaviour correctly: As the volume is decreased, the pressure exerted by the gas increases and when the pressure reaches a certain value the gas begins to condense to a liquid. On further decrease of the volume, the pressure remains constant as the gas liquefies, until the gas is totally liquefied. Because liquids are very difficult to compress, the pressure rises rapidly. At a critical temperature (T_c), the horizontal part appears as an inflection of the graph. At T_c the pressure and volume at this inflection point are called the critical pressure

(P_c) and critical volume (V_c) respectively. Under these conditions (P_c, T_c, V_c), the gas and the liquid may be thought of as a single phase and above this temperature the gas cannot be liquefied just by compressing it. The question then posed to students is whether this behaviour can be predicted by the van der Waal's equation (equation 1) and Figure 2(II) was given to them.

$$P = \frac{RT}{(V-b)} - \frac{a}{V^2} \qquad (1)$$

Students were asked to interpret the graph (which does not explain reality because the local minimum implies that the pressure decreases as the volume decreases) and then to calculate the conditions which represents the critical point. Students take a long time to realise that both the first and second derivative is zero at the point of inflection in this case. Identifying that the first derivative is zero at the point of inflection required them to directly link the chemistry concept and knowledge with a quantitative skill. They then easily progressed to solve the resulting two equations simultaneously to get the expressions for the critical constants in terms of the constants a and b.

Technology has played a major role in instruction innovations. The use of computers in the course reinforces the need for an appreciation of the diversity of mathematics tools that are applied to various areas. More than that, the use of computers actually makes it feasible to expose science students to a breadth of topics, both within other quantitative courses and within courses in their programmes. The use of computers and modelling increases the need for mathematics although many think the opposite! Because many "real-world" problems do not lend themselves to easy pen-and-paper solutions, the ability to use computers to visualize and analyse solutions is critical. No matter which way a solution is obtained, one's intuition as to whether the result makes sense is important, and doing this effectively may even require some pen-and-paper ability. Students in the course used a variety of computer packages in their projects (see section 4). However, it would be useful in the future to include the use of these and other computer packages as part of classroom examples and tutorial exercises as well.

4. Course structure and methods. The components of the course shown in Figure 3 were designed and integrated to teach quantitative skills using several reform-based instructional techniques– collaborative learning, active learning and the use of computers to aid students in the practice of quantitative skills.

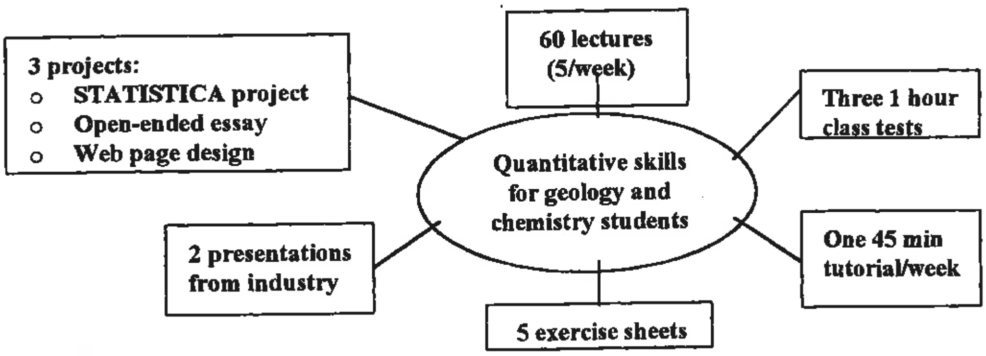

Figure 3: Components of the MAM 251F course

Table 1 below provides a list of some reform-based techniques and brief descriptions of how they were used. This list is not meant to be extensive but rather to demonstrate the variety of innovative instructional techniques that were employed in the course. Each week students attend one 45 minute tutorial session which involves working in groups of two to four students. The goals of the exercises given to the students at the beginning of each tutorial session are firstly to target possible misconceptions of material covered in the previous week of lectures and, secondly, to begin to develop students' concepts around material to be presented in the following few lectures. Exercise sheets have the same goals but students generally attempt these problems on their own. There is a need to develop more projects, because students found them extremely useful and they could use their developed skills in other courses.

Table 1. Examples of reform-based techniques used.

Reform-based technique	Examples of use
• Collaborative learning	Problems are set for tutorial sessions where students share ideas and understandings to solve these problems.
• Use of technology	Solving an ANOVA problem using a statistical software package (STATISTICA).
• Active learning	Students chose their own "open-ended" essays to write and place on their web page.
• Target misconceptions	Instruction is designed so that students will be confronted with their misconceptions and have the opportunity to reflect and derive a more coherent conceptual understanding.

