# Lighthouse
# Delta 2013

# *Shining* *through the fog*

The 9th Delta Conference of teaching and learning of undergraduate mathematics and statistics

# Proceedings

*Editors: Deborah King,
Birgit Loch, Leanne Rylands*

# LIGHTHOUSE DELTA 2013 CONFERENCE

*Proceedings of the 9th DELTA conference on the teaching and learning of undergraduate mathematics and statistics*

# FOREWORD

The first of the Delta series of conferences on the teaching and learning of undergraduate mathematics and statistics was held in Brisbane, Australia, in 1997. A Delta conference has been held every second year since then. All are held in the southern hemisphere and so far they have been held in South Africa, New Zealand, Argentina and Australia.

*Lighthouse Delta* is the ninth conference in the Delta series, held from the 24th to the 29th of November 2013 at the Pavilion, in Kiama, a small coastal town with a population of approximately 13,000 located 120km from Sydney.

The conference theme, "Shining through the fog" encapsulates the challenges faced by those with responsibilities for building the mathematics and statistics capacity needed for the 21st century. Indeed, it is a challenge. In some countries, including Australia, secondary school students are turning away from mathematics in their final years, yet we must do our best to prepare them for a technological world in which science, technology, mathematics and engineering are ever more important.

How do we make our way through the fog created by the many possibilities for teaching and engagement? The variety of technologies available to us, the many types of assessment, what to do online and what not, the use of e-resources, the many different approaches to teaching; all these provide many possibilities and hence there are many important decisions for those of us teaching mathematics. The passion, enthusiasm and expertise of the many *Lighthouse Delta* presenters can only help to improve our knowledge, to assist us to make good decisions and to improve tertiary mathematics teaching in the southern hemisphere and beyond.

The presentations at *Lighthouse Delta* include those from the authors of the papers in these proceedings, those whose papers appear in the Special Delta Issue of the *International Journal* of *Mathematical Education* in *Science* and *Technology* and many who chose just to give an oral presentation or a poster presentation. The *Lighthouse Delta* presenters came from more than a dozen countries.

These proceedings contain papers presented at *Lighthouse Delta*. Each submission was double blind reviewed by a minimum of two reviewers.

One of the organising committee members is working on a permanent web presence for Delta and it is hoped to have the previous Delta proceedings available on this site as well as these proceedings.

We give our heartfelt thanks to the various committees who made *Lighthouse Delta* happen, to our administrative support, the reviewers and to Cristina Varsavsky, who oversaw and coordinated it all. We also thank the International Steering committee for keeping Delta going.

We hope that you find *Lighthouse Delta* enjoyable and stimulating, and that you also have some time to enjoy the lovely Kiama surroundings.

*Lighthouse Delta Proceedings Editorial Team*
Deborah King, Birgit Loch and Leanne Rylands,

# *Shining through the fog*

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# Papers

# First year university students' understanding of functions: Over a decade after the introduction of CAS in Australian high schools, what is new?

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A strong understanding of the concept of function is vital for students studying calculus and has been seen as a unifying concept in mathematics and between mathematics and the real world. This paper reports on a pilot research study on first year university students' understanding of functions. Special attention has been given to students' background on the usage of technology at secondary school to look for potential impact on students' understanding of concepts. Results collected halfway through the project are analysed and implications for university teachers are discussed.

## Introduction

The mathematical concept of 'function' has been an integrating theme in the secondary mathematics curriculum for many years and is pursued in further depth at the tertiary level. As the review of literature below will show, findings in the past suggested that while many senior secondary, early tertiary students were operationally familiar and competent with simple functions (e.g. linear, quadratic, cubic) they did not necessarily have a conceptual understanding of functions that could be applied in a range of representations or new situations.

In the period since much of this research was conducted, new technology – in particular computer algebra systems – have become a common part of Australian secondary mathematics classrooms. Extensive research has been conducted to show that the technology shapes the way in which students perceive the mathematical concepts they manipulate, and as technology evolves, so does students' perception. Dynamic views of algebra seem to have flourished. Multiple representations are at the core (see Zbiek and Heid [1] for the notion of function) often taking the interrelationship between each representation for granted. However, recent studies (Pierce et al. [2]) pointed out some deep issues related to the interrelationship between environments, inherent to the computing system, that have significant impact on the teaching and on students' perception of algebra.

In the study reported in this paper we revisited the notion of function and focused on two key research questions. Can high achieving secondary students in their first semester of University describe what is meant by a function and identify functions in a range of representations? Does it seem that access at secondary school level to sophisticated mathematical technology, especially graphics calculators or Computer Algebra Systems (CAS – software that facilitates symbolic mathematics), impacts on students' understanding and recognition of functions?

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**Functions: previous research findings**

It is widely agreed that a strong understanding of the concept of function is vital for students studying calculus and has been seen as a unifying concept in mathematics and between mathematics and the real world. Research has shown, however, that an understanding of function develops over an extended period of time and that many undergraduate students have poorly developed notions of function.

History shows that this notion took several centuries before acquiring today's definition. For several decades, this notion was indeed tightly perceived 'geometrically'. In 1692 Leibniz first used the term 'function' with respect to aspects of curves such as the gradient at a point. Newton referred to variables associated with curves as 'fluents', a geometric image of points moving along curves. Early in the 18th century Bernoulli used 'function' to describe an expression made up of a variable and some constants and by 1748 Euler had refined his thinking on functions and wrote:

> If … some quantities depend on others in such a way that if the latter are changed the former undergo changes themselves then the former quantities are called functions of the latter quantities. (cited in Kleiner, 1989, p. 288 [3])

By the 19th century, new concepts of 'function' were developing, leading to Dirichlet's 1837 definition. By the early 20th century, discussions about the precise meaning of function and variable resulted in 1939 in Bourbaki's formal ordered pairs definition of function: a relation between ordered pairs in which every first element has a unique second element. This is often referred to as the Bourbaki definition or the Dirichlet-Bourbaki definition. A typical textbook definition is: "We define a function as a relation between two sets, called the domain and range, such that each element in the domain corresponds to exactly one element in the range." Kleiner [3] summarises the development of the function concept thus:

> The evolution of the function concept can be seen as a tug of war between two elements, two mental images: the geometric (expressed in the form of a curve) and the algebraic (expressed as a formula – first finite and later allowing infinitely many terms, the so-called "analytic expression"). Subsequently, a third element enters, namely, the "logical" definition of function as a correspondence (with a mental image of an input-output machine). In the wake of this development, the geometric conception of function is gradually abandoned. A new tug of war soon ensues (and is, in one form or another, still with us today) between this novel "logical" ("abstract", "synthetic" "postulational") conception of function. (p. 282)

Perhaps not surprisingly, then, the difficulty that students have with the function concept may be due in part to a mismatch between the definition of function that is presented to them and the nature of the tasks and applications they are required to complete.

Tall and Bakar [4] report on a study with twenty eight students (aged 16/17) who had studied the notion of a function during the previous year and had used functions in calculus, but with little emphasis on the technical aspects such as domain and range. The students were asked to: "Explain in a sentence or so what you think a function is. If you can give a definition of a function then do so." (p. 105)

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Tall and Bakar [4] note that none of the students gave satisfactory definitions, but all gave explanations, including the following:

- a function is like an equation which has variable inputs, processes the inputted number and gives an output.
- a process that numbers go through, treating them all the same to get an answer.
- an order which plots a curve or straight line on a graph.
- a term which will produce a sequence of numbers, when a random set of numbers is fed into the term.
- a series of calculations to determine a final answer, to which you have submitted a digit.
- a set of instructions that you can put numbers through. (p. 105)

Tall and Bakar [4] note that most of the students "expressed some idea of the *process* aspect of function – taking some kind of input and carrying out some procedure to produce an output – but no one mentioned that this only applies to a certain *domain* of inputs, or that it takes a *range* of values. Many used technical mathematical words, such as *term*, *sequence*, *series*, *set*, in an everyday sense, intimating potential difficulties for both students and teachers in transferring mathematical knowledge." (p. 105)

Research carried out by Oehrtman, Carlson and Thompson [5] also shows a strong procedural emphasis, where students think about functions only in terms of symbolic manipulations and procedural techniques. Vinner and Dreyfus [6] note that although a version of the Bourbaki definition is frequently presented in text-books and curricula, "the examples used to illustrate and work with the concept are usually, sometimes exclusively, functions whose rule of correspondence is given by a formula" (p. 357). Consequently, when asked to give a definition of function, students would be likely to give a Bourbaki-type definition, but their work on identification or construction tasks might be based on the formula conception.

Sierpinska [6] asserts that "the pedagogical conclusion… is that an early introduction of the general definition of function does not make sense; it will be either ignored or misunderstood." (p. 48). Cooney and Wilson [8], citing the work of Vinner and Dreyfus [6], Sfard [9] and others, claim that "the less abstract notion of functions as rules allows students to gain a strong conceptual background in functional thinking before progressing to the more abstract and general notion of functions as sets of ordered pairs" (p. 146).

Tall and Bakar [4] claim that even though curriculum documents include the definition of the function concept, the definition "is not stressed and proves to be inoperative, with student understanding of the concept reliant on properties of familiar prototype examples" (p. 104) with the result that students have many misconceptions. Tall and Bakar note, for example, that 44% of a sample of 109 students starting a university mathematics course considered a constant function not to be a function in at least one of its graphical or algebraic forms, usually because $y$ is independent of the value of $x$. Furthermore, 62% of the students thought that a circle is a function.

Blume and Heckman [10] note that despite the recognised importance of multiple representations in secondary school mathematics, many students are not understanding the connections among these representations.

About the use of technology and its impact on students' understanding of the notion of function, Bridger and Bridger [11] suggest that widespread use of technology to graph functions can obscure the function concept as a result of replacing the "dynamic idea of a rule, procedure or mapping with a static, automatically drawn picture" (p. 103).

Carlson [12] undertook a study which included a group of students who had just completed college algebra with high A scores and a group who had completed second

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semester calculus with high A scores. The college algebra course had included an introduction to functions. Carlson found that the college algebra students did not understand function notation, had difficulty understanding the role of the independent and dependent variables in a given functional relationship, could not explain what is meant by expressing one quantity as a function of another, and were unable to speak the language of functions. Whilst the calculus students had greater understanding of the function concept, when faced with more complex function expressions, they too experienced difficulties.

Oehrtman, Carlson and Thompson [5], referring to the same study [12], reported that many students believed that constant functions (e.g., $y = 5$) were not functions because they do not vary. When asked to give an example of a function all of whose output values are equal to each other, only 7% of A-students in college algebra could produce a correct example, and 25% of A-students in second semester calculus produced $x = y$ as an example." (p. 151). Oehrtman, Carlson and Thompson also found that 43% of the college algebra students attempted to find $f(x + a)$ by adding $a$ onto the end of the expression for $f(x)$ and many of them could not justify why the graph of $g(x) = f(x + a)$ is shifted to the left of the graph of $f(x)$ for $a > 0$.

Carlson [12] suggests that "curriculum developers underestimate the complexity of acquiring an understanding of the essential components of the function concept… and that current curricula provide little opportunity for developing the ability to: interpret and represent covariant aspects of functions, understand and interpret the language of functions, interpret information from dynamic functional events" (p. 142). She asserts that "the pace at which content is presented, the context in which it is presented as well as the types of activities in which we engage students have an enormous impact on what students know and what they can do" (p. 143).

**Methodology**

This paper reports on a one year pilot research study for which an online mathematics quiz (referred to as 'quiz' in what will follow) and a separate online background survey (referred to as 'survey' in what will follow) on students' background focusing on the use of technology at school were made available to first year mathematics students soon after the start of their first semester at The University of Melbourne. Students who participated to this study were undertaking at least one of the following subjects: Calculus 1, Calculus 2, Accelerated Mathematics 1, Experimental Design and Data Analysis, Introduction to Mathematics and Mathematics for Biomedicine. Students' attention was drawn to the quiz and survey through emails and verbally by their lecturers. The students accessed both quiz and survey through the university's Learning Management System. After the completion of semester 1 a purposive sample of the quiz respondents were invited to participate in a face-to-face interview where they were presented with similar questions but asked to explain their thinking. Students' participation in this research study was entirely voluntary and the quiz did not form a part of any subject they were studying. The quiz was limited on time (35 minutes) and students were given one attempt only. .

The quiz comprised 16 questions in total, focusing on students' understanding of functions, variation and variables. In this paper we will report findings about students' understanding of functions, in particular of four items related to this topic: questions 1, 2, 3 and 11 of the original quiz. As our study is framed around the notion of multiple representation of functions (Cuoco [13]), these items addressed: worded description (Question 1), symbolic/algebraic representation (Questions 2 and 3) and graphical representation of functions (Question 11).

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- **Question 1** is an open ended question: "Explain, in plain English, what a function is".
- Multiple choice **Question 2** provides a list of rules (see table 2) which do or do not describe a function and students were asked to "Identify those who describe functions (choose as many as apply)".
- **Question 3** is about gauging students' substitution skills for the function $f(x) = x^2 - x + 2$ where $x \in \mathbb{R}$. Items included numerical substitutions ($f(1)$), substitutions with a variable ($f(a), f(-a)$) with an ultimate question of multiple choice for the coefficient of $f$ ($f(x)$) (we wanted to avoid students having to type expressions without a proper math editor). The actual items of Question 3 are reported in table 3.
- Multiple choice **Question 11** is about exploring the graphical aspects of functions. Students were given four graphs (see figure 1) and were asked: "Among the following graphs, which one corresponds to a function? (Choose as many as they apply). Details of the questions and students' responses are expanded upon in the following section.

The background survey comprised six questions in total. In this article, we will focus on Question 4. Question 4 provided a list of 6 types of technology (CAS/ graphics/ scientific/ arithmetic calculator, computer based CAS, computer based spreadsheet) and a list of 5 different uses (not used, used in class only/out of class only/in and out of class/in final exams). Students were asked: "During your year 12 mathematics studies what technology, if any, did you use and where? (Choose as many as apply)".

## Results

Of the approximate 2000 students who could have accessed both the quiz and survey, 427 answered the quiz and over 600 completed the survey. In both cases, not all of the students answered every question. We will focus on the 383 students who both attempted the quiz and answered the survey. From the background survey Question 4, 69% of the 383 students made use of CAS in some way for their Year 12 mathematics subject(s) and 60% said that they had used this technology in their final examinations.

Students' responses to the maths quiz questions 1, 2, 3 and 11 are summarised in tables 1, 2, 3 and 4 (note: for simplicity percentages have been rounded to whole numbers). We will consider the distribution of responses and whether there is any statistically significant difference in the results of those who had or had not used CAS in their examinations at Year 12. Statistical significance has been tested using Chi-squared tests of independence with $p = 0.05$ as a critical value. Only the statistical values for the significant results will be reported in this brief paper.

### *Results for Questions 1, 2, 3 and 11*

Ninety-five percent of the 383 students answered Question 1. Of those who answered, 63% (228 students) were able to give one or more valid statements. The valid statements, categorised in Table 1, are: rule connecting dependent variable and independent variable; unique value of dependent variable for each value of the independent variable; vertical line test (a vertical line drawn anywhere on the graph of a function crosses the graph only once); mapping between values of independent and dependent variable; mapping with one-to-one or many-to-one correspondence; graph.

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**Table 1.** Responses to Question 1 (in categories).

| Concepts present in students' responses | Percentage of students including this concept ($N = 383$) |
|---|---|
| Rule | 51 |
| Unique *y* value | 19 |
| Vertical line test | 4 |
| Mapping | 4 |
| One-to-one or many-to-one | 5 |
| Graph | 6 |
| At least one valid concept | 63 |

While most students included at least one valid statement in their written description of a function, more than a third of students did not. To learn from a university mathematics lecture and engage in dialogue regarding mathematics students need suitable vocabulary and practice in verbalising mathematical ideas. Given that functions are privileged in secondary mathematics, it is of concern that so many students were apparently unable to give a simple (correct) explanation of this concept.

Only the specific mention of a graph did not appear to be independent of CAS use. In the case of including 'graph' (Chi-sq = 6.143, p = 0.013) more students who used CAS, than would have been expected if the two activities were independent, mentioned 'graph'. Mirroring this fewer non-CAS students than would be expected mentioned 'graph'.

**Table 2.** Percentage of responses (*correct answers) for each item of Question 2.

| **Rule** | **Percentage who marked as a function** ($N = 383$) |
|---|---|
| **a)** $f(x) = x^2 - \sqrt{2}$    where $x \in \mathbb{R}$ | 89* |
| **b)** $f(x) = ax + b$    where $x$, a, b $\in \mathbb{R}$ | 85* |
| **c)** $f(x) = \begin{cases} x^2 & if\ x \le 0 \\ 2 & if\ 0 < x \le 1 \\ 2x & if\ x > 1 \end{cases}$    for all $x \in \mathbb{R}$ | 76* |
| **d)** $f(x) = \begin{cases} 2x + 1 & if\ \ x < 2 \\ -5x + 3 & if\ \ x > 1 \end{cases}$    for all $x \in \mathbb{R}$ | 60 |
| **e)** Let $f$ be a function whose rule $f(x)$ is the area of a circle with circumference $x$ | 50* |
| **f)** Let $f$ be a function whose rule $f(x)$ is the area of a rectangle with perimeter $x$ | 39 |

All 383 students completed Question 2 (see table 2). Only 23 students (6%) correctly selected all four functions. In the first rule (a) the parameters/constants of this quadratic function were numbers but in the second rule (b) the parameters/constant for this linear function were letters. These were identified as functions by 89% and 85% of students respectively. We conjectured that students who typically worked with a graphics or CAS calculator may reject the second rule because their technology would not graph this rule without specified values for the parameters. Fewer students were able to determine if the hybrid rules were functions. Whilst 76% of students correctly marked the first hybrid rule (c), continuous on *x* in each section, as a function, the second hybrid rule (d), with discontinuities,

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was incorrectly identified as a function by 56% of students. The pair of statements (e and f) probed students' response to worded descriptions rather than symbolic expressions to describe a function as well as their recognition of the number of variables and awareness that functions link two variables. The first function described in words (circle area depending on circumference and therefore radius) was correctly marked as a function by only 50% of students, whilst 61% of students correctly did not mark the rectangle area (dependent on perimeter and therefore length and width) as a function. These are not typical 'school mathematics' questions that these students would have practised for their high stakes examinations but items which we felt were simple if students had a robust understanding of the concept of function.

There was no statistically significant difference between the observed results and those expected if the students' understanding was independent of CAS used. There was not sufficient evidence to reject our null hypothesis that responses were independent of CAS use in Year 12 examinations.

Less than one third of students (110 of the total of 383 students) answered all eight parts of question 3 correctly. The only response where the pattern did not appear to be independent of CAS use was the result of calculating $f(x+1)$ (Chi-sq=6.018, p=0.049). For both CAS and non-CAS students a correct, simplified answer was provided by roughly the number we could expect if these variables were independent. However fewer CAS students than expected actually gave incorrect answers while more than expected provided answers including $(x+1)^2$. These proportions were reversed for the non-CAS students.

**Table 3.** Items and students' responses to Question 3 for the function $f(x) = x^2 - x + 2$,

| Item | Percentage correct ($N = 383$) |
|---|---|
| What is the value of $f(1)$ | 97 |
| Find $f(a)$ | 96 |
| Find $f(-a)$ | 80 amongst whom 11% did not evaluate $(-a)^2$ |
| Find $f(2x)$ | 83 |
| Find $f(-x)$ | 91 |
| Find $f(x+1)$ | 52 |
| Find $f(x^2)$ | 90 |
| What is the coefficient of $x^2$ in $f(f(x))$? | 0   0%   1   29%   2   14%   3   0%   4 *   57% |

All 383 students completed question 11. Graphs 1 and 4 represented functions (see Figure 1). Only 47% of students responded correctly to all four graphs.



Graph 1     Graph 2     Graph 3     Graph 4
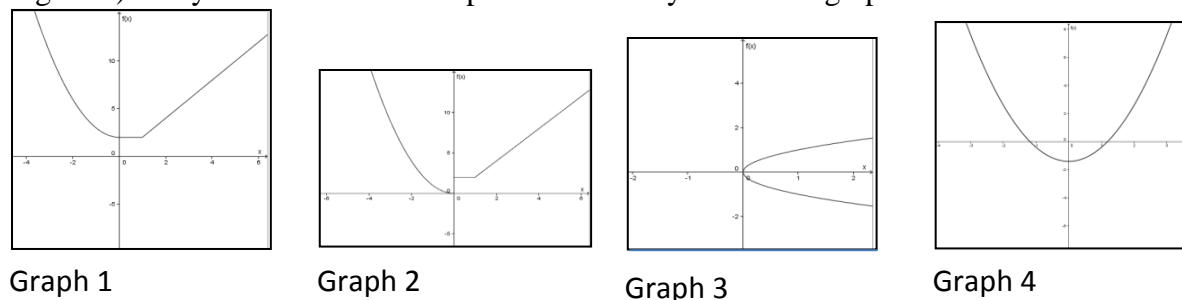
**Figure 1.** The four graphs of question 11.

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All 383 students completed this question. Only 47% of students responded correctly to all four graphs. Of the four graphs, the parabola was identified as a function by the greatest number of students (79%), followed by the hybrid function (Graph 1, 60%). Graphs 2 and 3 were incorrectly marked as functions by 49% and 33% of students respectively. Graph 2 is a variation of the graph of the $1^{st}$ hybrid function of Question 2. However, from the interviews conducted with students, we hypothesise that because there was no explicit graphical evidence of the *y* value for *x*=0, students might have discarded this graph Students were accustomed to seeing open or closed circles at such points to indicate if the point was included in the domain.

While the results for most items were independent of students' CAS use in Year 12, examination of the results for graph 3 showed a statistically significant relationship. Fewer CAS users, and more non-CAS students than would be expected if the variables were independent, incorrectly indicated that the sideways parabola was a function (Chi-sq=11.056, p=0.001).

### *Cross analysis of Questions 1, 2, 3 and 11*

The first option in Question 2 and Graph 4 in Question 11 represented the same quadratic function. Of the 383 students, 283 students (74%) were consistent in identifying both representations as a function, with only 6% believing that neither were functions. However, 15% indicated that only the rule represented a function and 5% indicated that only the graph was a function.

The third option in Question 2 and Graph 1 in Question 11 represented the same hybrid function. Of the 383 students, only 207 students (54%) were consistent in identifying both representations as a function, with 18% believing that neither were functions. A further 21% indicated that only the rule represented a function and 6% indicated that only the graph was a function.

In each of the three most common response patterns (a total of 71 students), incorrect responses were given for the fourth option of Question 2 (second hybrid rule) and Graph 2 of question 11. Details are provided in table 4.

**Table 4.** Three most common response patterns between rules and graphs

| Most common response patterns | Number of students | Percentage ( $N = 383$ ) |
|---|---|---|
| Q2 parts d, e*, f and Q11 graph 2 | 29 | 8 |
| Q2 part d and Q11 graph 2 | 23 | 6 |
| Q2 part d and Q11 graphs 2, 3 | 19 | 5 |
| **Total** | 71 | 19 |

Only five students made correct responses to all of the six rules (question 2) as well as the four graphs (question 11).

Of the 14 students who included the vertical line test in question 1, only three responded correctly to all four graphs, seven incorrectly identified Graph 2 as a function and four did not identify Graph 1 as a function. All 14 students correctly responded to Graphs 3 and 4. Of the 74 students who referred in Question 1 to a unique y value for each *x* value, 64 responded correctly to Graph 3 and 65 to Graph 4.

### Implications

In mathematics, it is of use to first define a concept before manipulating/working with it. In the teaching of mathematics, it looks as if the converse might be more useful –at least in what

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regards the concept of function. Although the majority of students from our study (61%), when asked to define/describe a function, did provide at least one valid statement, this did not reflect their mastery of the notion. When looking at the different aspects of a function (graphical, symbolic, etc.), results from this study suggest that students do not perceive the links between its multiple representations. And certainly the often over-emphasised 'teaching tool' of the 'vertical line test' is of little help –probably because it has been memorised rather than meaningfully understood. It is surprising how closely our results match previous research, despite the difference in schooling systems and the decades that separate our study from previous studies. It seems that important work has still to be done in the classroom to make the definition of a function (which is precisely what makes one determine what a function is and distinguish from what is not) meaningful and not just a concatenation of words. Every word that composes the definition has an implication/translation in each of the different representations. Our findings suggest that these connections are far from clear in students' minds and that teaching should aim to make more explicit the repercussion of each of the features of the concept of function in each of its representations. The definition of a function can be viewed as the worded synthesis of all its different aspects and certainly exploring all the different representations as much as possible sets good ground for the meaningful interpretation of this worded synthesis.

Technological tools that exploit multiple representation are nowadays easily available and widely adopted in Australia. Our results are, to this matter, somehow globally disappointing, suggesting that there is room for further exploration of potentialities of such technology in the classroom –especially when it comes to the teaching of functions. However in specific cases such as students acknowledging that a sideway parabola is not a function, the familiarity with CAS and its graphing properties seems to have had positive impact. Our findings suggest that CAS has also had benefits on the understanding of the underlying principle of substitution, or in other words on understanding the precise meaning of "$f$ of $x$" and its symbolic form $f(x)$, especially when it comes to substituting for $x$ an expression containing the variable $x$

The results presented here were collected and analysed halfway through a one-year pilot project and certainly need deeper investigation. However the results gathered so far provide us with already interesting information on students' conceptions of functions. As some of the findings may be surprising to university lecturers (e.g. 33% of students marked the sideway parabola as a function) and as this notion is critical in the different subjects students have still to encounter, it is of the utmost importance to inform lecturers and tutors of the actual landscape and provide them with appropriate teaching strategies and examples for use with students who are identified as having specific difficulties. This is precisely what the project will focus on, with the hope of improving students' learning and retention in the mathematical sciences.

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# Maximum problems without calculus: design, teaching and assessment using Maple

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At RMIT University, weekly computer laboratory sessions using Maple have been a component of an otherwise traditional first year calculus course at university. The Maple topics come from the senior school curriculum, but with innovative approaches to curriculum, pedagogy and assessment. A major objective is that students have a positive attitude to using Maple. Some of first year calculus repeats senior school calculus: including "Word problems" which have always been difficult for students. This paper focuses on Maximum problems. Our Maple topics have no lectures: students work collaboratively in small groups, and we take full advantage of the visualizations afforded by the Computer Algebra System (CAS). Students follow an explicit Polya approach to a maximizing an area problem, with an assignment on the Norman window problem that's individualized for each student group. We also introduced multiple representations and multiple solution methods: graphical (zoom-in), animation, proof without calculus and with calculus; with an accompanying parameterized assignment. Here we re-design some optimization problems where the maximum can be found without calculus. If the objective function is a quadratic polynomial, all that is needed is properties of quadratics. Whilst not quite so obvious, for cubic polynomials we show how to find the maximum (and use CAS for scaffolding: to handle some algebraic manipulation). Problems are parameterized so student groups see "different" problems and we use Computer Aided Assessment, CAA (within Maple or using MapleTA) to automatically assess these parameterized assignments. Surveys show students really like immediate automatic marking. Students are engaged, active and collaborative learners with these Maple sessions.

## 1. Introduction

Calculus in first year mathematics has a large component overlapping secondary school calculus (as well as extending school level calculus). This repetition (of the calculus from school) is deliberate, in part because many students have not fully understood their secondary level calculus. Unfortunately the consequence of this is that some of the first year work has been seen before and so lectures may well contribute to boredom: in a study of UK university students [1], it was reported "that 59% of students find their lectures boring half the time and 30% find most or all of their lectures boring." A range of different teaching methods were investigated: the most boring were lab sessions and computer sessions ranked as the second most boring. "Computer sessions too have the potential to be stimulating or tedious; the findings of this study suggest that too many fall into the later category. This could be due to

the manner in which sessions are conducted (e.g. are the computer tasks relevant and interesting?), …".

These findings seem to be hardly surprising since most courses use a traditional teaching approach (lectures) with "tutorials" and assignments which are mostly done without use of technology. When technology such as CAS is used (such as CAS calculators in the schools, or professional CAS such as Maple), it is often just a supplement where the usual "by hand" exercises are completed by using CAS with little or no deviation from the "by hand" approach. It is disappointing that there has been little innovative curriculum development: the CAS has been used primarily for visualization ("zooming in" in school curricula is valuable, but usually ignored at university level) and for "scaffolding" [2] which allows students to make progress with higher level work even if all the lower level skills are not yet fully mastered. For many decades in the Australian state of Victoria, it has been reported that school students have poor skills in algebra (a problem that persists at university level). Scaffolding with CAS does mean that students can proceed to higher level mathematics without needing to develop the high level of algebraic manipulation skills that was once prerequisite. Development of innovative teaching materials which exploit the strengths of the CAS for tightly integrated e-Learning and e-Assessment, such in the Maple component of our first year course generally, and in the Maximum problem as discussed in detail below, can be used to produce active and engaged learners of mathematics and the technology – at school and university.

This paper is organized as follows. Section 2 gives a brief overview of the first semester, first year Maple sessions preceding the optimization topics of this paper. Section 3 outlines how Polya's problem solving approach is applied to a classic maximum area problem, a farmer fencing an area alongside a river; and describes the Norman window assignment which is parameterized (that is "individualized" for different student groups): students are required to follow all of the Polya steps to complete their assignment. Additionally some preliminary remarks (with references) are made about Computer Aided Assessment (CAA).

Section 4 discusses how optimization problems with quadratic and cubic polynomial objective functions can be solved without calculus. For quadratics this gives a demonstration of the value of completion of the square. For cubics, we introduce a new "translate and in the small" approach which could easily be used at school level. In Section 5, multiple representation (visualization) of a standard calculus problem, the Open Box problem (which has a cubic polynomial objective function) is presented and solved without calculus. Section 6 outlines the use of parameterization of problems and explores possible parameterizations of the Open Box problem. Some brief remarks about the use of CAA are also presented. This is followed by a brief Conclusion.

## 2. Background before the optimization problems

The weekly lab sessions using Maple have no lectures: first year students work collaboratively in small groups. Initially, two sessions are allocated for students to work through an Introduction to Maple. An Animation worksheet is followed by an extended Animation Assignment (taking another three sessions): students choose a project from a provided list of projects that are on school level topics, for further details see [3].

With the students now being expert at preparing plots and animations, they are prepared for the next activity: Spot the Curve. Each student group is presented with a Maple file with several randomly generated plots of a curve and a translated copy of that curve: the task is to identify what each translation is. Visualization is used to identify the translation by plotting the student's guess on the same plot as the original curve and the translated curve.

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Also, animation is used to dynamically translate a curve to see the translation in action: resulting in the final result of the translated curve (which should coincide with the given translated curve). The Maple file downloaded from the web has hidden code to generate the random translated curves and also code to automatically mark the student answer. Thus, once the student group has identified the original translation (which is visually obvious), the students can use the provided automatic marking procedure to immediately mark their result. The activity cannot be marked by any other Computer Aided Assessment: for details, see [4].

Another style of automatically marked problem is the FishPond problem, a straightforward application of using the trapezoidal rule to approximate area. Although the problem is presented with a detailed Maple diagram for visualization, the solution method consists of multiple simple steps with numerical answers: the marking procedure (used within the Maple file) is used by students to immediately mark their work: for details, see [4]. The FishPond can also be automatically marked with CAA such as MapleTA. Students have completed all of these activities before undertaking the maximum problem(s).

### 3. Maximum (and minimum) problems

The optimization problems which typically ask for some function to be maximized (or minimized) are introduced at school level in Australia and many other countries. They are usually "word problems" and students struggle with these, so they are also part of the first year of an undergraduate mathematics course at university. These problems are introduced as part of calculus: with little visualization and little emphasis on the Polya approach on "How to Solve It" [5] which is so valuable.

We introduce an explicit Polya approach to maximizing an area problem. A Maple worksheet, follows the Polya method to solve a standard problem. The standard "Farmer Fencing" problem is to fence a rectangular block of land where one side is along a river, and the length of fencing for the other three sides is fixed: find the dimensions for the maximum area. The Polya steps are:

Step 0: Read the problem
Step 1: Draw a diagram
Step 2: Write the problem/information mathematically
Step 3: What is to be maximised (in terms of 1 variable)?
Step 4: What is the range for the one variable?
Step 5: Find the max
Step 6: Check
Step 7: The solution is …
(Step 8: Update the diagram?)

This problem is easy, almost trivial, using calculus for both secondary school students and undergraduates. The Maple file provided includes all details: students just work through this as preparation for undertaking the Norman window assignment. The Norman window has a semi-circular section on the top of a rectangular base. For a fixed perimeter, students are asked to find the dimensions which give the maximum area. Initially we parameterized the problem by assigning a value of the perimeter according to an average of the student group's last few digits of their student numbers, and students uploaded their Maple file which was eMarked: by a tutor who opened the student file; inserted a Marking Report and comments in the student file; and returned the marked file to the students. More recently, we wrote a CAS-immersed CAA: a Maple file used by the tutor to automatically mark all numerical and symbolic results in the student's Maple file. In addition, student plots and comments are placed alongside the correct plot or comment (in the tutor's file) so that the tutor can efficiently mark these elements. The tutor file then automatically prepares a full

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Marking Report (sent to students) and adds the student numbers and their mark to a cumulative Marks List (for upload to the Learning Management System, such as BlackBoard). For a full discussion, see [6].

No other CAA can mark plots (other than our CAS-immersed CAA done entirely within Maple), but provided that no marks are allocated for plots (or comments), CAS-enabled CAA can be used. CAA is widely used for routine drill and practice questions, but we have recently shown how to (easily) author Advanced Questions with MapleTA (the commercial CAA that uses Maple as the CAS). We consider Advanced Questions to be either 1. requiring the author to have some Maple expertise, or 2. implementing some problem requiring Higher Order Thinking skills, HOTs. We have implemented the Norman window problem (with no marks for the plot) as a (randomized) multiple response question in MapleTA with a full diagram (generated by Maple) provided for students, see [7].

## 4. Quadratic and cubic objective functions – no calculus

The impressive machinery of calculus is universally brought to bear on the introductory optimization problems such as the two problems mentioned above. However calculus is not required for theses problems. For the Farmer Fencing problem the objective function (that is, the function for which a maximum is sought) is the area, $A$, as a function of the two sides of length $y$, and one side of length $x$ (the other side is along a river). The part of the perimeter that is fenced, $p$, is fixed (and $p=1000$ m. in our introductory Maple file). Thus

$$A = x \, ( \, 500 - x/2 \, ) \, .$$

Finding the $x$ that makes $A$ a maximum is a trivial task since $A(x)$ is a quadratic function which obviously has zeros at $x=0$ and $x=1000$, so $x=500$ is the value of $x$ that maximizes the area, $A$.

We suggest that this problem could be used as part of a school student's first study of properties of quadratics, particularly completing the square. This problem provides an applied problem and good opportunities for visualization. If students have access to a professional level CAS such as Maple, some very attractive multiple representations are easy to provide and animations can be run by the students, see [6].

The Norman window problem (see [6] for a full discussion and visualization) has a rectangular base with vertical side $y$ and width $2\,r$, and a semicircle of radius $r$ placed on top. In our Maple file assignment, the perimeter, $p$, is a parameterized constant. The objective function is the area, A, which we want maximized (subject to the constraint that the perimeter has the given constant value). It is easy to eliminate $y$ to obtain

$$A = p \, r - (2 - \pi/2) \, r^2$$

which is a quadratic polynomial in $r$. While slightly more complicated than the previous example, this problem can be easily solved by completing the square, to give

$$A = \, - (2 - \pi/2) \, ( \, r - p \, /( \, 4 + \pi \, ) \, )^2 + \tfrac{1}{2} \, p^2 \, /( \, 4 + \pi \, ) \, .$$

Thus it is clear that a maximum A of $\tfrac{1}{2} \, p^2 \, /( \, 4 + \pi \, )$ is attained when $r = p \, /( \, 4 + \pi \, )$. Then the value of $y$ can be calculated (to be equal to $r$).

Note that Maple provides a CompleteSquare command, but we recommend that school students should initially complete the square "by hand", by which we mean by using paper and pen, or by the CAS but implementing each step in detail (without using the higher level command). Later at school, or within a university introductory or remedial program, when a student's understanding has matured, the higher level command can be used. This approach was first articulated as the White-Box/Black-Box Principle by Bruno Buchberger [8].

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It is interesting to note that, of the many hundreds of first year students who have completed this Norman window assignment, every single student has followed the standard approach and used calculus!

With an objective function that is a cubic polynomial, completing the square doesn't help and calculus is invariably used. Here we introduce another way – without calculus.

Consider a simple example: $f = (3 - x/3) \, x^2$.
A plot is provided: see Figure 1 (below). The minimum is at $x = 0$ and it is easy to see that, for $x$ small, $f \approx 3 \, x^2$. This is obtained by investigating the behaviour for $x$ small, and this skill is very important in applied mathematics.

From the plot (Figure 1), the location of the maximum looks like $x=6$. This can be explored by zooming in, which adds confidence to the hypothesis that $x=6$ is likely to be correct. However we can use a "translation and in-the-small" approach to both find the correct location and to prove this is correct. Let the value of $x$ at the maximum (or minimum) be denoted by $m$ and substitute $x = m+t$ (without knowing what $m$ is):

$$f\_t = 3 \, m^2 + 6 \, m \, t + 3 \, t^2 - \frac{1}{3} \, m^3 - m^2 \, t - m \, t^2 - \frac{1}{3} \, t^3$$

To have $x=m$ be at the maximum (or minimum), we require that the curve is locally "like a quadratic" but with no $t$ term. So we find the coefficient of $t$, which is

$$6 \, m - m^2 \, .$$

This is a quadratic in $m$ and we ensure it is zero. Thus $m=0$ or $6$. From our graph, we know that the second solution gives the maximum (and the first solution must be a minimum).

To illustrate further, we can substitute $m=6$ back into $f\_t$ to demonstrate the correct quadratic behaviour about $m$:

$$f\_t\_m = 36 - 3 \, t^2 - \frac{1}{3} \, t^3$$

For $t$ small, the $t^3$ is negligible, so the curve is like

$$f\_t\_m\_small = 36 - 3 \, t^2$$

This is a quadratic with a maximum value of 36. A plot $f\_t$-$m$ and $f\_t$_m_*small* is given in Figure 1.

Using Maple for the calculation above is straightforward. With $f$ already defined, the sequence of commands is:

```
> f_t:=expand(eval(f, x=m+t));
> coeff(f_t, t);
> m:=solve(%);  m:=m[2];
> f_t_m:=simplify(f_t);
> f_t_m_small:=f_t_m - op(3, f_t_m);
```

## 5. The Open Box Problem

The "original" problem, see Example 2, p. 72 of the school text [9], is:
An open box (a box with no top) is made from a sheet of cardboard which is a rectangle of length 8 and width 5. Each of the corners (squares of length $x$) are removed from the rectangular sheet which is then folded to form the box of height $x$. Find $x$ such that the volume of the box is a maximum. See Figure 2 for a diagram.

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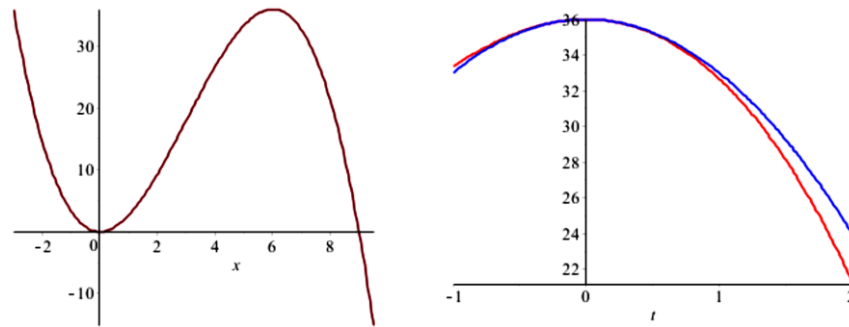
**Figure 1.** (a) A plot of the cubic example of an objective function and (b) the plots of the translated function x=m+t and the t small approximation.
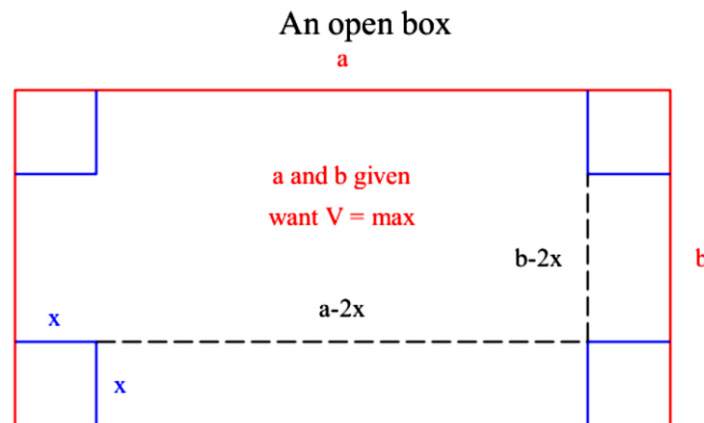


**Figure 2.** A diagram of the rectangular sheet of cardboard from which the Open Box is constructed (by folding). The original problem sets a=8 and b=5.

In this case the objective function is the volume, *V*, a cubic polynomial in *x*:

$$V = (a - 2x)(b - 2x)x, \quad a > b$$

with *a=8* and *b=5*. From the physical problem, $0 < x < b/2$. We provide multiple representation visualization, see Figure 3. Using Maple, we place the 3D plot and the 2D plot side by side, and can also run an animation using *x* as the variable (for both plots simultaneously) and add a Dot symbol on the *V* curve with the curve up to the current *x* value in a different colour (we use blue).
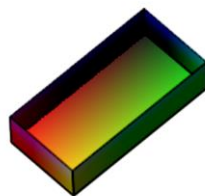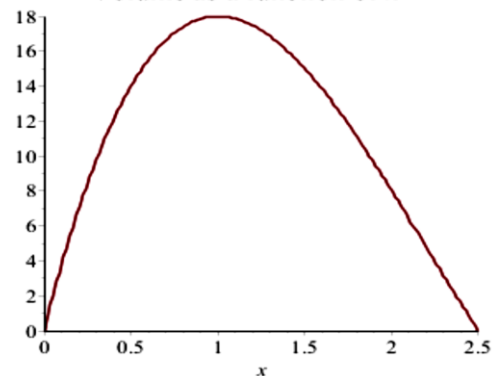


**Figure 3**. (a) A 3D plot of the folded up open box, and (b) The objective function Volume as a function of the height *x*.

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Students are able to solve this problem (as an assignment) by following the Polya approach and using calculus. However the problem can be solved without calculus following the approach described in the section above. In this case, graphical exploration indicates that the solution (to attain the maximum) is *x=m=1*, so a slightly simpler version of the "translation and in-the-small" approach can be used. Since we are confident that *x=m=1* is correct, translate the curve for *V(x)* to the left by setting *x=t+1* to obtain

$$V\_t\_1 = 18 - 14\,t^2 + 4\,t^3$$

so, for small *t*, this behaves like the quadratic polynomial $18 - 14\,t^2$. Thus, *x=m=1* is proven to be correct and the maximum volume is 18.

Students are active learners with this "applied mathematics" activity which uses the Polya approach for problem solving and includes the mathematically important skills of translation and an argument "in the small". Our first year students have studied translation of curves at school – but have not fully understood, so our Spot the Curve activity [4] uses multiple visualizations to discover the random translation of quadratic curves and cubic curves. Immediate automatic marking by the students is available (by using a provided marking command within Maple). Student feedback [4] has been very positive and includes, from 2008:

> I enjoyed a lot "Spot the Curve". The way functions were presented was really easy to understand and complete the assignment. I had a fun with my team.

Unsolicited student feedback included, from 2005:

> It was interesting task. I released I understand everything about quadratic function now. Visualization helped me understand. Can we have more of this?"

> AAAAAH now we understand quadratic functions. Thanks a lot, u helped heaps, and we understand everything. Thanks again

Our students work cooperatively in small groups, and are active learners. Smith [10] compares and contrasts surface learning and deep learning and says:

> Deep learning – the organized and conceptual learning described in the NRC study – is encouraged by
> - interaction with peers, especially working in groups,
> - a well-structured knowledge base with connections of new concepts to prior experience and knowledge,
> - a strong motivational context, with a choice of control and a sense of ownership, and
> - learner activity followed by faculty connecting the activity to the abstract concept.

Our collaborative learning with these CAS activities clearly satisfies David Smith's characterization of deep learning.

## 6. The Open Box Problem and parameterization

The "original" problem (see the previous section) has dimensions of *a=8* and *b=5* and has the solution of *x=1* (to maximize the volume of the folded open box). Our experience with many classes of various sizes from about 25 to more than 100, and various undergraduate year levels from first year to fourth year, is that students with a parameterized problem with different values of the parameter(s) behave as if they have different problems. If visualization

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is required, it is not possible to produce a master symbolic solution file: values of parameters must be used to produce the visualizations. In a collaborative learning environment, a generally lively and cooperative atmosphere is desired, but everyone working on the identical problem tends to encourage copying rather than collaboration. It is also clear that the marking load increases with parameterized problems, so CAA becomes very attractive! (For further discussion of the use of Maple for both the CAS and the CAA, and student feedback, see [11].)