5. Putting it all together and future directions. The educational paradigm shift from purely disciplinary to interdisciplinary implies that mathematics courses, not just those for the above mentioned science students, but all general-audience undergraduate courses, should be infused not with old examples, but with real ones. This means incorporating real-world data in mathematics courses and for geology and chemistry lecturers to challenge students to use the quantitative skills they have already developed, and to encourage those who have not developed them to do so. This goal required close collaboration between departments. There is a need to include more projects designed to complement lectures or lab work. Both lectures and projects should further emphasize how quantitative methods can enhance students understanding of concepts in geology and chemistry. It was the intention to further demonstrate basic multivariable calculus techniques because of the many examples in geology and chemistry but there was no time in the course to do so. It may be a good idea to develop a second one semester course which extends this material to include more multivariable techniques and the use of computer packages to visualize 3-D concepts of geoscience and chemistry phenomena.

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Students' Difficulties with Multiple Integration: A Preliminary Study

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ABSTRACT. This paper will present preliminary results of a study on students' learning of Calculus in Universiti Teknologi Malaysia. The focus will be on difficulties faced by students in understanding the concept of multiple integration. The sample consisted of two groups of second year undergraduates in Industrial Science majoring in Physics and Chemistry, respectively. 80 students were involved in the study, of which, 35 were females. Data on students' difficulties were collected from students' written work on selected questions in double and triple integration. Based on the analysis, students' difficulties can be classified into four main categories: visualisation of regions and surfaces, interpretation of graphs, algorithmic and algebraic manipulation errors. Students' performance based on gender will also be presented. The results indicate that the present teaching approach does not allow students to fully understand the meaning of the concepts. The paper will conclude with suggestions on how to modify current teaching practice.

1. Introduction. There have been various studies carried out to investigate students' difficulties in Calculus. Some of the difficulties identified were poor understanding of basic concepts, inability to formulate problems mathematically, lack of mastery in algebraic, geometric and trigonometric skills (Tall, 1993, Baldino, 1993; Norman & Prichard, 1994; Selden, Mason & Selden, 1994; Artigue, 1991; Cornu, 1991).

Similar problems were also found amongst Malaysian students (Ali & Tall, 1996; Bakar, 1991; Tall & Razali, 1993; Liew Su Tim & Wan Muhammad Saridan, 1991, Faridah, 1996). These studies found that students have difficulties in manipulating concepts, coordinating procedures, unable to manipulate symbols in a flexible way and showed poor performance in answering non-routine questions. Most of the studies carried out were on students' performance in Basic Calculus. Usually most universities will offer special programs or remedial courses to help the students in the first year. Universiti Teknologi Malaysia (UTM) has also offered such programs for their students. It was expected that they would be able to cope with subsequent mathematics in their studies. However, from our experience, students' achievements in higher Calculus courses were unsatisfactory. This gave an indication that students still encounter difficulties in the learning of Calculus.

This situation has motivated the current study to investigate students' learning of second year Calculus at UTM. This paper will present some preliminary results of the study. In particular, we will highlight some observed students' difficulties with multiple integration. We will begin with an overview of the students' mathematical background, the teaching and learning environment to give a clear picture of the setting of the study.

2. Mathematical Learning in UTM. There are two important characteristics of the mathematical learning environment in UTM that should be highlighted:

- UTM is one of two universities in Malaysia that takes students directly after the attainment of Sijil Pelajaran Malaysia (Malaysian Certificate of Education) which is equivalent to the O- Levels and follow a national curriculum. Thus, upon entry to UTM, students' mathematical experience is at the pre-Calculus level. The first year mathematics was designed to bridge the transition from school mathematics to mathematics at the university and should provide the necessary knowledge and competencies required by the students for further mathematics and for the mathematics in their own fields of study.
- Schools in Malaysia use the term system where the final examinations are given at the end of the year. Whereas UTM practises the semester system whereby students will sit for their final examinations at the end of the semester, after 15 weeks of study. However, the fast pace of mathematics teaching and the increasing complexity of the mathematical ideas makes it difficult for students to assimilate their learning in this short period. Thus, there is a tendency for teaching to be reduced to the learning of facts, solving routine problems and applying fixed mathematical procedures to collections of structurally identical

problems to help students cope with the course and examinations. This particular manner of learning mathematics was similar to their mathematics learning experiences in school with rote learning and drill exercises as the main modes of students''learning activities.

3. Method of study. The purpose of this study is to investigate UTM students' difficulties in second year Calculus with particular attention to the topic of multiple integration. The calculus syllabus consisted of topics in Series, Functions of several variables and Multiple integration. The data used in this study was collected from students' answers to examination questions at the end of the semester. The examination paper was set by a group of lecturers responsible for the teaching of the second year calculus course. The first named author was a member of the group.