The original problem could just be scaled. However to investigate possible parameterizations, we solve the problem with sides $a$, $b$ to find the height $x$ to give the maximum volume (of the open top box). We introduce the notation $c = \sqrt{b^2 - ab + a^2}$ and use calculus to obtain the solution as $x = \frac{(a + b + c)}{6}$ or $x = \frac{(a + b - c)}{6}$ .

Now $0 < x < b/2$, $a > b$ and $c^2 = b^2 - ab + a^2 = b^2 + a(a-b) > b^2$ , so $a+b+c > 3b$ and hence the first solution is outside the domain. Thus the solution is $x = \frac{(a + b - c)}{6}$ .

Note that if $a$ and $b$ are scaled by a factor $s$, then the solution is also scaled by $s$.

For simplicity of the solution, we would like to have integer or rational numbers. Using irrational numbers for $a$ and $b$ (the lengths of the sides of the rectangular sheet used for the construction of the box) seems to be impractical. For example, starting with a rectangular sheet with dimensions $20\sqrt{21}$ and $14\sqrt{39}$ would seem to be a little strange. Integer values for $a$ and $b$ generally give the solution for the length $x$ of the cut-out as a surd. For example, $a=8$ and $b=6$ gives $x = \frac{7}{3} - \frac{1}{3}\sqrt{13}$ . The solution procedure described above using Maple finds this irrational solution as easily as any other, but it seems to be more natural to avoid irrational lengths.

If desired, we can avoid irrational numbers, by finding integer values of $a$, $b$, $c$ and scaling these to obtain either integer or rational solutions. It is easy to show that if values of $a$ and $b$ give an integer value of $c$, then the values $a$ and $a-b$ do also. Thus the possible values of $a$ and $b$ are paired. Using Maple it is easy to search for all primitive solutions for $a \leq 100$ (which we could call box numbers): they are necessarily relatively prime and are listed in Table 1. Using scaling (with a rational number) and these primitive solutions, it is easy to create as many distinct versions of the problem with integer or rational solutions as desired.

If parameterized problems are solved with Maple we can provide hidden code to automatically mark students work (and add tutor marks for plots) as discussed above. Other CAA, such as MapleTA cannot mark plots. However if we don't mark any student plots, we can successfully use MapleTA to provide a diagram for the students and automatically mark their responses to multiple parts of the question (following the approach for the Norman window problem in MapleTA – for details on authoring, see [7]).

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**Table 1.** Primitive integer values of *a*, *b* for which *c* is an integer.

| | | | |
|---|---|---|---|
| **Original values** (original was the b=5 case) | a=8, b=3 or 5 | c=7 | x= 2/3 or 1 |
| **The 2nd solution** is | a=15, b=7 or 8 | c=13 | x= 3/2 or 5/3 |
| **The 3rd solution** is | a=21, b=5 or 16 | c=19 | x= 7/6 or 3 |
| **The 4th solution** is | a=35, b=11 or 24 | c=31 | x= 5/2 or 14/3 |
| **The 5th solution** is | a=40, b=7 or 33 | c=37 | x= 5/3 or 6 |
| **The 6th solution** is | a=48, b=13 or 35 | c=43 | x= 3 or 20/3 |
| **The 7th solution** is | a=55, b=16 or 39 | c=49 | x= 11/3 or 15/2 |
| **The 8th solution** is | a=65, b=9 or 56 | c=61 | x= 13/6 or 10 |
| **The 9th solution** is | a=77, b=32 or 45 | c=67 | x= 7 or 55/6 |
| **The 10th solution** is | a=80, b=17 or 63 | c=73 | x= 4 or 35/3 |
| **The 11th solution** is | a=91, b=40 or 51 | c=79 | x= 26/3 or 21/2 |
| **The 12th solution** is | a=96, b=11 or 85 | c=91 | x= 8/3 or 15 |

**Conclusion**

We favour cooperative learning with students using CAS and working in small groups on parameterized problems which follow an appropriate preliminary preparatory CAS activity. We look for every opportunity to take advantage of visualization: including animations and multiple representations. Following an introduction to the Polya problem solving method, we present a new approach to solving optimization problems without calculus: the objective function is a quadratic or cubic polynomial. In the quadratic case, since no calculus is required, this provides an applied environment for students to appreciate the value of being able to complete the square *at the time* that they meet completion of the square. For beginning university students, this is also a valuable activity, particularly for the cubic case where translation of the curve and approximation in the small is utilized. The Open Box (maximum volume) problem is used to demonstrate this new approach, and possible parameterizations are investigated.

With parameterized problems, the use of CAA becomes very attractive. If the professional level CAS Maple is used, then more powerful multiple representations are possible and also coding for CAA can be included within Maple (and students like immediate automatic marking). Otherwise a CAS-enabled CAA such as MapleTA can be used.

Preparation of good CAS teaching and learning materials with CAA requires considerable staff resources. I find that it takes approximately 10 hours to produce an innovative Maple file for students to use in a one hour lab session. On occasions, I have spent 30 hours to conceptualize and achieve something that is unusually innovative. Since I am now very experienced at designing and implementing CAA, I can provide CAA within Maple in about 6 hours for something basic. It is very much faster to produce CAA for routine drill and practice, particularly if a package such as MapleTA is used. For further comments and a call for staff collaboration across different universities, see the Engineering Mathematics and Applications Conference keynote address by Blyth [12]. In further work, we use binomial series to extend the class of objective functions for which the optimization problem can be solved without calculus. Our students enjoy these activities (and request more of the same) and are engaged as deep learners.

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**Table 1**. Participation in Delta conferences

|  | **97** | **99** | **01** | **03** | **05** | **07** | **09** | **11** |
|---|---|---|---|---|---|---|---|---|
| Host country | Aust. | Aust. | S.Afr. | N.Z. | Aust. | Arg. | S.Afr. | N.Z. |
| No. of participants | 106 | 65 | 109 | 115 | 100 | 53 | 157 | 122 |
| No. of countries | 5 | 8 | 19 | 9 | 15 | 15 | 32 | 15 |

## 2. Classification of papers

A book of abstracts only was published for Delta 97, but from 1999 full proceedings have been published. Since 2001, there have been two publications from each conference. Some papers are published in a special edition of a journal, and others in a book of proceedings. All published papers are peer-reviewed. The classifications in Table 2 include all full papers that have been published in either a journal or a book of proceedings, as well as the presentations from 1997. We have not included papers from other years for which only abstracts were published, nor have we included discussion papers or workshops.

**Table 2**. Classification of papers in Delta conferences from 1997 to 2011

| Category \ Year | **97** | **99** | **01** | **03** | **05** | **07** | **09** | **11** | **Total** |
|---|---|---|---|---|---|---|---|---|---|
| Teaching and learning | 13 | 9 | 24 | 17 | 17 | 8 | 9 | 12 | **109** |
| Technology | 19 | 8 | 14 | 11 | 8 | 5 | 4 | 11 | **80** |
| General pedagogy | 5 | 6 | 4 | 9 | 3 | 3 | 7 | 4 | **41** |
| Transition | 4 | 3 | 4 | 6 | 4 | 5 | 3 | 7 | **36** |
| Student resources | 3 | 6 | 7 | 3 | 4 | 2 | 2 | 2 | **29** |
| Professional development | 0 | 0 | 2 | 1 | 3 | 2 | 2 | 8 | **18** |
| Assessment | 2 | 3 | 3 | 5 | 2 | 1 | 1 | 0 | **17** |
| Pre-service teachers | 2 | 1 | 4 | 5 | 1 | 1 | 1 | 1 | **16** |
| Specific subsets of student cohort | 1 | 1 | 1 | 5 | 1 | 1 | 1 | 2 | **13** |
| Group learning | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | **9** |
| Miscellaneous | 5 | 2 | 1 | 6 | 6 | 6 | 6 | 9 | **41** |
| **Total** | **57** | **40** | **65** | **69** | **50** | **35** | **37** | **56** | **409** |

We found the list of categories in Table 2 to be a convenient way of classifying the 409 papers we considered, but do not suggest that the list is definitive. Any attempt at a classification such as this is bound to be personal to some extent. We note, however, that the categories chosen by Hubbard [2], are a subset of our categories, and that most of the categories chosen by Barry in his reports of Delta 07 and Delta 09 [3],[1], correspond to one of ours (albeit with different names). Similarly, in a review of research in tertiary mathematics education in Australasia between 2008 and 2011 by Barton et al. [4], transition, technology and specific mathematical topics are identified as major themes. (Note that this review was not restricted to Delta papers, and cites papers only from the journals associated with Delta 09 and Delta 11 [5], [6], not from the proceedings.) Of course, many papers overlap more than one category. The assignment of a paper to one category only was a decision taken by the authors based on what we considered to be the major thrust of the paper. Again, such decisions are bound to be personal. Despite these disclaimers, we believe that the numbers in the table provide a good overview of the relative importance Delta participants have afforded various issues over the years. In the following sections we discuss themes which emerged within the most common categories.

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### 3. Teaching and learning specific topics

Papers in this category include descriptions of initiatives taken in individual courses, studies of student responses to such initiatives, opinions on how a particular topic should be taught, and studies of how students learn (or fail to learn) specific topics. It is not surprising that this category contains the largest number of papers. As Hubbard [2], commented, "most participants work at the coalface in university education, teaching or facilitating courses, trying to improve their work and to help others do likewise". The 109 papers in this category have been further classified in Table 3. The focus of attention has been on calculus, DEs & modelling, and statistics, with calculus and statistics being discussed predominantly at the first year level. Given that most first year mathematics courses include linear algebra, the relatively small number of papers on that topic is somewhat surprising. Advanced mathematical topics have received scant attention, as has the topic of proof.

**Table 3.** Sub-classification of papers on teaching and learning specific topics

| Topic \ Year | 97 | 99 | 01 | 03 | 05 | 07 | 09 | 11 | Total |
|---|---|---|---|---|---|---|---|---|---|
| Calculus | 8 | 4 | 7 | 4 | 2 | 1 | 1 | 1 | **28** |
| DEs and modelling | 0 | 0 | 4 | 6 | 5 | 2 | 4 | 4 | **25** |
| Statistics | 0 | 3 | 4 | 3 | 4 | 4 | 1 | 3 | **22** |
| Linear algebra | 2 | 0 | 1 | 1 | 2 | 1 | 3 | 1 | **11** |
| Mathematics for non [maths/eng] | 2 | 1 | 3 | 2 | 2 | 0 | 0 | 0 | **10** |
| Proof | 0 | 1 | 3 | 0 | 2 | 0 | 0 | 1 | **7** |
| Advanced maths | 1 | 0 | 2 | 1 | 0 | 0 | 0 | 2 | **6** |
| **Total** | **13** | **9** | **24** | **17** | **17** | **8** | **9** | **12** | **109** |

The calculus reform movement was in full swing in the 1990s, and the world-wide interest in the movement was reflected in the early Delta conferences. At Delta 97 Deborah Hughes-Hallett (one of the leaders of the reform movement) was a keynote speaker, and there were presentations from Australia, South Africa and the USA describing the implementation of calculus reform in particular first year courses. Delta 99 saw an argument for further reform from Belward (in [7]), as well as a report from Ruxton (in [7]) on calculus reform in a second year multi-variable calculus course. Harman (in [8]) speculated on the future of the calculus reform movement, and argued that it is important that reform projects be thoroughly researched and evaluated. Such research does not appear to have occurred, and since 2003 there have been very few papers devoted to either calculus reform or to student learning in calculus. Papers that do focus on student learning in calculus include an investigation of students' performance and confidence in handling procedural as opposed to conceptual problems by Engelbrecht et al. (in [8]), and a report of students' difficulties with volumes of solids of revolution by Mofolo et al. (in [9]).

Student learning is the focus of almost all of the small number of papers on linear algebra. Stewart and Thomas (in [8]) found that first year linear algebra students had difficulties in understanding definitions. Hillel and Mueller (in [10]) investigated whether or not students are able to make links between different interpretations of the solution space of a system of homogeneous equations. Further papers by Stewart and Thomas (in [5] and [11]) study student thinking about various linear algebra concepts by applying both APOS theory and Tall's three worlds of mathematics. Hannah (in [12]) also used Tall's theory in an attempt to improve his students' understanding of span and subspace. Britton and Henderson (in [5]) investigated why second year students have particular difficulty with the concept of closure

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of a set, and with the idea of functions forming vector spaces. They followed this with a paper (in [13]) in which suggestions were made as to how students could be helped to construct a simple proof of closure.

Most papers on DEs and modelling are of quite a different type from those on other topics. With only a few exceptions, these papers present particular DEs and their solutions, and a small number of presenters are responsible for almost all of them. (For example, Fay and Joubert (in [8], [10] and [12]), Fay (in [12] and [13]), Joubert (in [14]), Martinez-Luaces (in [8], [9], [10] and [12]), Martinez-Luaces and Cobs (in [10] and [14]).) Papers that deal with modelling in a more general way include Harding's study of student performance in a modelling course (in [10]), and a description of four different approaches to teaching modelling by Anaya and Cavallaro (in [9]).

The number of papers on statistics has remained reasonably consistent since 1999, and most papers address the issue of how statistics should be taught, either in general or with respect to a specific topic. Most recently, Pfannkuch et al. (in [6]) argued for major changes in the way statistical inference is taught, suggesting a move away from a norm-based mathematical approach to a computer-based approach. Sowey (in [15]) put forth his belief that statistics should be taught with a perspective on the logic of statistical inference, while Varsavsky (in [16]) addressed the issue of a relevant mathematics and statistics curriculum for science students. A new course using ideas from gambling, sport and medicine was described by Croucher (in [10]). The challenge of explicitly teaching the interpretation and critical evaluation of statistically-based reports in a large introductory statistics course was addressed by Pfannkuch et al. (in [8]). Papers that focus on teaching a specific topic include Roberts and Pierce on the central limit theorem (in [7]), Stewart on Bayesian statistics (in [9] and [13]), Sinay on stochastic processes (in [9]) and van Staden and Loots on kurtosis (in [12]). There are no studies that focus exclusively on student learning in statistics, but Petocz and Reid (in [15]) investigate students' conceptions of statistics.

The topic "mathematics for students other than engineering or mathematics students" includes descriptions of courses for business students, agriculture students and liberal arts students. In a plenary address in 2005 Briggs (in [10]) discussed the type of mathematics courses that should be taught to students of fine arts and the humanities, and concluded that the development of such courses should be a collaborative and interdisciplinary effort in which mathematics departments take the lead.

## 4. Technology

The use of technology has been one of the major themes at each Delta conference, with the overwhelming majority of papers describing the introduction of a particular piece of technology into the teaching and learning of an individual course. Technological change has been rapid since 1997 and the use of technology in mathematics teaching and learning has increased enormously. On-line delivery of material and assessment in one form or another is now commonplace. Hence, many of the papers from early Deltas now seem out-of-date, and so we restrict our discussion to papers from 2005 onwards.

Tablet PCs have captured the interest of several presenters in recent years. Dekkers and Shepherd (in [10]), Loch (in [10]), Bonnington et al. (in [9]), Loch et al. (in [6]) and Dekkers et al. (in [13]) describe the use of tablet PCs in a variety of courses, and all claim that the technology is useful and effective. Student feedback is reported as positive, and Loch et al. found that the tablets promoted collaborative learning. However, whether or not learning outcomes are improved by the use of this technology has not been reported, leaving room for further research. Four papers discuss the use of classroom response systems (CRS, or 'clickers'). Stewart and Stewart (in [12]) argue that the clickers engaged students and allowed

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lecturers to better understand students' level of learning. Lozanovski et al. (in [13]) found that education students reacted favourably to the use of clickers, but caution against assuming that this is associated with any measurable improvement in overall performance. The use of mobile phones as clickers is reported by Dunn et al. (in [13]), and a teaching model for the use of clickers is proposed by Stewart and Stewart (in [13]). The authors of all four papers on CRS argue for more research in this area.

Several papers discuss the use of computer algebra systems (CAS), although the number of such papers has declined significantly since the early Deltas. For example, Tonkes et al. (in [15]) report on the development of a learning model using Matlab; Blyth and Labovic (in [5]) report on the development of a learning system that incorporates computer-aided assessment using Maple; Tobin and Weiss (in [13]) discuss ways of teaching DEs using CAS in general; and Mofolo (in [10]) describes teaching volumes of solids of revolution using Mathematica. A paper by Ponce-Campuzano and Rivera-Figueroa (in [13]) gives examples of the variety of solutions that different CAS provide when finding anti-derivatives, and argues that these can be used to encourage students to be more thoughtful when using CAS. The ability of students to understand the output from CAS is clearly important and is explicitly studied by Berger (in [12]). Rather alarmingly, Berger found that more than half the students in the study could not interpret the output appropriately.

## 5. General pedagogy

At each Delta conference there have been some papers that take a broader view and deal with general issues related to teaching and learning mathematics. Early Delta conferences included a number of papers that argued a need for change in undergraduate mathematics curricula and the ways in which mathematics is delivered to students. The first of these was the plenary address given in 1997 by the inimitable Jerry Uhl (in [17]). Uhl argued that student learning is improved if long lectures are abandoned and replaced by time in a computer lab where students learn by exploration and guided discovery. The need for change in order to cope with an increasingly diverse student population, and/or to make mathematics accessible to a greater diversity of students, was argued by Oldknow in [7], Smith in [7], Zevenbergen in [7], and Steen in [8]. Others recommended changes in order to attract students to mathematics (Kagesten in [7], Pierce et al. in [8]), to cope with large classes (Chacko in [14]) or simply to improve student learning (Coutis et al. in [7]). All these articles advocate some or all of the following: flexible delivery, presentations from students, peer tutoring, use of up-to-date technology, co-operative learning, explicit aims and objectives for courses, use of realistic problems, self-test online quizzes. Papers such as these do not generally appear after 2003, possibly because many of these improvements have been adopted to some extent by most universities.

Several interesting and thoughtful theories related to the way students learn have been presented. Tall (in [10]) presented his "3 worlds of mathematics" framework in 2005. Mason's plenary address (in [13]) in 2011, on definitions and how mathematicians use them, is thought-provoking, as are Easdown's two papers (in [5] and [13]) on the interplay between syntactic and semantic reasoning. The complex role played by visualisation was explored by Sierpinska (in [8]) in her plenary address in 2003. The question of whether or not students learn anything as a result of attending a traditional lecture was addressed in several papers (Hubbard in [17], Cretchley in [10], Wood et al. in [11]), and each of these provide evidence that the answer is in the affirmative. While tutorials are generally considered to be useful to learning, Henderson and Britton (in [8]) found that students have widely differing views on how a tutorial should be conducted. The use of historical anecdotes to enrich and enliven mathematics teaching has been advocated (Maritz in [14], Montelle in [12]), and for those

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wishing to do so the latter paper provides some wonderful and lesser-known anecdotes. Interdisciplinary teaching is an area that has been identified in several papers (Wood et al. in [14], Matthews et al. in [5], Matthews in [13]) as one in which further research would be valuable.

## 6. Transition

The problems encountered by under-prepared students when they commence tertiary mathematics have been documented at every Delta conference. Many factors contribute, including fewer students studying advanced mathematics in high school, a disconnect between the style of high school mathematics teaching and that encountered at university, the opening up of tertiary education to far greater numbers of school leavers and more adult learners commencing degree study.

In the first four Delta conferences, researchers reported on particular bridging/preparatory courses (Brooks and Hewett in [17]), the importance of developing independent learning skills (Snyders in [7]), the use of diagnostic testing to identify students at risk (Barry et al. in [14]), the persistence of rote learning at university (Barnard in [8]), and strategies to scaffold learning (Hartnall in [14]). In an overview article, Persens (in [14]) pointed out that student difficulties in transition were universal, and appealed to academics to share best practice on all successful remediation initiatives.

From Delta 05 onwards, there is increasing emphasis on students' beliefs, perceptions and attitudes, and on examining the context of tertiary study rather than the content. Several articles described the benefits of reflective writing exercises and strong pastoral care (Parnell in [10], Parnell and Statham in [11], Miller-Reilly in [11]). Gordon and Nicholas (in [13])–studied bridging course students' perceptions of their main challenges. The role of mathematical self-confidence and the benefits of peer support, collaborative learning and good study skills were the subject of several papers (for example Carmichael and Taylor in [15] and D'Arcy-Warmington in [10]). Opinions of lecturers and high school teachers were investigated by Hong et al. (in [5]), who concluded that the professionals in the two groups generally worked in ignorance of the other's teaching practices. The challenge of supporting students in transition in an era of increasing student numbers is to manage the process sustainably. A paper describing the introduction of the University of Missouri-Kansas City Supplemental Instruction System (Harding et al. in [6]) gives an interesting insight into the benefits and drawbacks of implementation when student numbers are large. An article by Tularam and Amri (in [13]) reminds the reader that while universities have a duty to offer support systems to students to enable them to develop into mature learners, students also have to accept responsibility for their learning.

## 7. Other categories

Papers dealing with specific subsets of the student cohort (for example, ESL students), group learning and the education of school teachers are relatively low in number and have remained fairly stable, while the number dealing with assessment has declined markedly over the years. This last fact is surprising, given the huge influence on student behaviour exerted by assessment in all its forms. The categories of student resources and professional development have the highest overall representation and also the highest numbers since Delta 03, so our discussion will concentrate mainly on these areas.

The student resources category encompasses a wide variety of tools, both real and virtual, whose purpose is to enhance student learning. Topics include projects to bolster essential skills (for example, Barry and Davis in [17] and Craig in [12]), projects designed to upgrade study skills and examination technique (Coutis and Wood in [17]), the building of

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on-line resources such as quizzes and flash animations, and improved design of course web pages. Papers by Bulmer (in [7]), Varsavsky (in [7]), Broughton et al. (in [7]), Norton and Ovens (in [10]), Jacobs (in [15]) and Bukhatwa et al. (in [13]) provide examples of these web-based initiatives. The role of pastoral care in large statistics classes was investigated by Bulmer and Miller (in [14]) and Bulmer and Rodd (in [15]), while an account of how the use of on-line teaching and pastoral care rescued a statistics course affected by the Christchurch earthquakes is provided by David and Brown (in [13]). Engineering students often require specific resources to support their mathematics. A South African initiative (Steyn and Du Plessis, in [11]) targeting both mathematical and professional skills not only resulted in good mathematical outcomes but also improved retention rates. In the UK, the Helping Engineers Learn Mathematics project provides purpose-built support materials and facilities, resulting in improved student motivation and, in some cases, better exam performance (Green et al. in [10], Harrison and Petrie in [12]).

Many universities now require new academics to undergo some formal training in teaching and to provide evidence of good teaching skills for promotion. This has been reflected in Delta conferences, with the number of papers on professional development trending upwards. Barton (in [6]) constructs a framework to support a self-examination of his lecturing. Other similar studies come from Hannah et al. (in [6]), Paterson et al. (in [6]) and Kensington-Miller et al. (in [13]). One feature of interest in this research is the supportive way in which mathematicians and mathematics educators work together to analyse lecturers' classroom practice and find ways to implement change, a rare partnership indeed. Views of tutors, lecturers, coordinators and heads of school were sought by Wood et al. (in [6]) in a survey to determine the most useful content for a new professional development course designed for Australian academics. Main themes identified were the management of large classes, the lack of knowledge of modern teaching methods, how to handle gaps in student knowledge and the tension between teaching and research. A paper by Wood and Harding (in [11]) gives a checklist of factors which can help to define and measure good lecturing, providing a useful tool for academics seeking to have their professional development efforts formalised. Perceptions of teaching assistants/tutors on the nature of their work and the developing understanding of their students have been examined by Froman et al. (in [9]), Bieseigel (in [13]) and Scataglini-Belghitar and Mason (in [13]). Oates et al. (in [15]) describe the success of a for-credit tutoring course whose graduates (student-tutors) then teach younger students in collaborative tutorials. Graduates of this course are reported to have enhanced mathematical understanding, deeper knowledge of good teaching practice and increased personal satisfaction.

The papers categorised as miscellaneous cover a wide spectrum of topics and just a small number can be mentioned here. Surveys on the nature of undergraduate mathematics education in various countries are reported by Varsavsky and Anaya (comparing Australia and Argentina, in [12]) and by Varsavsky et al. (in [13]), comparing many of the countries which have traditionally been well-represented at Delta conferences. Varsavsky (in [14]) and Craig (in [6]) studied aspects of students' writing and concluded that writing tasks can provide valuable insights into the struggles faced by students as problem-solvers. Wood and Reid (in [10]) wrote of the need to reform the curriculum to improve communication and inter-personal skills of mathematics graduates. Varsavsky (in [9]) examined survey data from Australian graduates, showing that students value the provision of a supportive social environment and the development of generic skills. Reid et al. (in [8]) reported on student notions of professional work in the mathematical sciences, the way they learn mathematics and their conceptions of mathematics. A paper from Delta 11 comes somewhat as a reality check. Bartholomew et al. (in [6]) explored the ways in which students talk about studying

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mathematics. The students' narratives appeared to focus on notions of natural ability and enthusiasm for studying mathematics for its own sake, leading to a pervasive view that success in mathematics confers membership of an elite club. Bartholomew et al. suggest that this view be challenged, and that students who struggle to succeed in mathematics, or who see the subject simply as a sensible career choice, should be actively encouraged to achieve their goals.

## 8. Discussion

The first Delta gathering posed the question: "What can we do to improve learning?" Subsequent Deltas have had various themes, but the fundamental focus remains a search for answers to that question. We have provided here a brief overview of the multitude of ways in which past participants have attempted to provide such answers. In doing so, we have recognised afresh the value of the publications from Delta conferences as a resource for mathematicians and statisticians with an interest in teaching and learning. Unfortunately, most of the publications are not readily available. The iJMEST journals ([5], [6], [11], [15]) are available online, as are the proceedings from Delta 11 ([13]). However, none of the other proceedings is available electronically nor are the journals [8] and [16]. The construction of a website on which all publications are available would be a worthwhile enterprise, in our view.

Teaching and learning specific topics, and the use of technology, have remained the dominant themes over all eight conferences. The reporting of teaching initiatives in calculus has declined significantly in recent years, and there is scope for future research that addresses developments in the teaching and learning of calculus in the post-reform era. It is disappointing that more advanced mathematics topics, including proof, have to a large extent been neglected in this forum. The number of students majoring in mathematics is small compared with the number in first and second year, but they also face challenges in learning in their senior years and should not be ignored. While many papers on technology report positive student responses to its use in a particular course, the wider implications of technology use, such as its impact on student learning and on the curriculum, have been largely ignored by Delta presenters.

Interdisciplinary teaching between mathematicians and other scientists and engineers, the involvement of mathematicians in developing courses for liberal arts students, and communication and collaboration between university and high school teachers are all areas in which it has been suggested that further research would be fruitful. As noted previously, assessment has not been a focus of attention in recent conferences. The fact that many universities are now moving towards standards-based assessment should provide some interesting opportunities for research on assessment in the future. The influence of university policies can also be seen in other emerging trends: the increasing emphasis on professional development, provision of resources for students in transition and investigations of students' perceptions and opinions.

The name Delta was chosen to imply change. Over the sixteen years of Delta's existence, the Delta community has grown and evolved, and the types of conference presentations have also changed. Participants at Delta 97 were, by and large, mathematicians with an interest in teaching and learning but possibly little expertise in educational theory. Most presentations were reports of a particular teaching experience or initiative. The conferences now see many more formal research presentations, based on educational theories and methodologies. However, many mathematicians and statisticians with research interests in their mathematical discipline attend Delta conferences and value the experience. Hopefully Delta will continue to accommodate academics who are not formally involved in research in mathematics

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education, and to accept papers which allow for the dissemination of interesting teaching initiatives.

Delta conferences remain invaluable forums bringing together academics from diverse countries. Sixteen years on, it is timely to remember the debt of gratitude owed to Patricia Cretchley, whose vision gave birth to the Delta community.

## Acknowledgement

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# Students' mathematical preparation Part B: Students' perceptions

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**Keywords:** Tertiary mathematics education; mathematical under-preparedness; diversity of student population

There is a growing concern worldwide over the decrease in the number of mathematics and science graduates. Associated with this are factors such as the mathematical under-preparedness of students entering the university system, and a lack of emphasis on the importance of mathematics for post-secondary studies.

This paper analyses students' perceived preparation in mathematics encountered in their first semester of undergraduate study at the University of Southern Queensland. The excerpts are drawn from surveys and interviews of students enrolled in first year courses that have mathematical content. The survey showed that there was a broad response of being "well prepared" for mathematics across first year courses. Interviews generally confirmed the responses to the survey in terms of students' confidence in their mathematical preparation. However a significant number of students in the science based degrees acknowledged that they were inadequately prepared

## 1. Introduction

The student population of Australian universities has become more diverse than ever before [1, 2]. This diversity covers not just cultural or socio-economic diversity but also academic background and approach to study. The federal government's acceptance of the Bradley Review [3] and the resulting policy changes including uncapped places and more graduates from low socio-economic areas are going to heighten this diversity and hence the difficulties faced by students, lecturers and university policy designers.

Students' preparation in mathematics and statistics is of particular interest in this study, because a smooth transition to tertiary education can, for many students, be hindered by less than adequate mathematical background. It has been shown that students who face difficulties with their first year mathematics may not continue or may fail some courses [4].

At the University of Southern Queensland (USQ), which is a regional university with strength in distance education, over 80% of students are not recent school leavers [5] and about 23% are designated as having low socio-economic status (SES) [6]. Low SES has been shown to be associated with low academic achievement [7]. In addition, about 53% are from regional or remote areas [6]. This diversity not only means a range of academic abilities but also means a variety of mathematical preparation which can result in academic difficulties for universities and students. The situation at USQ is compounded by the multiplicity of pathways for entry into first year, each with their own level and coverage of mathematical concepts. For example students may enter from school directly or via the Technical and Further Education (TAFE) system. USQ also provides entry pathways which include the Tertiary Preparation Program (TPP) and, English for Academic Purposes (EAP). EAP is a

suite of English language courses. Many students may also have a considerable time gap between finishing formal study and beginning university and some of these gain entry without a preparatory program, entering on their past experience often with very dated qualifications.

In order to ameliorate these difficulties, USQ staff need to be aware of what first year students feel about their mathematical preparation coming into undergraduate study. So the aim of this project is to examine first year students' perceptions of their mathematical preparedness for tertiary study. This information was obtained by student questionnaires and interviews. At the same time, the views of lecturers of first year courses were sought. They were asked what requirements, in terms of topics and level of mathematics, are necessary for their course even though there may not be a formal prerequisite of a certain level of mathematics. Lecturers were also asked their opinions of how well their students were mathematically prepared. These results can be found in a companion paper.

The results from this study will assist the planning of lectures and assessment in all programs, particularly in first year mathematics service courses, such as Engineering Mathematics, and enabling programs such as the TPP and the EAP. Academic support, in the form of The Learning Centre, also would benefit from this information for developmental purposes.


## 2. Literature review

Mathematics, along with the other STEM (Science, Technology, Engineering and Mathematics) areas, is crucial to the development of modern society. However there is a world-wide shortage of graduates in these fields [8-10].

Research in mathematics education at the tertiary level is still modest [10], and does not adequately cover the secondary-tertiary transition. A review of the literature produces clear evidence that this transition in mathematics is a complex problem [11, 12]. For example, in Hong Kong there are four factors that are making the transition problematic: students' lack of preparation; adjustment to the pace of courses; mathematical rigour; and examination processes [13]. In particular, other researchers have also identified mathematical under-preparedness as problematic and suggest this has an influence on students' achievement in university mathematics [13-16]. Part of the problem is that parents and guidance officers influence their students' ideas about the significance of mathematics and its applicability [17, 18].

Researchers from around the world have provided a number of reasons for the under-preparedness in mathematics such as inadequate funding and a recent trend of moving to mass university education and hence to a greater diversity of student backgrounds. The approval of the findings of the Bradley Review [3] means there will be more graduates from low socio-economic areas. The increase in numbers of students that have low SES has meant that greater support is needed to maintain academic achievement [14, 19, 20]. There has been a lowering of the mathematics standards at school and hence at universities [20, 21]. This has also led to students having problems in areas such as biology and nursing [21-25].

Confusion can be caused by the assumed knowledge for a university course not being made explicit to students. Students may be accepted into a university even though their mathematical background does not match published assumed knowledge for their degree program [23, 24].

It is clear that a problem of mathematical preparedness at university has existed for some time. However it is unclear if these perceptions about preparation for first year university study are changing. It is also unclear if lecturers and students are in accord when it

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comes to beliefs of students' preparation for first year study. The current project investigates these issues.

## 3.    Method

Students from the main first year courses were surveyed. Ethics clearance was obtained to survey and communicate with students from the Faculties of Arts, Business and Law, and Sciences. However, students from the Faculties of Education and Engineering and Surveying were included in the survey because many were enrolled in the courses surveyed.

All the examiners (coordinators) of the key first year courses within the three faculties were asked to email their students with our invitation to fill out the survey. All these examiners agreed to do so. Students were encouraged to participate by offering them the chance of winning a $100 book voucher at the University bookshop. Their consent to a brief interview about their perceptions of their mathematical preparedness was sought within the survey. The survey elicited 122 responses and 20 student interviews.

Information was gathered from the students after the end of Semester 1 and in the first weeks of Semester 2. This was after the students had received their results for Semester 1 in order to obtain a clearer picture of their perceptions.

The questions that the students were asked covered topics such as the degree they were enrolled in, their major subject and pathway into university and what level of mathematics that entailed. Another question was what courses they had studied in Semester 1 that required some mathematics even if it was elementary mathematics. Perceptions of their preparation in various topics were sought using a Likert scale with the options "Well prepared", "Prepared", "Poorly prepared", "Very poorly prepared" and "Not applicable" in this order. Analysis of the survey responses revealed a possible ambiguity with the term "Not applicable". Adjustments will be made to the survey for future use. The topics included percentages, ratio, algebra, statistics, decimals, problem solving and calculus. There was an opportunity for them to add a topic. They were asked if their overall mathematical preparation was adequate for first year study followed by a possibility to make any comments. Finally they were asked to suggest what USQ could do better in terms of mathematical preparation.

## 4.    Results

All first year students enrolled in key first year courses were sent the survey resulting in a broad spectrum of views about their mathematical preparation. These data are categorised by the level of mathematics required for their study because students following different pathways have different mathematical needs. The students were grouped into three areas; Engineering and Science, Nursing and Business, Arts and Education.

Figure 1 and Figure 2 show the pre-university level of mathematics completed by respondents. As can be seen respondents came from diverse mathematical backgrounds. Year 12 Level A mathematics is the basic course. Level B introduces calculus. Most science and engineering degrees assume their students have this level of mathematical knowledge. Level C covers more advanced topics. Those that did not state their school level have either completed their secondary education outside Australia or have entered university through an alternative pathway. Within each area of study, approximately 30% have completed the equivalent of Level A Mathematics. Some of the students that did not complete Year 12 may have also completed TAFE courses or one of USQ's tertiary preparation courses.

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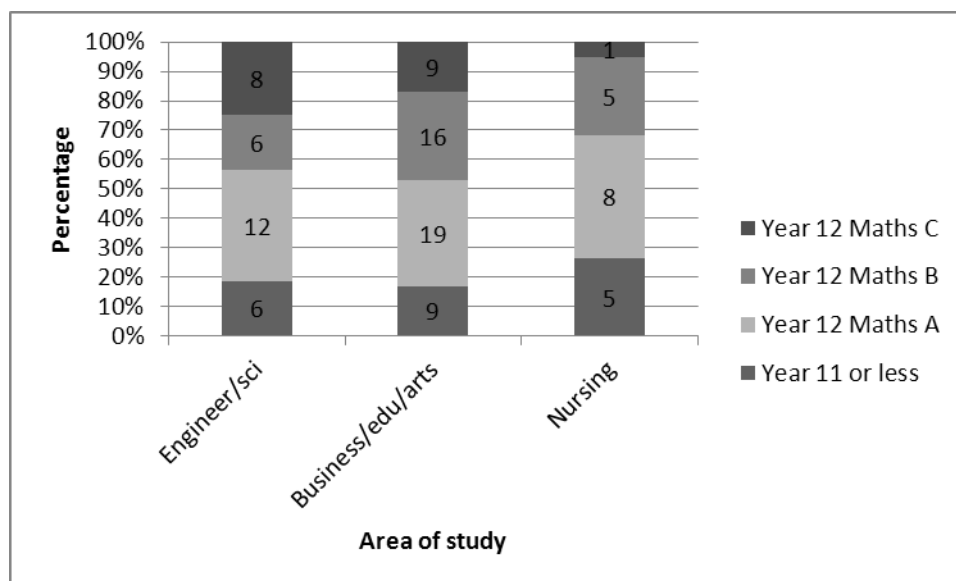
**Figure 1.** School level completed as a percentage within area of study (n = 104).
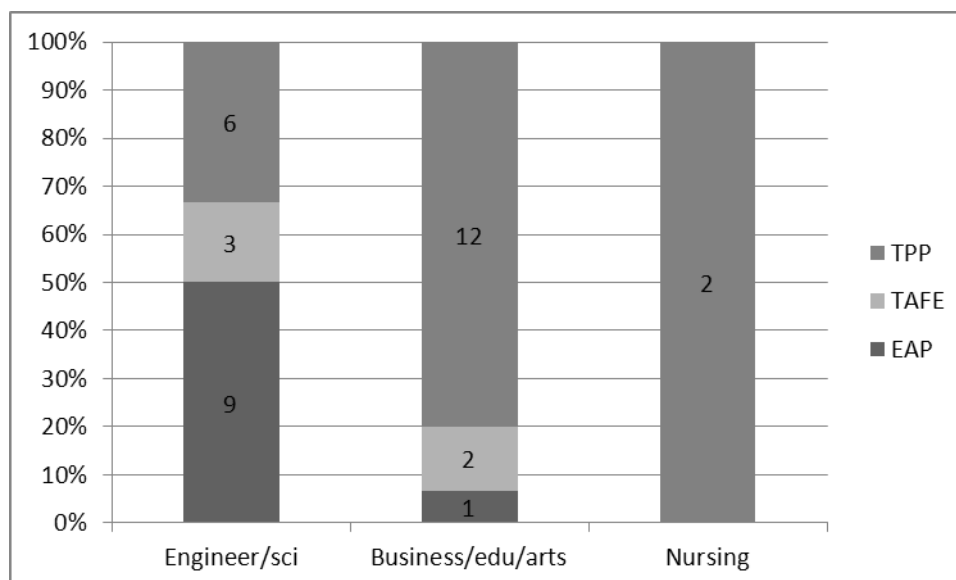


**Figure 2.** Alternative pathway completed as a percentage within each area of study (n = 35).

The following graphs relate to students' perceptions of how well prepared they were in the area of basic arithmetical concepts through to the more complicated mathematics of trigonometry and calculus, when encountered in first year courses. To be noted is that a response of "not applicable" should imply that the student felt that he/she had not encountered this area of mathematics in any of his/her first year courses. This option was not fully explained and may have led to some students responding with "Prepared" or "Not prepared" when their choice should have been "Not applicable".

Figure 3 shows the responses from engineering and science students. Apart from being well prepared for the topics of decimals and percentages, responses from 20% to 30% of the students indicated that they felt poorly prepared.

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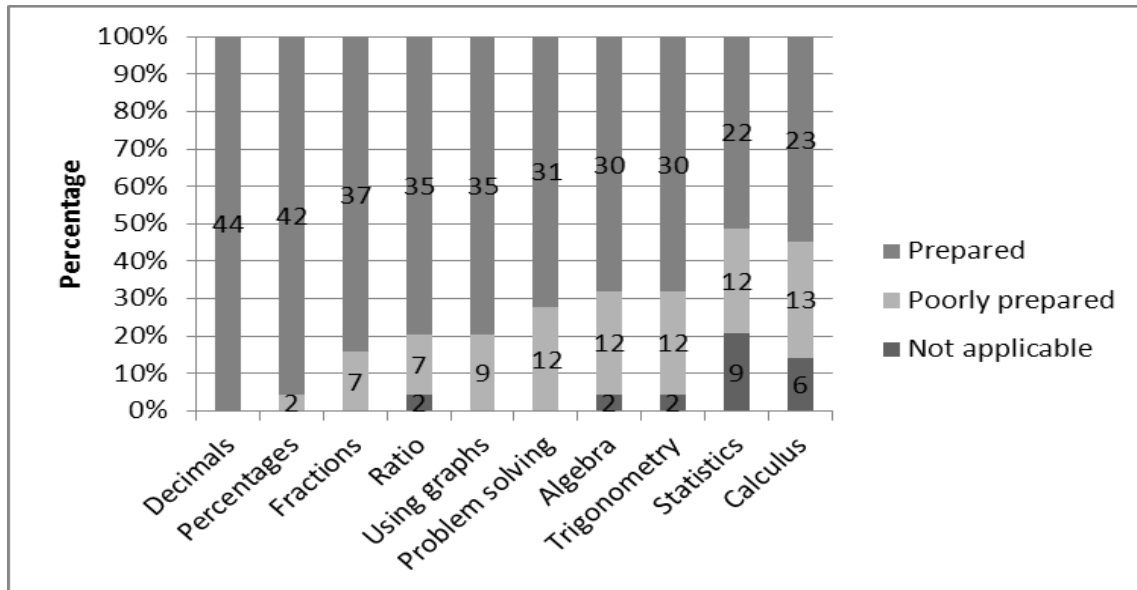
**Figure 3**. Engineering and sciences student responses to preparedness in mathematical topics (n=44).

Figure 4 indicates that on the whole, business, education, arts and general studies students felt well prepared for the mathematics encountered within their first year courses. Many of the concepts had not been experienced in first year courses by some of these students.
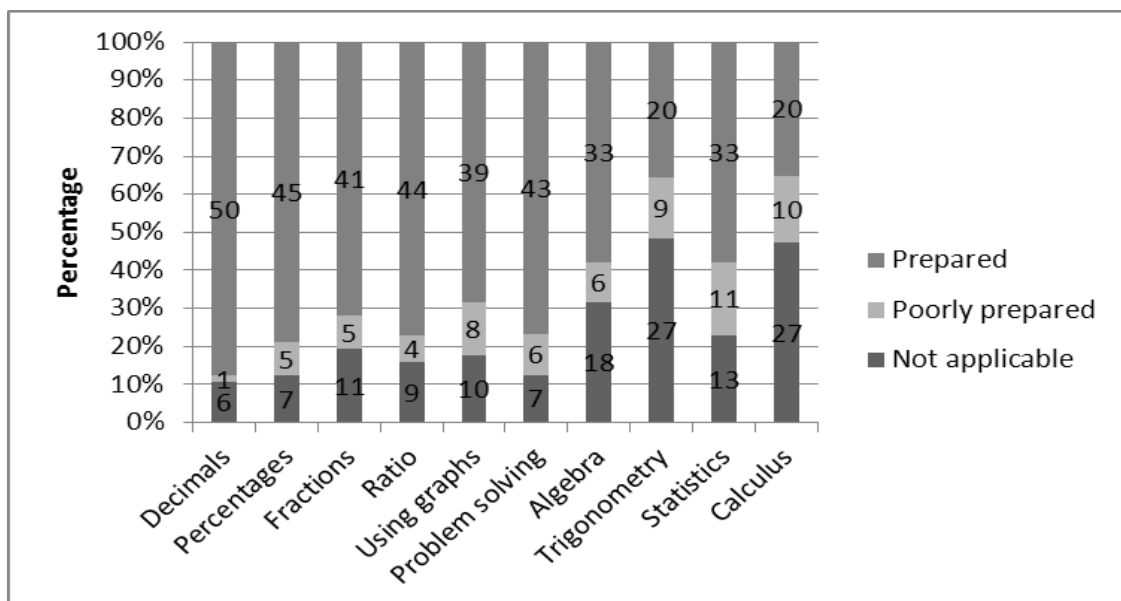


**Figure 4.** Business, education, arts and general studies student responses to preparedness in mathematical topics (n=57).

Nursing students generally enrol in a compulsory mathematics course in their first semester. Responses shown in Figure 5 show that most felt well prepared for the basic mathematics that they encountered, except for ratios, trigonometry and calculus where 25% of students felt poorly prepared.