The total number of students who sat for the examination was 480 candidates. However, only eighty answer scripts were used in the exploratory study. These were from Industrial Science students majoring in Physics and Chemistry and were in their second semester of study. In this study, we had examined students' written work to 4 questions on multiple integration. Each of the questions consisted of two parts (a) and (b). Part (a) focuses mainly on carrying out procedures whilst part (b) consist of routine application problems which requires the construction of solution methods. The questions are given in Appendix A. The students' scores and performance in each question were analysed.

4. Analysis and results. We will begin with a brief description of the types of questions asked. The topics covered in multiple integral are double integrals, double integrals in polar form, finding areas, volumes, moments and centre of mass; triple integrals in rectangular coordinates, cylindrical and spherical coordinates, finding volumes, centre of mass and moments of solids. The course coordinator has provided a standard marking scheme to all concerned lecturers. This scheme consisted of allocation of marks for specified students' responses. Based on this scheme, we concluded that the specific students' abilities to be assessed were as summarised below.

Table 1: Description of questions and abilities assessed

Item	Description	Abilities assessed
1a	Evaluation of double integral with a given region	sketching the given region finding the limits of integration evaluating the integral
1b	Evaluation of double integral in polar coordinates	converting Cartesian to polar coordinates evaluating the integral
2a	Evaluation of triple integral	evaluating the integral
2b	Finding the volume of a solid using triple integral	setting up the algorithm sketching the solid identifying the region finding the limits of integration evaluating the integral
3a	Evaluation of triple integral in cylindrical coordinates	converting rectangular to cylindrical coordinates evaluating the integral
3b	Finding the centroid of a solid in three dimensions	setting up the algorithm sketching the solid identifying the region finding the limits of integration evaluating the integral

4a	Evaluation of triple integral in spherical evaluating the integral	converting rectangular to spherical coordinates evaluating the integral
4b	Finding the volume of a solid in spherical coordinates	setting up the algorithm sketching the solid identifying the region finding the limits of integration evaluating the integral

Students should be familiar with questions 1(a), 2(a), 3(a), 4(a) and 4(b) as these were mainly routine questions. The total number of answer scripts examined was 80, of which 35 were females. Each question consisted of two parts (a) and (b) and carried 6 marks respectively. The total possible score is 48. The results obtained were tabulated according to students' (1) overall performance, (2) performance in each question and (3) achievement according to gender.

4.1. Students' overall performance.

Table 2: Distribution of students' overall scores

Maximum score = 48	Female	Male	Total no. of students
No. of students whose total marks are < 24	24	22	46
Total no. of students	35	45	80

From Table 2, we found that the number of students with total score of less than 24 marks is 57.5% which consisted nearly equal numbers of both genders (52 % females to 48 % males).

4.2. Students' performance in each question.

Table 3: Number of students whose item score is < 3 for each question

Item	Female	Male	Total	Percentage
1a	12	11	23	28.8
1b	26	24	50	62.5
2a	16	14	30	37.5
2b	16	21	37	46.3
3a	22	20	42	52.5
3b	32	29	61	76.3
4a	25	24	49	61.3
4b	21	17	38	47.5

It can be seen that a majority of the students performed poorly in Questions 1(b), 3(a), 3(b) and 4(a). Some attention should also be given to questions 2(b) and 4(b).

There are two groups of students, students with a total score greater than or equal to 24 make up Group A and those with a total score less than 24 form Group B. For each group, we now tabulate the number of students whose item score is less than 3 marks according to gender.

Table 4: Group A (total score 24) Number of students with item score < 3

	Female	Male	Total no. of students
Total no. of students	11	23	34
Item	**No. of students with item score < 3 (out of 6 marks)**		
1a	1	3	4
1b	5	7	12
2a	4	3	7
2b	2	0	2
3a	2	3	5
3b	8	8	16
4a	4	6	10
4b	0	2	2

Table 5: Group B Group A (total score < 24) Number of students with item score < 3

	Female	Male	Total no. of students
Total no. of students	24	22	46
Item	**No. of students with item score < 3 (out of 6 marks)**		
1a	11	8	19
1b	21	17	38
2a	12	11	33
2b	21	14	35
3a	20	17	37
3b	24	21	45
4a	21	18	39
4b	21	15	36

As expected, students in Group B performed poorly in all of the questions. Although students in group A have a satisfactory overall performance, there are indications that some students found Questions 1(b) and 3(b) difficult.

4.3. Achievement according to gender. The performance of the female and male students was considered separately and comparisons made within the gender group. It was found that the overall performance of female students was unsatisfactory. 68.6% of the total number of female students compared to 48.9% of the total number of male students failed. When the performance for each question was examined, we found that female students in Group B did badly in all the questions. However, the female students in Group A performed slightly better but most found questions 1(b) and 4(b) difficult. Similar results were found amongst the male students in Groups A and B. However, in Group B, the percentage of failure is slightly lower when compared to the female students.

Using Table 1 as a guide, the students' problem solving performance were examined and each of the abilities to be assessed were studied in detail according to gender. We will highlight some of the findings that are of concern and deserved further attention.

- More than 70% of students of both genders had difficulties in finding the limits of integration in question 1(b).
- 51% of the students of both genders were not able to evaluate the integral in 2(a).
- 51% of the female students could not sketch and identify the region in 2(b) and 3(b) as compared to 27% of the male students.
- For 3(b), students were required to set up the necessary algorithm to solve the question. In general, more than half of the students were not able to set up the algorithm or did not show any attempt to do so. However, when comparison was made within each gender grouping, 77% of the female students fared badly as compared to 58% of the males.

- 51% of the female students could not change the Cartesian integrals to either the cylindrical or spherical form (3(a) and 4(a)).
- For 4(b), the disparity in performance between the females and males were wider. 74 % females had difficulties in setting up the algorithm and failed to answer the question as compared to 29% of the males. This is a surprising result as this was a routine question. On the other hand, this could also explain why the males showed better performance.

5. Summary of findings. The students' difficulties can be grouped into 4 main categories, as described below.

5.1.Visualisation of regions and surfaces (1(a), 2(b), 3(b), 4(b)). There are two types of questions in this category. In type I, the students were expected to sketch the given region and identify the limits of integration from their sketch. In type II, based on the given information, students had to identify the region/surface, sketch the required region/surface and determine the limits of integration. It was observed that most students had difficulties with type II questions as compared to type I. In this category, the male students appeared to perform better than the female students. 67% of the male students gave the correct sketch as compare to 49% of the female students.

5.2. Interpretation of graphs. From the results, it could be seen that those students who were not able to sketch the region/surface required could not answer the questions (32% of the total number of students). A small group of students managed to give the correct sketches but were not able to identify the correct limits of integration (15%).

5.3. Algorithmic errors (2(b), 3(b), 4(b)). Only 22% of the students could not set up the algorithm to evaluate the volume of the solid in Cartesian coordinates (2(b)) and slightly more students (39%) could not do so in spherical coordinates (4(b)). However, nearly half the students (49.5%) could not provide the correct algorithm for finding the centroid of a solid. It was observed that the students faced difficulties in setting up the algorithm that involved several procedures.

5.4. Algebraic manipulation errors. The most common errors found in the students' answer scripts were errors in computation, incorrect recall of facts, arbitrary application of techniques of integration and incorrect conversion from one coordinate system to another.

6. Conclusion and suggestions. Based on the data collected it was found that students still had difficulties in solving the routine problems. There are indications that students could cope with simple procedures but were overwhelmed when several procedures were required to solve a problem. Similar findings were documented in an earlier study on UTM students in Basic Calculus (Tall & Razali, 1993). The class consisted of students with varied mathematical abilities. The current teaching methods does not promote students' mathematical understanding of procedures and concepts but concentrated mainly on learning of procedures to solve a set of structurally identical problems. Most lecturers cited reasons such as this method was the most suitable to cope with the wide range of students' mathematical abilities, time constraints and in helping students to be successful in the examinations. However, students' performance showed that the current teaching practice appeared detrimental to students' mathematical learning at this level.

There is a need to change the teaching methods to those that encourage and develop students' understanding of concepts, procedures and the relationship between the two. In particular, teaching must focus on students' problem solving skills and mathematical thinking. Problem solving courses could be taught separately or integrated within the mathematics class. An investigation on the implementation of the proposed teaching methods will be conducted as a follow-up to this study.

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Appendix A: Examination questions

1 (a) Evaluate $\iint_R (x+xy)dA$ where $R=\{(x,y): x^2+y^2 \le 4\}$

1(b) Convert the integral $\int_0^2 \int_0^{\sqrt{1-(y-1)^2}} xy\,dx\,dy$ to polar coordinates and evaluate.

2(a) Evaluate $\int_0^{\pi} \int_{-2}^{1} \int_{1}^{4} yz\cos(xy)\,dz\,dx\,dy$

2(b) Use a triple integral to compute the volume of the solid in the first octant that is bounded by the planes $y + z = 2$, the coordinate planes and the parabolic cylinder $x = 4 - y^2$.

3(a) Convert the integral $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{x^2+y^2}^{4} x\,dz\,dy\,dx$ to cylindrical coordinates and evaluate.

3(b) Find the centroid of the solid that is bounded above by the sphere $r^2 + z^2 = 9$, on the side by the cylinder $r = \sqrt{5}$ and below by the plane $z = 0$.

Convert the integral $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\,dzdydx$ to spherical coordinates and evaluate.

Use spherical coordinates to find the volume of the solid bounded by the hemisphere $z=\sqrt{8-x^2-y^2}$ and below by the cone $z=\sqrt{x^2+y^2}$.