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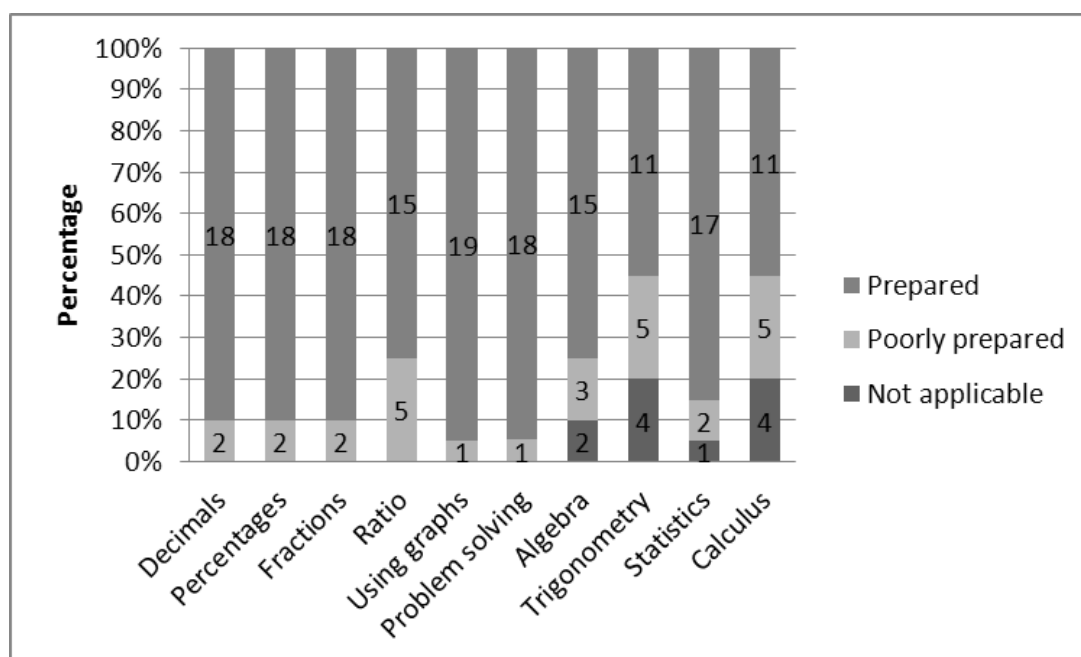
**Figure 5.** Nursing student responses to preparedness in mathematical topics (n = 20)

Figure 6 indicates whether students agreed or disagreed that their pre-university mathematics had prepared them for mathematical concepts encountered in their university studies.
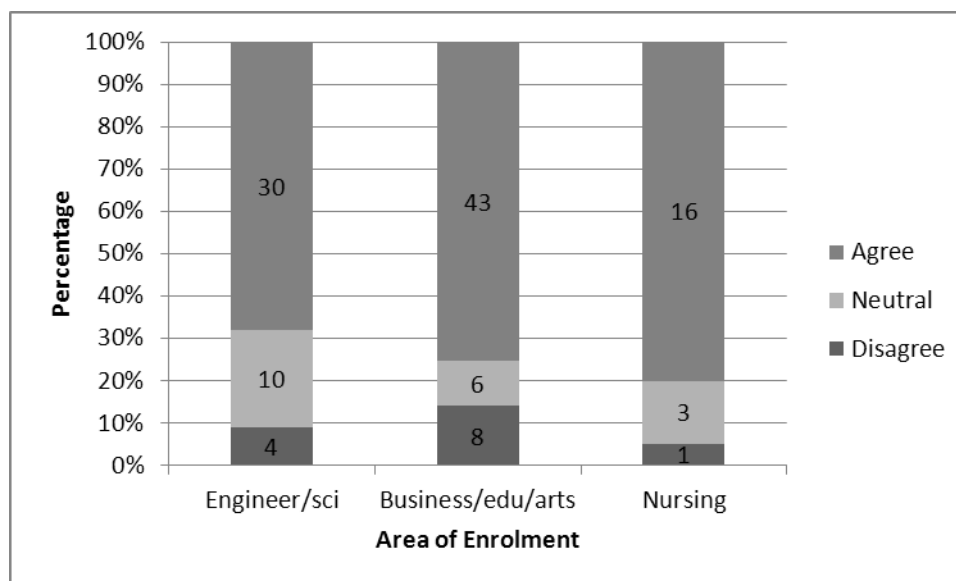


**Figure 6.** Pre-university mathematics preparation was adequate by degree of enrolment (n = 121)

Figure 7 shows a basic analysis of student responses to an open ended question about recommendations for mathematics preparation for undergraduate degrees. Responses could be categorized into three groups: those that recommended pre-course/degree preparation, those that recommended within course/degree preparation and a third group that did not feel that the University needed to do anything further.

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Ten respondents recommended a whole semester Mathematics non-credit revision course for students that were underprepared. Five respondents felt that they had benefited from doing the Tertiary Preparation Mathematics course prior to Undergraduate degree enrolment. Several comments were made about the difficulties faced by mature-aged students who had not done mathematics for many years. Further submissions were that students should be given more detail on the mathematics content of a course so that they could better prepare themselves along with a few suggestions for some sort of self-assessment prior to enrolling in a course. There were seven proposals for pre-course preparation that would target specific courses. Stand-alone workshops were recommended for mastering the use of calculators and graphing packages.

Recommendations from fifteen students that were considering mathematical preparation from an in-course perspective were for a greater number of tutorials, online videos, practice quizzes, workshops and video conferencing tutorials as part of the course offering. Five respondents also recommended a basic mathematics course as part of a degree.

Twelve positive comments indicated that the respondents have accessed the current support structures in place at the university Four respondents indicated that the in-course support was sufficient.
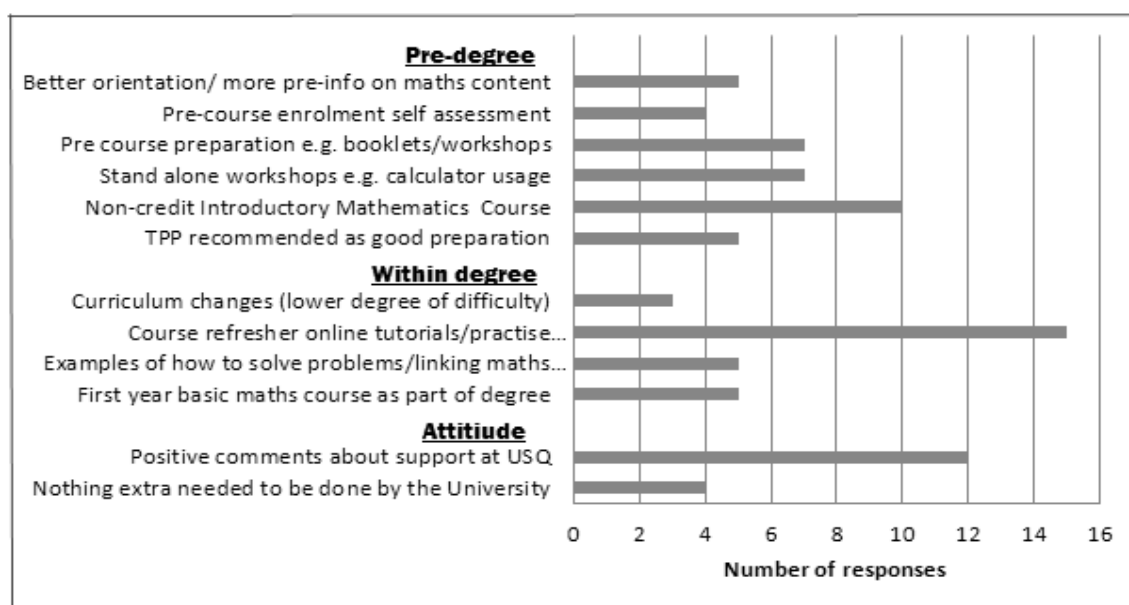


**Figure 7.** Student recommendations for mathematics preparation for undergraduate studies

Eighteen students were interviewed about their perceptions of their mathematical preparedness for their first semester. A summary of notable responses to the interview questions is shown below.

To the question "Broadly, are you happy or unhappy with your mathematical preparation for first year study?" sixteen students responded positively, with only two being cautious in their response.

Suggested topics that students would have pre-university maths teachers place greater emphasis on were logs, trigonometry, algebra, higher level calculus and statistics. Interviewees were hesitant to recommend topics that should be included or dropped from pre-university preparation. One student commented that no topics could have been dropped as she has found that she is using maths that she thought she would never use.

Students were aware that when providing solutions to a mathematics related problem in an assignment or exam the method should be to "Write out all the steps". The majority of interviewees stated that they had no difficulties with word problems.

Response to the question "What about the approach to learning – were you surprised at the differences (between pre-university and university)?" received 13 responses of "no" with five exceptions:

- I find differences as an external student. The lecturers put up the problem but do not break it down into steps. I have only had one good teacher who has done this. I find they don't take time to explain things. I find they have a different attitude towards external students and can be rude when you ask questions.
- At uni I know there is help there but you have to ask for it. At school someone will help. Maths and physics were explained differently at school.
- Not surprised - find a way to work it out and seek answers for yourself - about the same.
- I am no good at maths but understand all the accounting things. Lecturers do not explain steps, do not take the time to break it down.
- big jump - big!

Responses to the question "mathematics assessment – did you find any differences between pre-university study and university?  Do you think you needed to be better prepared?" elicited only three positive responses:

- Yes, because there are different between studies in my country and Australia.
- Needed to be more prepared - school to uni was a 15 year gap.
- A lot of difference, mainly formal qualities of assessment, harder and is of a higher quality.

To the question "Can you think of ways where the teaching of pre-university maths could be improved?" some students replied that courses in high school should be more aligned to university studies. All but two of the students responded positively to whether their pre-university subject choice was the best one. Two comments were

- TAFE course was a good choice as I did not get the OP to go to uni from school.
- From school I did a trade certificate and apprenticeship in mechanics/ spare parts used a lot of maths - addition, estimation, stocktaking. Did Maths B and C at school but dropped to Maths A.

## 5.    Limitations

As the survey was voluntary, the group of respondents may not be representative of the entire first year student body and will not have included any students that did not complete their first semester of study. Because these students were out of the enrolment system, the authors had no access to their details, including how many there were. Moreover, it is evident that there are contradictions in some of the students' interpretations of the questions on mathematics topics.

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## 6.    Discussion and conclusion

Comparing the more mathematically based degrees, Engineering and Science, against Business and Nursing, there is a higher percentage of students who felt underprepared for the mathematics that they encountered in their first semester at university. Current research literature indicates that many students enter university with an inadequate mathematical background for their intended degree. This survey shows that a significant percentage of respondents in mathematically based degrees acknowledge this.

The interview question that elicited the most interaction was the difference in the approach to learning at University. In addition, there were useful written suggestions from the students for self-assessment, workshops, practice quizzes and online tutorials. Such support is available in many courses, but perhaps a university approach to such support is needed [26].

Overall the survey showed that there was a general response of being "well prepared" for mathematics encountered in first year subjects, particularly in the Social Sciences and Nursing, although if 20 to 30% of students feel poorly prepared for the quantitative components in their courses, then there is a role for enabling and support programs. Interviews generally backed up the responses to the survey in terms of students' confidence in their mathematical preparation.

Lecturers, particularly in support and large first year service courses, may see a higher percentage of these underprepared students which could skew their view of students' preparedness. A closer analysis of the connection between the lecturers' perception (Part A of this project) and students' perception has yet to be undertaken.

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# Students' mathematical preparation Part A: Lecturers' perceptions

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This paper analyses first year lecturers' perceptions of mathematics topics and skills needed in the respective courses that they teach and their perceptions of students' preparedness for these topics. Surveys and interviews with lecturers were conducted at a regional university in Australia and showed many lecturers regarded that little mathematics was needed in their courses and that mathematics was compartmentalised into specific courses. However, when mathematics was evidenced, lecturers often perceived their students having poor skills and some have adjusted their courses accordingly.

**Keywords:** tertiary mathematics education; mathematical under-preparedness; diversity of student population

## 1. Introduction

The last fifteen years has seen Australian universities' student population become more diverse than ever before [1]. The federal government's acceptance of the Bradley Review [2] and the resulting policy changes including uncapped places and more graduates from low socio-economic areas is heightening this diversity. As a consequence, difficulties in addressing this diversity are being faced by students, lecturers and university policy designers [3]. This diversity covers not just cultural or socio-economic diversity but also academic background, experiences, and views [1].

The situation is compounded by multiple entry pathways into first year. For example students may enter directly from school, from Technical and Further Education (TAFE), complete a preparatory program for Australian students, or an English language program for international students. Many students may also have a considerable time gap between finishing school and beginning university. Some of these gain entry with dated qualifications, and enter on their prior experience. For each pathway there are issues around the level and coverage of academic skills required.

Of particular interest to this study is students' preparation in mathematics and statistics, because a smooth transition to tertiary education can be hindered by an unsatisfactory mathematical background. This lack of preparation is evident in quantitative programs such as engineering and science [4-7] but also other programs such as business and education [8-10]. It has been argued that all students need to have some level of numeracy to be successful at university [11, 12].

At the University of Southern Queensland (USQ) the majority of students are not recent school leavers (median age 28 years) [13] and many are from a low socio-economic status background. In 1997, a USQ study found that the mathematics knowledge expected by lecturers did not always match quantitative aspects of the course material ("course" being one quarter of a full-time load in one semester), degree program "recommendations" or "assumed knowledge" [14]. This current study replicates the previous study completed 16 years ago and

extends it to include the students' perspective of their mathematical background. There were three aims in this project. The first aim was to determine lecturers' perceptions of students' mathematical preparedness. The second aim was to examine first year students' perception of their mathematical preparedness for tertiary study. The final aim was to discover lecturers' perceptions of the mathematical topics needed for study in their respective first year courses. This information was obtained by student and staff questionnaires and interviews. This paper (Part A) will report on the first and last aim. A companion paper (Part B) reports on the student perspective.

The outcomes of this study will assist planning in all programs, particularly in the enabling programs, first year mathematics service courses, and academic learning support retention and progression.

## 2.      Literature Review

This section provides a background to the paper discussing the nature of student diversity, mathematics diversity at university, and responses to this diversity from universities.

### 2.1.      *Diversity of students*

Students entering university present with a diversity of academic abilities [15] as well as expectations and experience [1]. Many students also manage study with other commitments such as paid work and family obligations [16]. This diversity has been recognised as an issue in Australia with the commission of a series of reports from 1994 [1]. Students also have a variety of mathematical preparation which can result in a range of academic issues that universities and students need to address. For example, should universities provide free support to underprepared students that they have accepted via multiple entry pathways? What starting point in mathematics is suitable in a university undergraduate degree? To address this diversity there is a range of support and preparation that can be given to students from one-on-one to embedded support in courses or programs [12].

Diversity and university preparation issues were highlighted in the Australian university transition reports. In the 2009 report [1], while it was found that about half the respondents felt that school prepared them well for university:

> This was not the case for students from rural areas and those from lower socioeconomic backgrounds. There continues to be a disparity in the level of university preparedness of students from certain demographic subgroups. Enhancing the quality of targeted pre-enrolment support and information continues to be a challenge for universities. [p. 33]

These concerns are prevalent in a range of subject areas but mentioned particularly are students' abilities to write and perform mathematics [17, 18].

### 2.2.      *Diversity of necessary mathematics*

Research into the mathematics needed in university is critically under-researched [19, 20], despite a need for numeracy as an explicit graduate skill. The seven generic capabilities that appear common in higher education [21] do have some reference to numeracy. This is sometimes specifically highlighted at individual universities. For example Murdoch University has numeracy as an explicit sub-section of Communication Skills in their Graduate Attributes [22] and is part of their Foundation program. James Cook University [23] has developed a Post-Entry Numeracy Assessments (PENAs) with follow-up support for people with identified numeracy needs. A report from the University of Tasmania recommended in 2010 that "numeracy be included as a graduate attribute by explicit

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incorporation within one of the existing UTAS graduate attributes or academic standards" [11], but currently only has "numerical and graphic communication" under their communication section. In response, a call has been made for a national approach "in cross-disciplinary mathematics and statistics learning support to enhance student learning and confidence" (p. i). [24]

Currently, each discipline at university has its own mathematics prerequisites, culture, expectations and characteristics. There appears to be little debate that basic mathematics such as fractions, decimals, percentages, data representation, analysis, and interpretation is needed across all disciplines [25]. For anything more, there is some debate around the level of mathematics needed. In a recent Wall St Journal article it was argued that "For many young people who aspire to be scientists, the great bugbear is mathematics. Without advanced math, how can you do serious work in the sciences? Many of the most successful scientists in the world today are mathematically no more than semiliterate." [26] Along a similar vein, some in the engineering profession argued that: "their courses did not reflect what the workplace required and this was an issue for them in the workplace (p.8) " [27]. They believed that "that much of the mathematics included in their courses was never required in the workplace and that the time could have been better spent on other areas". More broadly, a 2008 National Numeracy review panel concluded "that at all levels of education up to and including university, many students (and their parents) are not persuaded that the mathematics education they are receiving will serve them well in their future workplaces." [27]

However, at many universities there is a belief that a problem exists in students' understanding of mathematics, even a crisis. In a Western Australian study [28] researchers found that coordinators of scientific and quantitative courses noted that their students were nervous and lacked confidence about the mathematical content, often withdrew early in the semester, were mathematically poorly prepared and reluctant to enrol in their quantitative courses. Similarly, UTAS [11] described a "mismatch between the numeracy requirements of UTAS courses and the students' ability to meet those demands" and concluded that "numeracy is required in every area of study" (p. 1). There is a concern that this lack of participation in mathematics (and science) disciplines may lead to problems with sustained economic and productivity growth and development [29, 30]. This concern is shared by others around the world [31, 32].

In engineering, evidence suggests that mathematics is still considered to be a core skill for a student enrolling in engineering, while the targeted level of mathematics within university can be too high for many graduates entering industry, but too low for those pursuing research-intensive careers. A 2008 engineering report concluded that data analysis and statistics were grossly under-taught for all graduates [33]. The results of a survey of 31 engineering staff covering 63 courses identified nineteen broad mathematical topics in engineering programs [34]. This paper and others suggested that deep basic mathematical knowledge is essential as it underpins much of the engineering curriculum [35]. Other programs including chemistry, computer science, economics, business and finance, suggest that, for some, mathematics is still an important part of many degrees [36]. In addition, it has been shown that students who face difficulties with their first year mathematics may not continue or may fail some courses [5].

It is not just the content but the expectations and culture around university learning that influence students' behaviour and performance. Matthews, Adam and Goos [5] suggest that students bring strong beliefs about science and mathematics from high school in which they are treated as two different subjects. When they reach university, mathematics is often embedded in other courses, and in this interdisciplinary approach, students with weak

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mathematical skills are more likely to disengage from learning than those students who have a stronger mathematics foundation.

For staff in academic support or service programs, there is a crucial need to understand this meta-knowledge around the different mathematics skills. However, it is also important to have an institutional structure around development of such knowledge [12] and a shared knowledge of what that knowledge is. For example, in science:

> it is crucial for mathematicians to better understand the skills, interests and requirements of the student audience, who are generally not studying mathematics for its own sake. It is equally crucial for scientists to recognize and understand the scope, relevance and usage of mathematics in science. [5]

### 2.3. *University response to diversity*

Universities have strategies to address diversity issues of expectation, experience and preparation. In particular, a variety of support, particularly for under-represented groups in the community, is available to students in most universities. While a national approach to this issue has not been developed [12], support does include a mix of upfront bridging courses, parallel support in courses where it is integrated into the teaching [12] and one-on-one support [25]. Often this support includes testing students on specific mathematics skills where early detection may bridge perceived deficiencies [20]. While those in mathematics support or in mathematics related disciplines recognise students' quantitative under-preparedness, those outside are often unaware of the extent of the problem.

### 3. Method

Ethics clearance was obtained to survey both students and lecturers with permission given to communicate with students and staff by the Faculties of Arts, Sciences and Business and Law. Similar information from Engineering and Surveying lecturers had already been obtained and some results have been published [34] (permission was not granted from the Faculty of Education).

All first year lecturers were asked to fill in a survey and if they would consent to a brief interview. The Queensland Junior and Senior Mathematics Syllabi for Mathematics A, B and C were used to develop questionnaires detailing mathematics topics studied in Years 10, 11 and 12 Queensland Secondary Schools. Questionnaires were designed to ascertain academics' perceptions of mathematics topics expected for study in their respective courses and their perceptions of on- and off-campus students' performance in these topics. An open-ended question allowed academics to make personalised comments. This survey is a partial replication of a survey conducted at USQ in 1997 [14]. This replication was to allow comparisons to be made and changes in perceptions analysed (not part of this paper).

### 4. Results

Results of the quantitative data from survey, open-ended comments from surveys and staff interviews are now outlined.

A total of 131 questionnaires were sent. The response rate is shown in Table 1. There were thirty-six responses with a total of 17 lecturers who responded to open-ended questions. Of these, only four indicated that mathematics was not required in their courses, with two of the four stating that mathematics would be required at higher levels of their subject area. Six lecturers commented that a portion of their students did not have the assumed prerequisite mathematics skills and another four lecturers acknowledged that many of their students showed an aversion to mathematics. To be noted is that four lecturers stated that they had

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altered their course or teaching of their course to accommodate for the lack of mathematical skills.

**Table 1.** Number of surveys returned by Faculty **Error! Reference source not found.**

## General Topics and Skills
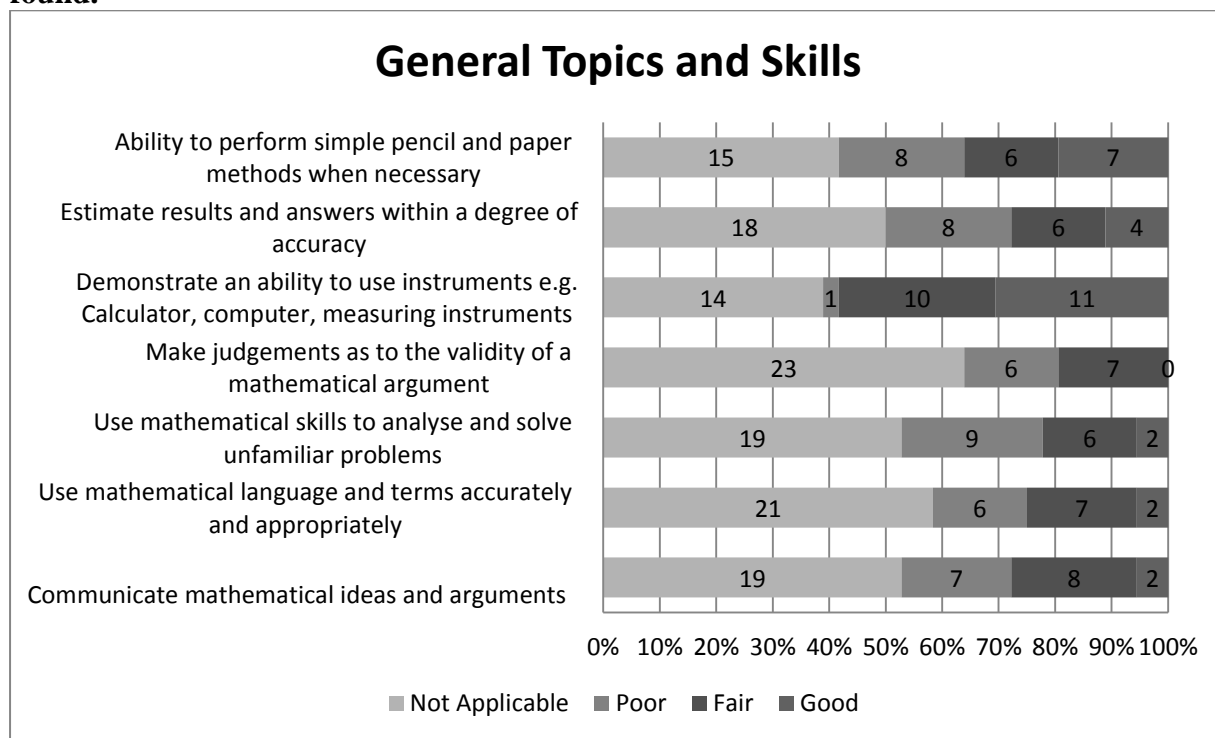


**Figure 1.** Lecturers' responses to "perceived student proficiency" in general topics and skills. The bars show the fraction of responses and numbers within bars show a count.

Figure 1 shows the lecturer's response to the perceived level of proficiency for general topics and skills that students require in order to complete their course. Of significance in these results is the high response that is recorded as "not applicable". This indicates that lecturers perceive that these general topics and skills are not necessary for their course. Given that lecturers were surveyed across three faculties, Business, Science and Arts this is a result of interest. In addition, some of the respondents from the open-ended comments explicitly stated that mathematics was not required in their course, but was necessary in other courses. Despite this, seven of those respondents who stated that mathematics was unnecessary rated their students' capabilities in a significant number of topics or skills in at least the Year 10 proficiencies. While one hypothesis may be that the lecturers are largely unaware of how pervasive mathematics is in their courses, from the corresponding open-ended comments it seems more likely that the lecturers have low expectations of the students' abilities and the course content is appropriately targeted as a consequence:

> Respondent 3 (Psychology, No expected background, Some skills rated): Maths only explicitly relevant to one module. This requires understanding (not calculating) of, central tendency, variability, and inferential statistics. Later courses in the program require more advanced stats. Students sometimes change major due to concerns about these skills, pre-emptively before they even try to understand.
>
> Respondent 12 (Arts, No expected background, No skills rated): Because of a lack of competency in students in general, there has been a shift away from teaching history

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students statistics. This trend is long-term though, and no historian (from this university) would be willing to teach a "statistics for history" course.

Despite all of this negativity, there are some lecturers who are acting positively to address the issue for the benefit of their students:

Respondent 17 (Arts): students tell me they hate, don't understand or can't do maths. Knowledge of basic maths is as important to a good journalism as having news sense, knowing the rules of grammar, and write clear, accurate and interesting stories. …in their first year, students in my course will learn that if they know their way around numbers they will be in a much better position to tell their readers what's going on.

Looking into the results, the first four statements are similar in that they are assessing mastery of concepts, which is fundamental for enabling synthesis and analysis. This higher level of thought capability is where students struggle the most, which causes difficulties in terms of introducing newer, more advanced concepts that require mastery. Unsurprisingly, in this modern age of pervasive gadgets, the students' ability to use gadgets is their strongest asset; this does not necessarily mean that the students are competent overall:

Respondent 9 (Science): Students can sometimes solve a problem because they know how but they don't know why. Without a calculator they can('t) calculate, can't roughly estimate and can't even tell the order of the result.

Finally, investigating the last two statements, there is a similar proportion of students who struggle with basic skills as those who struggle with more advanced skills, which has a causal link: basic skills scaffold advanced skills. However, there is a higher proportion of students with mastery over the basic skills, so this means that a substantial proportion of the cohorts are able to learn the new content that they will face.

The remainder of the survey focussed on the capabilities taught at the various year levels of high school. **Error! Reference source not found.** shows the results for expectations of students who have completed Year 10, where a total of 25 specific topics and skills were examined, with the data agglomerated based on a categorisation of the skill, so each bar shows data for a number of statements. Again, the content not required (Not Applicable) response rate is surprisingly high, particularly when the "Graphs" category includes interpreting graphs and "Statistics" includes data collection. The "Finance" statements were very specific to the field and so it is to be expected that these were restricted to a small sample. Assessing the actual responses, the overall good results for "Numbers", "Graphs" and "Unit comparison/conversion" correspond to the reasonable level of basic abilities that were shown in **Error! Reference source not found.**. The moderate capabilities in "Statistics" are to be expected because many people find these concepts difficult to master. The poor abilities in "Trigonometry" and "Solving Equations" have been experienced by the authors regularly and are of considerable concern because mastery of these fundamental areas leads to capability in the more advanced concepts.

Lecturers who indicated some mathematics in their courses, commented on students' lack of skills, for example

Respondent 30 (Chemistry): Basic numeric skills are lacking. I dread times when I introduce logarithmic & exponential functions... much of my time is spent teaching maths, it gets very frustrating especially when it is yr 8 level!

The extra teaching that these students receive mean that the fundamental mathematical knowledge in Geometry, Trigonometry, Solving Equations and Graphs is improved overall in terms of a reduced fraction demonstrating *Poor capability*. Nonetheless, as an absolute measure, these results are substandard. By contrast, the capability in statistics

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appears to remain largely unchanged. That integration is considered to be weaker than differentiation is consistent with long experience of this phenomenon, but the capability in differentiation is insufficient:

> Respondent 34 (Science): My class has a mix of student maths backgrounds, with many it seems being uncomfortable with any aspect of calculus. I may to have resort to returning to an algebra-based physics textbook.
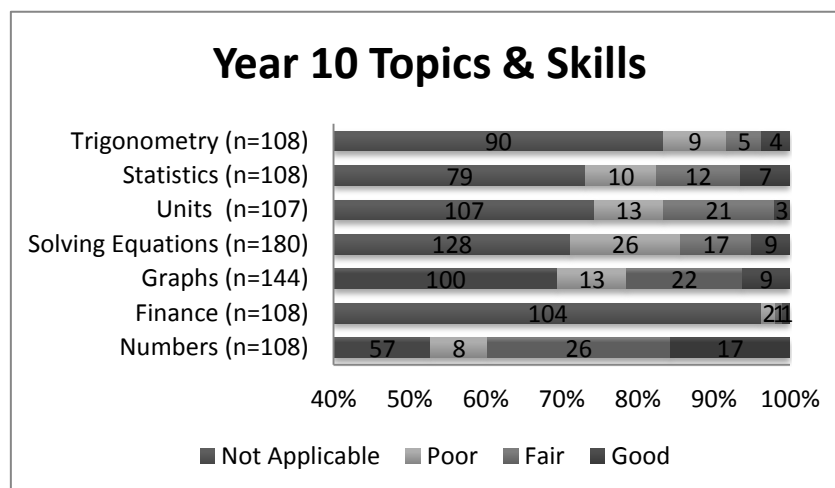


**Figure 2.** Lecturers' responses to perceived student proficiency in topics and skills from the Year 10 curriculum. The bars show the fraction of responses and numbers within bars show a count. Note that each bar is an agglomeration of a number of statements, so the total number of responses (n) for each category is listed.

For Year 11/12 (Figure 3), the most alarming result is the overwhelming majority of Poor responses for Functions. This is indicative of mathematics not being a field that is understood, rather one where steps must be followed in a particular order to produce an answer. With this lack of mastery comes the inability to attempt any problem that falls outside these strict protocols.

Only two respondents considered that their courses contained material pertaining to Mathematics C capabilities and so there is limited value in the data other than the overall capabilities were below standard, with no topic considered to have "Good" capability.

Individual interviews were held with some of the survey respondents. The three science lecturers voiced a strong opinion from their colleagues that they were not satisfied with some students' mathematics preparedness and that students have the impression that they can do mathematically-based courses without basic mathematics, as one lecturer stated; "Students need to realise they cannot do statistics in 5 minute grabs… Students can't do the building blocks." Other lecturers' opinions were that students are unaware of their own lack of mathematics skills and enrol in higher level courses because they are unable to recognise that their set of skills is deficient compared with those required. The students are then unprepared because they do not understand their lack of skills and therefore unsuccessfully try to cram what they do not know instead of building upon a foundation. Furthermore, responses indicated that students have an acceptance that they cannot do mathematics because Australian society is quite happy to accept the catch phrase "I am no good at maths."

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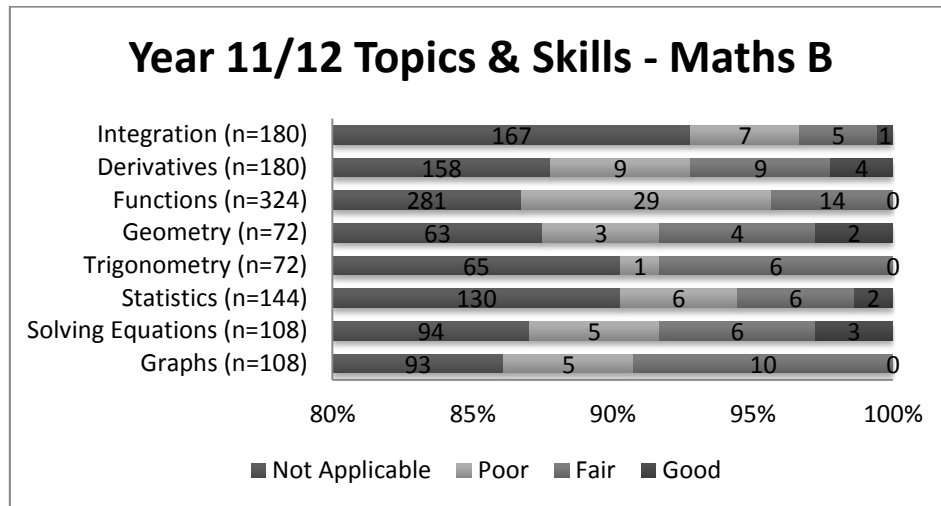
**Figure 3.** Lecturers' responses to perceived student proficiency in topics and skills from the Mathematics B curriculum. (as per Figure 2).

## 5.    Discussion and Conclusion

Diversity is an issue in most universities, however at USQ the multiple facets of diversity, i.e. in abilities, expectations, experience, commitments, and mathematical preparation, creates challenges that need a range of solutions at the institutional, faculty, program, and course level [12]. While the response rate to this survey was less than 30%, it does represent data about numeracy aspects across a whole university. A full report is currently being written. The investigation reported in this and the complimentary paper (Part B) and our other investigations [34] provides evidence to inform program planners, particularly in the enabling programs, first year mathematics service courses, and academic learning support at USQ and in tertiary institutions with similarly diverse cohorts of students. Further analysis is being undertaken to compare this data to the 1997 and analyse changes in perceptions. Further research would need to be undertaken to see if the results are replicated in more traditional universities.

Specifically, this paper found many academics saw little need for students to have any level of mathematics in their courses. For those that did, there is a perception of lack of capabilities in the fundamental skills shown by students entering university, which translates into a difficulty in scaffolding advanced knowledge. This problem is partly due to a compartmentalisation of knowledge that mathematical skills are only required when studying a course titled "Mathematics" and a lack of awareness that mathematics permeates into many aspects of life. We conjecture that this compartmentalisation translates internally in terms of strategic learning, whereby a student might target specific topics to achieve a passing grade and thereby not devote the requisite energy to understand the breadth of mathematics.

Coincidentally, while the current data was being collected, a number of changes occurred. A new statistics for social science course was introduced in the Business, Education, Sciences and Arts disciplines to provide students with basic concepts on how data is evaluated. Unfortunately, as it is only an optional course in programs, much of the numeracy requirements are still going to have to be taught within courses, (like journalism) or expectations reduced (like history). Another approach is to address the issue at entry level. Engineering instigated a reorganisation of the engineering mathematics courses, citing a lack of competency in the topics described here when undertaking fundamental engineering courses. The approach taken has been to increase the entry-level requirements for the two-

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year degree program and simultaneously include introductory material for higher-end topics. With the implementation of these changes commencing in 2013, it remains to be seen what influence this will have on the engineering programs.

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# Issues and trends: a review of Delta conference papers from 1997 to 2011

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**Keywords:** Tertiary mathematics education; review of research; issues; trends

From the first Delta conference in 1997 to the eighth in 2011, a total of 409 papers have been published in special issues of journals or in conference proceedings. These papers cover a wide variety of topics in the teaching and learning of tertiary mathematics and statistics, largely within universities in southern hemisphere countries. This article reviews and classifies past papers, reflects on apparent trends and attempts to identify ideas for future research projects and initiatives in teaching and learning that are suggested by past Delta papers.

## 1. Introduction

Since its inception in 1997, the series of Delta conferences has established itself internationally as one of the most important forums for the discussion of the teaching and learning of undergraduate mathematics and statistics. As Mike Barry wrote in a report on Delta 09 [1], "Delta has for many years united the widely disparate Southern Hemisphere in a quite unique form of conference. Maybe not totally education and certainly not totally mathematics but definitely a forum of challenge that has no obvious parallel further North."

At Delta 05 Ruth Hubbard [2], presented a paper in which she looked back over the first four conferences and discussed trends in the topics that had been presented in those conferences. At this, the ninth conference in the series, it seems timely to reflect on the issues that have been raised and the topics that have been discussed from 1997 to 2011.

Delta is a community of academics committed to improving tertiary mathematics and statistics learning and teaching. The community was established in Brisbane, Australia in 1997, when Patricia Cretchley and a group of Queensland colleagues organised the first Delta symposium. Patricia had been involved in the formation of the South African Mathematics Education Reform Network in 1994, and had moved to Queensland in 1996. After attending a conference organised by the Sydney University Tertiary Mathematics Education Group in 1996, she saw a need for a new forum which could bring together academics from South Africa and Australia, and so Delta was born. Since 1997, Delta conferences have been held biennially. Each conference has been held in a different location and has had a different theme. The 1999 conference, "The challenge of diversity", was held at Laguna Quays, Queensland. South Africa hosted Delta 01, "Gearing for flexibility", in the Kruger National Park. Statistics was included in the title for the first time in 2003, when Delta 03, "From all angles", was held in Queenstown, New Zealand. In 2005 Delta returned to Queensland, this time to Fraser Island, with the theme "Blending beyond boundaries". Two years later the conference moved to South America when Delta 07, "Vision and change for a new century", took place in El Calafate, Argentina. In 2009 Delta returned to South Africa for the largest conference so far. Delta 09, "Mathematics in a dynamic environment", was held at Gordon's Bay, near Cape Town. Delta 11, "The long abiding path of knowledge", took place in Rotorua, New Zealand.

**Table 1**. Participation in Delta conferences

|  | **97** | **99** | **01** | **03** | **05** | **07** | **09** | **11** |
|---|---|---|---|---|---|---|---|---|
| Host country | Aust. | Aust. | S.Afr. | N.Z. | Aust. | Arg. | S.Afr. | N.Z. |
| No. of participants | 106 | 65 | 109 | 115 | 100 | 53 | 157 | 122 |
| No. of countries | 5 | 8 | 19 | 9 | 15 | 15 | 32 | 15 |

## 2. Classification of papers

A book of abstracts only was published for Delta 97, but from 1999 full proceedings have been published. Since 2001, there have been two publications from each conference. Some papers are published in a special edition of a journal, and others in a book of proceedings. All published papers are peer-reviewed. The classifications in Table 2 include all full papers that have been published in either a journal or a book of proceedings, as well as the presentations from 1997. We have not included papers from other years for which only abstracts were published, nor have we included discussion papers or workshops.

**Table 2**. Classification of papers in Delta conferences from 1997 to 2011

| Category \ Year | **97** | **99** | **01** | **03** | **05** | **07** | **09** | **11** | **Total** |
|---|---|---|---|---|---|---|---|---|---|
| Teaching and learning | 13 | 9 | 24 | 17 | 17 | 8 | 9 | 12 | **109** |
| Technology | 19 | 8 | 14 | 11 | 8 | 5 | 4 | 11 | **80** |
| General pedagogy | 5 | 6 | 4 | 9 | 3 | 3 | 7 | 4 | **41** |
| Transition | 4 | 3 | 4 | 6 | 4 | 5 | 3 | 7 | **36** |
| Student resources | 3 | 6 | 7 | 3 | 4 | 2 | 2 | 2 | **29** |
| Professional development | 0 | 0 | 2 | 1 | 3 | 2 | 2 | 8 | **18** |
| Assessment | 2 | 3 | 3 | 5 | 2 | 1 | 1 | 0 | **17** |
| Pre-service teachers | 2 | 1 | 4 | 5 | 1 | 1 | 1 | 1 | **16** |
| Specific subsets of student cohort | 1 | 1 | 1 | 5 | 1 | 1 | 1 | 2 | **13** |
| Group learning | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | **9** |
| Miscellaneous | 5 | 2 | 1 | 6 | 6 | 6 | 6 | 9 | **41** |
| **Total** | **57** | **40** | **65** | **69** | **50** | **35** | **37** | **56** | **409** |

We found the list of categories in Table 2 to be a convenient way of classifying the 409 papers we considered, but do not suggest that the list is definitive. Any attempt at a classification such as this is bound to be personal to some extent. We note, however, that the categories chosen by Hubbard [2], are a subset of our categories, and that most of the categories chosen by Barry in his reports of Delta 07 and Delta 09 [3],[1], correspond to one of ours (albeit with different names). Similarly, in a review of research in tertiary mathematics education in Australasia between 2008 and 2011 by Barton et al. [4], transition, technology and specific mathematical topics are identified as major themes. (Note that this review was not restricted to Delta papers, and cites papers only from the journals associated with Delta 09 and Delta 11 [5], [6], not from the proceedings.) Of course, many papers overlap more than one category. The assignment of a paper to one category only was a decision taken by the authors based on what we considered to be the major thrust of the paper. Again, such decisions are bound to be personal. Despite these disclaimers, we believe that the numbers in the table provide a good overview of the relative importance Delta participants have afforded various issues over the years. In the following sections we discuss themes which emerged within the most common categories.

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## 3. Teaching and learning specific topics

Papers in this category include descriptions of initiatives taken in individual courses, studies of student responses to such initiatives, opinions on how a particular topic should be taught, and studies of how students learn (or fail to learn) specific topics. It is not surprising that this category contains the largest number of papers. As Hubbard [2], commented, "most participants work at the coalface in university education, teaching or facilitating courses, trying to improve their work and to help others do likewise". The 109 papers in this category have been further classified in Table 3. The focus of attention has been on calculus, DEs & modelling, and statistics, with calculus and statistics being discussed predominantly at the first year level. Given that most first year mathematics courses include linear algebra, the relatively small number of papers on that topic is somewhat surprising. Advanced mathematical topics have received scant attention, as has the topic of proof.

**Table 3.** Sub-classification of papers on teaching and learning specific topics

| Topic \ Year | 97 | 99 | 01 | 03 | 05 | 07 | 09 | 11 | Total |
|---|---|---|---|---|---|---|---|---|---|
| Calculus | 8 | 4 | 7 | 4 | 2 | 1 | 1 | 1 | **28** |
| DEs and modelling | 0 | 0 | 4 | 6 | 5 | 2 | 4 | 4 | **25** |
| Statistics | 0 | 3 | 4 | 3 | 4 | 4 | 1 | 3 | **22** |
| Linear algebra | 2 | 0 | 1 | 1 | 2 | 1 | 3 | 1 | **11** |
| Mathematics for non [maths/eng] | 2 | 1 | 3 | 2 | 2 | 0 | 0 | 0 | **10** |
| Proof | 0 | 1 | 3 | 0 | 2 | 0 | 0 | 1 | **7** |
| Advanced maths | 1 | 0 | 2 | 1 | 0 | 0 | 0 | 2 | **6** |
| **Total** | **13** | **9** | **24** | **17** | **17** | **8** | **9** | **12** | **109** |

The calculus reform movement was in full swing in the 1990s, and the world-wide interest in the movement was reflected in the early Delta conferences. At Delta 97 Deborah Hughes-Hallett (one of the leaders of the reform movement) was a keynote speaker, and there were presentations from Australia, South Africa and the USA describing the implementation of calculus reform in particular first year courses. Delta 99 saw an argument for further reform from Belward (in [7]), as well as a report from Ruxton (in [7]) on calculus reform in a second year multi-variable calculus course. Harman (in [8]) speculated on the future of the calculus reform movement, and argued that it is important that reform projects be thoroughly researched and evaluated. Such research does not appear to have occurred, and since 2003 there have been very few papers devoted to either calculus reform or to student learning in calculus. Papers that do focus on student learning in calculus include an investigation of students' performance and confidence in handling procedural as opposed to conceptual problems by Engelbrecht et al. (in [8]), and a report of students' difficulties with volumes of solids of revolution by Mofolo et al. (in [9]).

Student learning is the focus of almost all of the small number of papers on linear algebra. Stewart and Thomas (in [8]) found that first year linear algebra students had difficulties in understanding definitions. Hillel and Mueller (in [10]) investigated whether or not students are able to make links between different interpretations of the solution space of a system of homogeneous equations. Further papers by Stewart and Thomas (in [5] and [11]) study student thinking about various linear algebra concepts by applying both APOS theory and Tall's three worlds of mathematics. Hannah (in [12]) also used Tall's theory in an attempt to improve his students' understanding of span and subspace. Britton and Henderson (in [5]) investigated why second year students have particular difficulty with the concept of closure

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of a set, and with the idea of functions forming vector spaces. They followed this with a paper (in [13]) in which suggestions were made as to how students could be helped to construct a simple proof of closure.

Most papers on DEs and modelling are of quite a different type from those on other topics. With only a few exceptions, these papers present particular DEs and their solutions, and a small number of presenters are responsible for almost all of them. (For example, Fay and Joubert (in [8], [10] and [12]), Fay (in [12] and [13]), Joubert (in [14]), Martinez-Luaces (in [8], [9], [10] and [12]), Martinez-Luaces and Cobs (in [10] and [14]).) Papers that deal with modelling in a more general way include Harding's study of student performance in a modelling course (in [10]), and a description of four different approaches to teaching modelling by Anaya and Cavallaro (in [9]).

The number of papers on statistics has remained reasonably consistent since 1999, and most papers address the issue of how statistics should be taught, either in general or with respect to a specific topic. Most recently, Pfannkuch et al. (in [6]) argued for major changes in the way statistical inference is taught, suggesting a move away from a norm-based mathematical approach to a computer-based approach. Sowey (in [15]) put forth his belief that statistics should be taught with a perspective on the logic of statistical inference, while Varsavsky (in [16]) addressed the issue of a relevant mathematics and statistics curriculum for science students. A new course using ideas from gambling, sport and medicine was described by Croucher (in [10]). The challenge of explicitly teaching the interpretation and critical evaluation of statistically-based reports in a large introductory statistics course was addressed by Pfannkuch et al. (in [8]). Papers that focus on teaching a specific topic include Roberts and Pierce on the central limit theorem (in [7]), Stewart on Bayesian statistics (in [9] and [13]), Sinay on stochastic processes (in [9]) and van Staden and Loots on kurtosis (in [12]). There are no studies that focus exclusively on student learning in statistics, but Petocz and Reid (in [15]) investigate students' conceptions of statistics.

The topic "mathematics for students other than engineering or mathematics students" includes descriptions of courses for business students, agriculture students and liberal arts students. In a plenary address in 2005 Briggs (in [10]) discussed the type of mathematics courses that should be taught to students of fine arts and the humanities, and concluded that the development of such courses should be a collaborative and interdisciplinary effort in which mathematics departments take the lead.

## 4. Technology

The use of technology has been one of the major themes at each Delta conference, with the overwhelming majority of papers describing the introduction of a particular piece of technology into the teaching and learning of an individual course. Technological change has been rapid since 1997 and the use of technology in mathematics teaching and learning has increased enormously. On-line delivery of material and assessment in one form or another is now commonplace. Hence, many of the papers from early Deltas now seem out-of-date, and so we restrict our discussion to papers from 2005 onwards.

Tablet PCs have captured the interest of several presenters in recent years. Dekkers and Shepherd (in [10]), Loch (in [10]), Bonnington et al. (in [9]), Loch et al. (in [6]) and Dekkers et al. (in [13]) describe the use of tablet PCs in a variety of courses, and all claim that the technology is useful and effective. Student feedback is reported as positive, and Loch et al. found that the tablets promoted collaborative learning. However, whether or not learning outcomes are improved by the use of this technology has not been reported, leaving room for further research. Four papers discuss the use of classroom response systems (CRS, or 'clickers'). Stewart and Stewart (in [12]) argue that the clickers engaged students and allowed

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lecturers to better understand students' level of learning. Lozanovski et al. (in [13]) found that education students reacted favourably to the use of clickers, but caution against assuming that this is associated with any measurable improvement in overall performance. The use of mobile phones as clickers is reported by Dunn et al. (in [13]), and a teaching model for the use of clickers is proposed by Stewart and Stewart (in [13]). The authors of all four papers on CRS argue for more research in this area.

Several papers discuss the use of computer algebra systems (CAS), although the number of such papers has declined significantly since the early Deltas. For example, Tonkes et al. (in [15]) report on the development of a learning model using Matlab; Blyth and Labovic (in [5]) report on the development of a learning system that incorporates computer-aided assessment using Maple; Tobin and Weiss (in [13]) discuss ways of teaching DEs using CAS in general; and Mofolo (in [10]) describes teaching volumes of solids of revolution using Mathematica. A paper by Ponce-Campuzano and Rivera-Figueroa (in [13]) gives examples of the variety of solutions that different CAS provide when finding anti-derivatives, and argues that these can be used to encourage students to be more thoughtful when using CAS. The ability of students to understand the output from CAS is clearly important and is explicitly studied by Berger (in [12]). Rather alarmingly, Berger found that more than half the students in the study could not interpret the output appropriately.

## 5. General pedagogy

At each Delta conference there have been some papers that take a broader view and deal with general issues related to teaching and learning mathematics. Early Delta conferences included a number of papers that argued a need for change in undergraduate mathematics curricula and the ways in which mathematics is delivered to students. The first of these was the plenary address given in 1997 by the inimitable Jerry Uhl (in [17]). Uhl argued that student learning is improved if long lectures are abandoned and replaced by time in a computer lab where students learn by exploration and guided discovery. The need for change in order to cope with an increasingly diverse student population, and/or to make mathematics accessible to a greater diversity of students, was argued by Oldknow in [7], Smith in [7], Zevenbergen in [7], and Steen in [8]. Others recommended changes in order to attract students to mathematics (Kagesten in [7], Pierce et al. in [8]), to cope with large classes (Chacko in [14]) or simply to improve student learning (Coutis et al. in [7]). All these articles advocate some or all of the following: flexible delivery, presentations from students, peer tutoring, use of up-to-date technology, co-operative learning, explicit aims and objectives for courses, use of realistic problems, self-test online quizzes. Papers such as these do not generally appear after 2003, possibly because many of these improvements have been adopted to some extent by most universities.

Several interesting and thoughtful theories related to the way students learn have been presented. Tall (in [10]) presented his "3 worlds of mathematics" framework in 2005. Mason's plenary address (in [13]) in 2011, on definitions and how mathematicians use them, is thought-provoking, as are Easdown's two papers (in [5] and [13]) on the interplay between syntactic and semantic reasoning. The complex role played by visualisation was explored by Sierpinska (in [8]) in her plenary address in 2003. The question of whether or not students learn anything as a result of attending a traditional lecture was addressed in several papers (Hubbard in [17], Cretchley in [10], Wood et al. in [11]), and each of these provide evidence that the answer is in the affirmative. While tutorials are generally considered to be useful to learning, Henderson and Britton (in [8]) found that students have widely differing views on how a tutorial should be conducted. The use of historical anecdotes to enrich and enliven mathematics teaching has been advocated (Maritz in [14], Montelle in [12]), and for those

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wishing to do so the latter paper provides some wonderful and lesser-known anecdotes. Interdisciplinary teaching is an area that has been identified in several papers (Wood et al. in [14], Matthews et al. in [5], Matthews in [13]) as one in which further research would be valuable.

## 6. Transition

The problems encountered by under-prepared students when they commence tertiary mathematics have been documented at every Delta conference. Many factors contribute, including fewer students studying advanced mathematics in high school, a disconnect between the style of high school mathematics teaching and that encountered at university, the opening up of tertiary education to far greater numbers of school leavers and more adult learners commencing degree study.

In the first four Delta conferences, researchers reported on particular bridging/preparatory courses (Brooks and Hewett in [17]), the importance of developing independent learning skills (Snyders in [7]), the use of diagnostic testing to identify students at risk (Barry et al. in [14]), the persistence of rote learning at university (Barnard in [8]), and strategies to scaffold learning (Hartnall in [14]). In an overview article, Persens (in [14]) pointed out that student difficulties in transition were universal, and appealed to academics to share best practice on all successful remediation initiatives.

From Delta 05 onwards, there is increasing emphasis on students' beliefs, perceptions and attitudes, and on examining the context of tertiary study rather than the content. Several articles described the benefits of reflective writing exercises and strong pastoral care (Parnell in [10], Parnell and Statham in [11], Miller-Reilly in [11]). Gordon and Nicholas (in [13])–studied bridging course students' perceptions of their main challenges. The role of mathematical self-confidence and the benefits of peer support, collaborative learning and good study skills were the subject of several papers (for example Carmichael and Taylor in [15] and D'Arcy-Warmington in [10]). Opinions of lecturers and high school teachers were investigated by Hong et al. (in [5]), who concluded that the professionals in the two groups generally worked in ignorance of the other's teaching practices. The challenge of supporting students in transition in an era of increasing student numbers is to manage the process sustainably. A paper describing the introduction of the University of Missouri-Kansas City Supplemental Instruction System (Harding et al. in [6]) gives an interesting insight into the benefits and drawbacks of implementation when student numbers are large. An article by Tularam and Amri (in [13]) reminds the reader that while universities have a duty to offer support systems to students to enable them to develop into mature learners, students also have to accept responsibility for their learning.

## 7. Other categories

Papers dealing with specific subsets of the student cohort (for example, ESL students), group learning and the education of school teachers are relatively low in number and have remained fairly stable, while the number dealing with assessment has declined markedly over the years. This last fact is surprising, given the huge influence on student behaviour exerted by assessment in all its forms. The categories of student resources and professional development have the highest overall representation and also the highest numbers since Delta 03, so our discussion will concentrate mainly on these areas.

The student resources category encompasses a wide variety of tools, both real and virtual, whose purpose is to enhance student learning. Topics include projects to bolster essential skills (for example, Barry and Davis in [17] and Craig in [12]), projects designed to upgrade study skills and examination technique (Coutis and Wood in [17]), the building of

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on-line resources such as quizzes and flash animations, and improved design of course web pages. Papers by Bulmer (in [7]), Varsavsky (in [7]), Broughton et al. (in [7]), Norton and Ovens (in [10]), Jacobs (in [15]) and Bukhatwa et al. (in [13]) provide examples of these web-based initiatives. The role of pastoral care in large statistics classes was investigated by Bulmer and Miller (in [14]) and Bulmer and Rodd (in [15]), while an account of how the use of on-line teaching and pastoral care rescued a statistics course affected by the Christchurch earthquakes is provided by David and Brown (in [13]). Engineering students often require specific resources to support their mathematics. A South African initiative (Steyn and Du Plessis, in [11]) targeting both mathematical and professional skills not only resulted in good mathematical outcomes but also improved retention rates. In the UK, the Helping Engineers Learn Mathematics project provides purpose-built support materials and facilities, resulting in improved student motivation and, in some cases, better exam performance (Green et al. in [10], Harrison and Petrie in [12]).

Many universities now require new academics to undergo some formal training in teaching and to provide evidence of good teaching skills for promotion. This has been reflected in Delta conferences, with the number of papers on professional development trending upwards. Barton (in [6]) constructs a framework to support a self-examination of his lecturing. Other similar studies come from Hannah et al. (in [6]), Paterson et al. (in [6]) and Kensington-Miller et al. (in [13]). One feature of interest in this research is the supportive way in which mathematicians and mathematics educators work together to analyse lecturers' classroom practice and find ways to implement change, a rare partnership indeed. Views of tutors, lecturers, coordinators and heads of school were sought by Wood et al. (in [6]) in a survey to determine the most useful content for a new professional development course designed for Australian academics. Main themes identified were the management of large classes, the lack of knowledge of modern teaching methods, how to handle gaps in student knowledge and the tension between teaching and research. A paper by Wood and Harding (in [11]) gives a checklist of factors which can help to define and measure good lecturing, providing a useful tool for academics seeking to have their professional development efforts formalised. Perceptions of teaching assistants/tutors on the nature of their work and the developing understanding of their students have been examined by Froman et al. (in [9]), Bieseigel (in [13]) and Scataglini-Belghitar and Mason (in [13]). Oates et al. (in [15]) describe the success of a for-credit tutoring course whose graduates (student-tutors) then teach younger students in collaborative tutorials. Graduates of this course are reported to have enhanced mathematical understanding, deeper knowledge of good teaching practice and increased personal satisfaction.

The papers categorised as miscellaneous cover a wide spectrum of topics and just a small number can be mentioned here. Surveys on the nature of undergraduate mathematics education in various countries are reported by Varsavsky and Anaya (comparing Australia and Argentina, in [12]) and by Varsavsky et al. (in [13]), comparing many of the countries which have traditionally been well-represented at Delta conferences. Varsavsky (in [14]) and Craig (in [6]) studied aspects of students' writing and concluded that writing tasks can provide valuable insights into the struggles faced by students as problem-solvers. Wood and Reid (in [10]) wrote of the need to reform the curriculum to improve communication and inter-personal skills of mathematics graduates. Varsavsky (in [9]) examined survey data from Australian graduates, showing that students value the provision of a supportive social environment and the development of generic skills. Reid et al. (in [8]) reported on student notions of professional work in the mathematical sciences, the way they learn mathematics and their conceptions of mathematics. A paper from Delta 11 comes somewhat as a reality check. Bartholomew et al. (in [6]) explored the ways in which students talk about studying

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mathematics. The students' narratives appeared to focus on notions of natural ability and enthusiasm for studying mathematics for its own sake, leading to a pervasive view that success in mathematics confers membership of an elite club. Bartholomew et al. suggest that this view be challenged, and that students who struggle to succeed in mathematics, or who see the subject simply as a sensible career choice, should be actively encouraged to achieve their goals.

## 8. Discussion

The first Delta gathering posed the question: "What can we do to improve learning?" Subsequent Deltas have had various themes, but the fundamental focus remains a search for answers to that question. We have provided here a brief overview of the multitude of ways in which past participants have attempted to provide such answers. In doing so, we have recognised afresh the value of the publications from Delta conferences as a resource for mathematicians and statisticians with an interest in teaching and learning. Unfortunately, most of the publications are not readily available. The iJMEST journals ([5], [6], [11], [15]) are available online, as are the proceedings from Delta 11 ([13]). However, none of the other proceedings is available electronically nor are the journals [8] and [16]. The construction of a website on which all publications are available would be a worthwhile enterprise, in our view.

Teaching and learning specific topics, and the use of technology, have remained the dominant themes over all eight conferences. The reporting of teaching initiatives in calculus has declined significantly in recent years, and there is scope for future research that addresses developments in the teaching and learning of calculus in the post-reform era. It is disappointing that more advanced mathematics topics, including proof, have to a large extent been neglected in this forum. The number of students majoring in mathematics is small compared with the number in first and second year, but they also face challenges in learning in their senior years and should not be ignored. While many papers on technology report positive student responses to its use in a particular course, the wider implications of technology use, such as its impact on student learning and on the curriculum, have been largely ignored by Delta presenters.

Interdisciplinary teaching between mathematicians and other scientists and engineers, the involvement of mathematicians in developing courses for liberal arts students, and communication and collaboration between university and high school teachers are all areas in which it has been suggested that further research would be fruitful. As noted previously, assessment has not been a focus of attention in recent conferences. The fact that many universities are now moving towards standards-based assessment should provide some interesting opportunities for research on assessment in the future. The influence of university policies can also be seen in other emerging trends: the increasing emphasis on professional development, provision of resources for students in transition and investigations of students' perceptions and opinions.

The name Delta was chosen to imply change. Over the sixteen years of Delta's existence, the Delta community has grown and evolved, and the types of conference presentations have also changed. Participants at Delta 97 were, by and large, mathematicians with an interest in teaching and learning but possibly little expertise in educational theory. Most presentations were reports of a particular teaching experience or initiative. The conferences now see many more formal research presentations, based on educational theories and methodologies. However, many mathematicians and statisticians with research interests in their mathematical discipline attend Delta conferences and value the experience. Hopefully Delta will continue to accommodate academics who are not formally involved in research in mathematics

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education, and to accept papers which allow for the dissemination of interesting teaching initiatives.

Delta conferences remain invaluable forums bringing together academics from diverse countries. Sixteen years on, it is timely to remember the debt of gratitude owed to Patricia Cretchley, whose vision gave birth to the Delta community.

## Acknowledgement

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# Use and perceptions of worked example videos for first-year students studying mathematics in a primary education degree

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**Keywords:** Worked example videos; primary teaching; mathematics for hearing-impaired students

Worked example videos have great potential to be useful for students when learning mathematics as they can work through the questions at their own pace, pausing as needed, but still learn from the way the demonstrator thinks and solves problems. We created worked example videos each week for a mathematics subject taught in the first year of a primary education degree and investigated student perceptions and their usage patterns. An additional aspect of this undertaking was the inclusion of subtitles to make the videos accessible to hearing impaired and ESL students. This report will reflect on the process of creating these videos, as well as some initial findings on their success.

## 1. Introduction

In the current educational climate there is increasing demand for online learning resources to help provide flexible learning options for students enrolled at university. Many universities are expanding their online environments to include resources such as audio/video recorded lectures [1-9], lecture notes, discussion boards, review quizzes etc. As in-class demonstration of how to solve sample routine and non-routine problems is at the heart of many mathematics teaching styles, undergraduate mathematics units lend themselves particularly well to the inclusion of worked example videos. Of recent interest to teachers at the university level is the use of Tablet PCs either in the production of lecture recordings [1,9] or shorter self-contained examples [10]. However, as pointed out by Trenholm *et al.* [11], although students generally respond positively to the availability of such resources, a number of empirical studies have shown them to have little bearing on student learning outcomes (e.g. Cascaval et al. [3]). Nonetheless, we contend that there are certain mathematics subjects where working to improve overall student attitudes is quite important. In particular, the focus of this study is a compulsory first year mathematics subject for primary education students, some of whom have an aversion, negative attitude, or even fear of mathematics [12]. Whilst we have made lectures available online for some time (referred to as iLectures, these show the power point slides or document camera view from the lecture along with the audio), in practice it did not seem like students were making great use of these. Re-listening to an hour-long lecture is not always useful for consolidating mathematical concepts (some authors have suggested that students need training on how to *effectively* use the resources available to them [13]). In an attempt to provide a more focussed resource that students might be more likely to use, we undertook the production of short (4-10 minute) worked example videos using a mobile phone to record the demonstrator solving problems on a white board, which were then uploaded in HD quality to YouTube. Between 1 and 3 videos were created each week. The videos were also subtitled in English, primarily to make them inclusive and accessible to hearing impaired students completing the subject, however also with anticipated benefit to

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students with English as a second language (ESL). This work will reflect on the creation of these videos and present results gained from usage statistics and a short online survey, which students were invited to respond to at the end of the teaching period. Overall, students responded positively, with many of the anticipated benefits of worked example videos being mentioned in the feedback they gave. The particular benefit to hearing impaired students is something that deserves further attention in the future teaching of similar units.

## 2. Background

Him *et al.* note that "the use of fully worked-out examples to teach mathematical concepts and procedures has been the predominant instructional technique in mathematics classrooms" [14]. The availability of worked example videos can hence aim to replicate this classroom experience. Dekkers *et al.* [7] report on the integration of Tablet PC recorded instructional videos into an electronic study guide, in particular drawing attention to the benefit of seeing the example solutions 'unfold' dynamically along with the demonstrator's explanation of the mental processes involved. Student feedback suggested that this was preferred to having the whole solution presented and being required to decipher the reasons behind each step.

Much of the focus for incorporating worked example videos has been on the use of Tablet PCs. This could be in part due to the small file size requirements and relative ease of producing such videos, however, the creators of the Macquarie University Mathematics Skills (MUMS) modules noted that their choice to use Tablet PC worked example videos over other forms was also motivated by a desire to reduce the number of distractions [10], e.g. unconscious lecturer habits and idiosyncrasies. Findings from *Dey et al.* [6] were that, while students were not generally distracted by 'personalised' presentations (in this case, Tablet PC demonstrations were accompanied with an embedded video of the lecturer's face), the presence of the lecturer also did not tend to enhance learning outcomes. When using Tablet PCs, however, some aspects of the tutorial room experience are clearly lost. In particular, the use of gestures, i.e. using hands to emphasise notions like small, large, steep, is a potentially important part of mathematical communication and mathematical thinking [15]. The *Summertime Maths* website of University of Wollongong [16] is one existing project that has opted for the use of 'live' videos (available to anyone) showing the lecturer in front of a white board, solving problems and giving overviews of some fundamental topics. As the technology to undertake live recordings becomes more convenient, their incorporation into mathematics teaching may become more common.

The inclusion of subtitles into worked example videos has seen little attention in current studies, perhaps due to the associated difficulty and cost. If such resources are to be made accessible to deaf and hearing-impaired students, however, their inclusion is a necessity [17]. In fact, worked example videos have particular potential for addressing a number of problems faced by deaf students in the university context. Several mathematical concepts do not have specific signs associated with them, or are seldom known by interpreters who do not usually have a focused mathematical background. Mathematics terminology is hence usually signed 'on the fly', with the nuances in how to convey things mathematically lost on learners. Hearing-impaired students may have additional difficulty in reading comprehension, and miss out on much of the incidental mathematics learning that comes from everyday life [18,19]. The addition of subtitles to worked example videos hence really helps to reinforce mathematical concepts and relationships, perhaps even beyond what can be achieved in a live classroom context or with reading materials alone. They may also have benefits for ESL students and even native English speakers, by enabling learners to be exposed to the subject specific terminology that might otherwise be lost in the stream of speech [20].

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Although there is rich literature on the use of worked examples in mathematics and a number of existing implementations of worked example videos, to the best of our knowledge the availability of worked examples, created with live video and subtitles, is not something that has been studied to date.

## 3. Methodology

### 3.1 Participants

There were 400 students originally enrolled (and 369 that completed) the first year mathematics subject which is core to the primary education degree. Most of the students were female (314 out of the 400) and just over one quarter of those enrolled were not straight from high school (i.e. mature age or course transfer students). The subject involved a 1 hour introductory lecture, a 2 hour tutorial and a number of activities to be completed online from the study text, as well as online practice quizzes. The unit was assessed with two online multiple choice assignments, a third assignment comprised of online and written components and a short answer final exam. The subject ran from July to October in 2012.

### 3.2 Video Production

The videos were created using an iPhone and then uploaded directly to a specifically created YouTube page. The videos were between 4 and 10 minutes long and in most cases showed the lecturer demonstrating the process of solving a previous year's exam question relevant to the topic that had just been covered (See Fig. 1). A video giving an overview of the final assignment's criteria was also made available.

The decision to create the videos in this way was primarily for the sake of convenience (i.e. purchase and familiarisation with a new technology was not required as it would have been for Tablet PC videos), however this format also allows the lecturer to use gestures and draw attention to multiple aspects of the problem space at once similarly to a tutorial situation. As these videos were only made available to students undertaking the subject, the presence of the lecturer was not anticipated to be an unnecessary distraction. Furthermore, it was hoped that short videos with demonstrations that matched the teaching of the subject would be perceived by the students to be more relevant and useful than if they were directed to an external resource.

The videos were also subtitled to make them accessible to hearing impaired students. An assistant was employed for a few hours each week to add subtitles to the videos by firstly writing out a transcript, using Captiontube to make the *srt* file and then reviewing to make sure the text was aligned with the speech. Ensuring the resources and teaching in this subject were inclusive to hearing impaired students was another motivation for 'live' videos rather than screen capture type videos, as these students' ability to understand what is being said is improved if they can see the lecturer's lips and pick up on expressions, gestures etc.

### 3.3 Data Collection

Data was collected from three sources. Firstly, individual student data was available from the university's online Desire2Learn (D2L) environment. Such data included statistics on how often students accessed a particular link, their average time spent on each page, their practice quiz scores and assignment scores. Students were considered to have 'watched' one of the videos if their duration of access to the relevant link was greater than 2 minutes. The first and second assignments both required the students to complete an online test of 30 multiple choice questions. Each test was available for a period of about 10 days, however students had just one hour to answer all the questions once they started. The tests each contributed

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15% of the students' overall mark. Other resources available to students for which we collected download statistics were the online lectures (iLectures), lecture notes (both the PowerPoint presentations and static PDFs of these were made available) and the Study Guide solutions, i.e. answers and worked solutions to questions in the study guide.

Secondly, statistical summaries on video views per day and total views were collected from YouTube. These were used to identify trends in when students tended to access the videos.

Thirdly, students were given a survey with 14 closed (multiple choice and Likert scale) questions and 2 open questions about their perception of the videos.

## 4. Results and Analysis

We now give a summary of some of the results we obtained. Firstly, we can briefly mention the average assignment scores for students who watched the videos compared to those that did not. For the first assignment, 18.9% (72 out of the 381 students who completed the online test) watched one or more of the relevant videos beforehand. For the second assignment, 14.4% (52 out of 360) of students watched a relevant video prior to completing the test. The mean scores for assignments 1 and 2 for students who had viewed the videos were only slightly higher than those who had not (24.75 and 24.1 compared to 23.63 and 23.8 out of 30 respectively) and had slightly lower standard deviations (3.816 and 3.733 compared to 4.306 and 3.991). We should note, however that we do not see these results as particularly significant or indicative of the videos' *success*. In a subject such as this one, students enter with various backgrounds. Some would already have understood the concepts quite well, and not needed to make use of additional resources. On the other hand, some strong students were also more likely to be diligent and make use of every resource available to them even if they didn't need to. Students who struggled may have used the videos to aid their study, however still may not have performed well, while those who really would have benefited did not have the motivation to make time to view them. In other words, measuring real effectiveness without a controlled study at best provides a rough guess. Rather, since the motivation in creating these videos was to provide a relevant resource that would actually be used by students, we focused on answering the following questions:

> Did students make use of the worked example videos and how did this use compare to other available resources?
> Did students find the worked example videos useful?
> What factors influenced when and whether students would use worked example videos?

We note that whilst we were interested in the process of creating the subtitles and whether they too were considered useful for hearing impaired and ESL students, the small number of these students enrolled in the subject meant we could not collect sufficient data to investigate these aspects properly.

### 4.1 Student Usage Patterns

A comparison of how many students accessed the worked example videos to other available online resources is shown in Table 1. In a small number of cases (weeks 7, 9 and 11), more students watched the worked example video(s) than accessed the iLecture. Certainly the number of students is comparable to the number that downloaded the study guide solutions, suggesting that incorporating such a resource into the subject was a worthwhile venture. Some videos were longer than others, however the average time spent on each video also

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suggests that some students tended to watch the videos more than once, or utilise the pause and rewind functions. This can also be seen in Fig. 2, which shows the unique number of viewers and total views over time for one of the videos.

The viewing trend in Fig. 2 is fairly typical of all videos produced. The videos were watched as they became available, in some cases with increased views toward the due date of the relevant online assignment. Views then stopped until students resumed watching them about two weeks leading up to the exam, always hitting the peak the day before.

Table 2 also shows this tendency, with a comparison between views over the period in which topics 1-11 were covered (July 9 - September 23) and over the period that includes the one week break from classes, revision week, and the study period leading up to the exam (September 24 - October 19). Although this usage can partly be explained by students becoming more aware of the videos as the teaching period progressed, it is clear that students were particularly interested in the videos as a study resource for the exam.

## 4.2 Student Evaluation

The survey was completed anonymously online by 27 students, 3 of whom had not yet watched any of the worked example videos. Students were offered the chance to win a $20 gift-voucher for participating. The survey was short, and designed to gain some insight into whether the students found the videos useful, why they tended to use them and how they would like to see them improved. The gender break-down of the respondents was similar to that for the subject, with 23 females and 4 males, while 20 out of the 27 were aged between 18-24. No respondents identified as having English as their second language, and one of the respondents was known to be hearing impaired. Of the students that watched the videos, 7 said they watched 1-2 videos, 3 said they watched 3-4, 8 watched 4-6 and 6 estimated that they watched 7 or more. Only 3 of the participants responded that they wanted to use the subtitles (See Table 3).

Table 4 shows the responses to five Likert scale questions. Students were varied on whether they watched videos more than once and many students did not find the subtitles particularly useful or user-friendly, however all respondents were positive about the videos. We needn't overemphasise this result, as the response rate of the survey was low and it could well be assumed that students more positive about the videos were more likely to respond to the survey and hence can't be seen as an unbiased sample. Previous studies have also shown that students tend to be positive about additional resources being made available to them [11].

Table 5 gives an idea as to why students tended to use the videos. All respondents who had watched videos indicated that they used the videos to clarify a concept they were having trouble understanding, while only a quarter wanted to see what they had missed by not attending the tutorial. As these are only short examples, the videos were not intended to substitute for tutorial attendance, however there is often speculation among lecturers that providing too much content online will lead to lower tutorial numbers. It is our belief that, as supported by the findings in [9], availability of online material rarely factors in to a student's decision to miss class. For teaching this subject, providing a supportive and flexible learning environment for all students was seen as important.

When asked why students chose not to watch some of the available videos, the most common response was that they already understood the topic, while a number also noted lack of time or lack of awareness (See Table 6). Although the videos were promoted in the lecture and on the online noticeboard, their availability *after* the week's classes rather than at the same time that the rest of the material became available may have contributed to the frequency of this response.

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Students were also given the opportunity to provide comments about the videos. Of the 22 comments provided, 11 made reference to the videos being clear or easy to understand, 4 appreciated the availability to follow up on something if they didn't understand or missed it in class, while 3 made reference to the ability to pause or rewind the videos. One student liked the ability to "see and hear the work not just read about it". Another student responded that it was "better then just re-reading notes in [the] study guide or a written explanation."

For suggested improvements, although 18 comments were provided, 5 of these stated that the videos were fine as they were. There were 4 that commented on the quality or camera set-up, while 5 suggested that more videos could be included, either longer or with varying difficulty.

### 4.3 Subtitle Availability for Hearing Impaired Students

From 2009-2012, there have been between one and three hearing impaired students undertaking the mathematics subject (and its previous versions) in which these videos were introduced. Clearly the numbers are not large enough to gather substantial evidence, however it is worth noting the positive informal feedback provided regarding the videos from the student enrolled in 2012. An online discussion board comment posted by this student read:

> Thanks for doing the first week with captions! They did help a LOT, I found that I could get more out of it than watching my interpreters, as sometimes they interpret things [in a way] that doesn't quite make sense to me. It is hard to explain, but the important thing is that I 'got it' when watching the video.

As was mentioned in the introduction, it is sometimes difficult for hearing impaired students to make effective connections between written information and that conveyed from the lecturer through their interpreters. The worked example videos with subtitles are an ideal resource for supporting these students.

### 5. Conclusion and Future Implementation

This paper has reflected on the creation of worked example videos for primary education students undertaking a first year mathematics subject as part of their degree. Although much of the recent research has concerned the use of Tablet PCs, these videos showed the demonstrator in front of a white board and included subtitles for hearing impaired students. We found that many students made use of the videos for exam and assignment preparation and that their reactions were quite positive. Whilst in the past the undertaking of such videos may have required low-scale camera crews or expensive equipment, current personal technologies and online processing allow for relatively easy and inexpensive videos to be created. Although the use of online lectures is common to many universities, short videos allow lecturers to provide easily accessed and focused presentations to complement other modes of teaching.

In future, these videos will continue to be made available for the primary education students as well as other subjects taught by the current lecturer. Of interest are whether students would prefer 'live' presentations given by their own subject teachers, or whether other formats such as Tablet PC presentations and flash files, or externally available resources, would be just as welcome. There is also the question of whether the videos would be received differently by students of first to third year subjects of a mathematics major sequence.

### Acknowledgement

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**Table 1.** Student use of web resources.

| Week | Worked example video | No. users | Avg. time | iLecture | Lec. notes (pdf) | Lec. notes (ppt) | study guide sols. |
|---|---|---|---|---|---|---|---|
| 1 | Roman to dec. conversion | 127 | 0:07:18 | 194 | 343 | 166 | 141 |
|  | Greek to dec. conversion | 85 | 0:02:54 |  |  |  |  |
|  | Time to hours:min:seconds | 141 | 0:16:16 |  |  |  |  |
| 2 | Divisibility tests | 102 | 0:11:13 | 136 | 293 | 91 | 97 |
| 3 | HCF and LCM using primes | 94 | 0:12:47 | 120 | 256 | 57 | 104 |
|  | Determining primeness | 99 | 0:11:00 |  |  |  |  |
| 4 | Fractions close to a half | 92 | 0:10:37 | 135 | 242 | 60 | 124 |
| 5 | Converting fractions to dec. | 79 | 0:14:49 | 93 | 228 | 42 | 91 |
|  | Mixed recurring dec. to frac. | 71 | 0:12:26 |  |  |  |  |
| 6 | Converting % to fractions | 61 | 0:13:14 | 85 | 205 | 38 | 90 |
|  | Eval. % w/ first principles | 68 | 0:08:05 |  |  |  |  |
| 7 | Solving linear equations | 132 | 0:13:26 | 87 | 205 | 66 | 106 |
| 8 | (no worked example video) | - | - | 89 | 223 | 44 | 79 |
| 9 | 5-num summary, box plots | 79 | 0:11:10 | 75 | 203 | 46 | 66 |
|  | Mean and sample st.dev | 78 | 0:11:01 |  |  |  |  |
| 10 | Int. angles of reg. polygon | 77 | 0:10:12 | 84 | 197 | 49 | 77 |
| 11 | Assignment 3 | 111 | 0:18:50 | 85 | 219 | 50 | 58 |
| 12 | (revision week) | - | - | 47 | 85 | 37 | - |

*Access statistics can be slightly higher than actual since these are obtained from the university online environment whereas those in the following table are obtained from YouTube which does not double count repeated access during the same session.

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**Table 2.** Comparison of views during teaching period and leading up to exam.

| Week | Worked example video | views 9 Jul - 23 Sep | views 24 Sep - 19 Oct |
|------|----------------------|----------------------|-----------------------|
| 1 | Roman to dec. conversion | 63 | 46 |
|   | Greek to dec. conversion | 40 | 26 |
|   | Time to hours:min:seconds | 83 | 84 |
| 2 | Divisibility tests | 56 | 58 |
| 3 | HCF and LCM using primes | 35 | 62 |
|   | Determining primeness | 53 | 60 |
| 4 | Fractions close to a half | 48 | 49 |
| 5 | Converting fractions to dec. | 28 | 55 |
|   | Mixed recurring dec. to frac. | 30 | 61 |
| 6 | Converting % to fractions | 23 | 47 |
|   | Eval. % w/ first principles | 27 | 41 |
| 7 | Solving linear equations | 58 | 54 |
| 9 | 5-num summary, box plots | 25 | 40 |
|   | Mean and sample st.dev | 34 | 45 |
| 10 | Int. angles of reg. polygon | 17 | 64 |
| 11 | Assignment 3 | 23 | 91 |
| **Total** | | **643** | **883** |

**Table 3**. Responses to question regarding use of subtitles.

| All the videos used in this unit were subtitled, and in some cases, these would have been activated automatically.  Have you watched any of the worked example videos using the subtitles feature? | No. Students |
|---|---|
| Yes, I wanted to view subtitles | 3 |
| Yes, however the subtitles loaded for me automatically, I would not have chosen to use them | 3 |
| No, I either turned the subtitles off, or did not activate them | 19 |

**Table 4.** Student responses to Likert scale questions on worked example videos.

| Question | SD | D | N | A | SA |
|----------|----|---|---|---|----|
| For the worked example videos I watched, I tended to watch them more than once. | 3 | 9 | 5 | 4 | 3 |
| The worked example videos I watched assisted with my understanding of the given topic. | 0 | 0 | 0 | 10 | 14 |
| The worked example videos were user-friendly, i.e. accessibility, loading-time, easy to use. | 0 | 0 | 0 | 10 | 14 |
| The availability of subtitles in the worked example videos assisted me with my understanding of the topic.* | 2 | 4 | 12 | 2 | 2 |
| The subtitle feature was easy to access | 2 | 1 | 9 | 7 | 3 |
| * Note responses in Table 3 regarding subtitles feature | | | | | |

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**Table 5.** Student responses to the question: Please indicate how and why you tended to use the worked example videos (tick all that apply)

| Purpose | No. Students |
|---|---|
| To revise for the assignment/exam | 17 |
| To clarify a concept I was having trouble understanding | 24 |
| To reinforce a concept that I wanted to understand better | 17 |
| To help me gain an idea of what was expected for assignments/exams | 7 |
| I didn't attend the tute and wanted to see how the concepts were explained | 6 |
| I wanted to see how problems could be solved in different ways | 10 |

**Table 6.** Student responses to the question: If you have seen none or only a few of the worked example videos, what are the main reasons you did not watch them? (tick all that apply)

| Purpose | No. students |
|---|---|
| Lack of time | 5 |
| Already understood the topics | 9 |
| Wasn't aware they were available | 6 |
| Accessibility (didn't know how or had difficulty with download speed etc.) | 1 |
| Other | 5 |
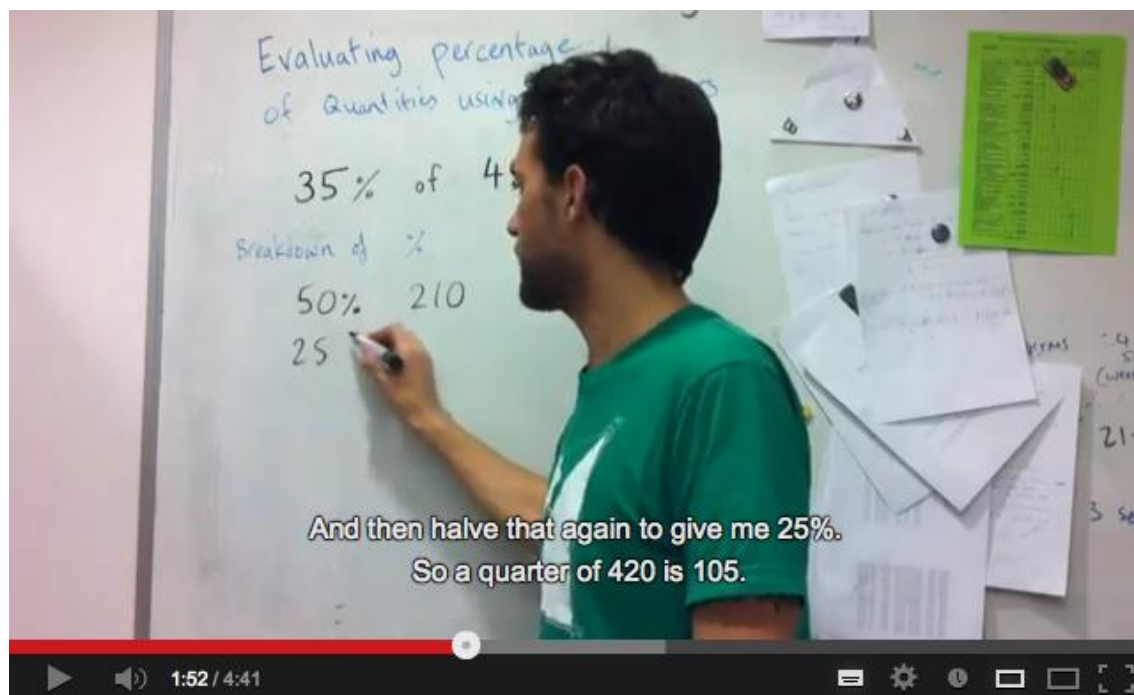


**Figure 1.** Screen capture from YouTube showing the subtitle display and demonstration.

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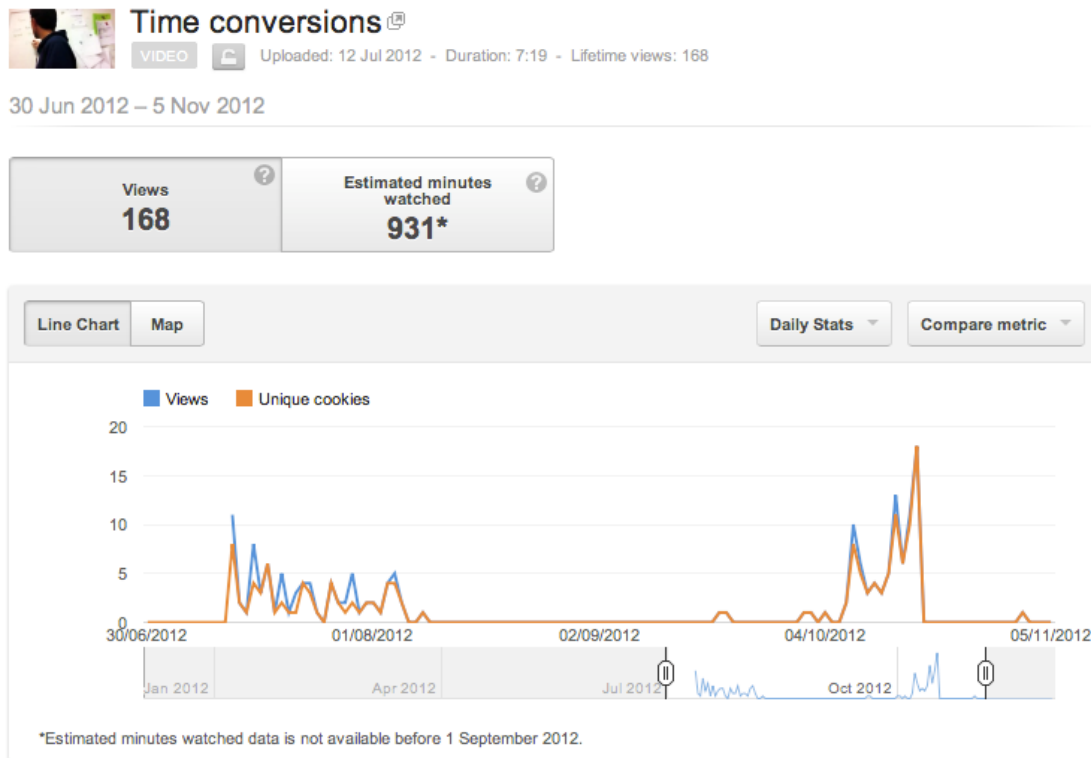
**Figure 2.** Screen capture from YouTube. Views over time for worked example video on time conversions.

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# WeBWorK CLASS: Fostering design experiment research on concept development

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We introduce the WeBWorK Collaborative Learning and Active Support System (CLASS), an extension to the popular online homework system, WeBWorK, available for any touch-enabled device. WeBWorK CLASS is a powerful tool for use in and out of the classroom which engages students, aids instructors, and allows researchers to explore student thinking. Research enhancements allow us to analyze both quantitative and qualitative data from in-class sessions, online homework, and other assessments. We use the system in a first semester calculus class for ongoing iterations of a design experiment focusing on function composition as it relates to calculus concepts such as the chain rule. Examples from collected data demonstrate the utility of CLASS as a research tool. Further, we describe how an approach to calculus based on differentials was used to teach the chain rule and how students interpret it.

**Keywords:** Calculus; design experiment; online assignment; WeBWorK

## 1. Introduction

As online assignment systems[1] gain popularity in undergraduate mathematics courses, the effective incorporation of such systems in classroom teaching and educational research studies addressing students' development of mathematical concepts needs more attention. Research studies on traditional homework indicate that there is a positive association between assignments and achievement [1]. Students who complete online assignments perform as well as those who complete traditional paper and pencil assignments [2,3]. Compared to traditional assignments, students prefer the immediate feedback on answers from online systems and this contributed to higher persistence rates while solving problems [4]. Moreover, online assignment systems provide students additional learning opportunities by providing problem variations, multiple attempts per question, and help via online tutorials. However, online assignment systems could do more.

Online assignment systems have the potential to provide more information on students' understanding of concepts. We enhanced the free, open-source online assignment system, WeBWorK[2], to create WeBWorK Collaborative Learning and Active Support System (CLASS). The newly incorporated tools include: the integration of a whiteboard to capture students' written work (Figure 1), an integrated graphing utility (no need to carry a separate calculator or access an external website), digital work maps (Figure 2), CLASS performance map (Figure 3), and anonymous classroom sharing of student generated work.

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[1] Our use of *online assignment system* includes other assessment aspects, such as online exams.
[2] More information about WeBWorK can be found at http://webwork.maa.org/
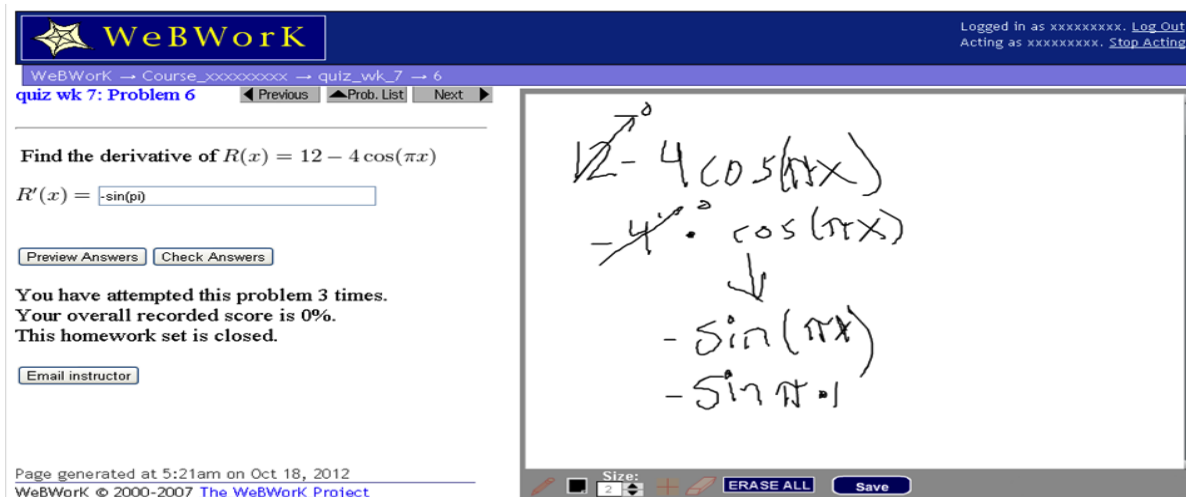
**Figure 1.** Integrated whiteboard.

CLASS works on all touch-enabled platforms including tablets, ipads, laptops, and phones - technologies students increasingly already own. CLASS can be used for any course that has a WeBWorK component. CLASS, together with the existing capabilities of WeBWorK, provides an all-in-one system that goes beyond static online assignment systems and creates an interactive in-class teaching tool and collaborative research platform.

CLASS contributes to research efforts in the mathematics classroom by collecting and organizing quantitative data. Furthermore, inclusion of the annotation tool helps analyze qualitative data. We use these capabilities in a design experiment [5], intended to address two general questions:

How does CLASS allow researchers to learn more about students' learning and understanding of mathematical concepts?

How does CLASS contribute to concept development and student success in calculus?

In this paper we provide results from the first iteration of the project, which focuses on use of the annotation tool. We also share our implementation of CLASS in an ongoing cycle of the project.

As a research tool, CLASS efficiently gathers and processes quantitative and qualitative data that are especially useful for mathematics education researchers and teachers. Since CLASS is able to capture all of a student's work, including blind alleys and backups, the researchers can identify when a student is blindly parroting previous work or when she is incorporating old concepts with new ideas correctly. Identifying such student actions allows researchers to track concept development. Moreover, CLASS' annotation tool provides a mechanism for researchers to investigate students' understanding collaboratively. We will provide evidence of how collaborating researchers at institutions across the globe can create codes with descriptions, modify existing codes, and link codes to particular times in students' solutions.

We begin with a brief review of the literature on online assignment systems and in-class technology used to promote engagement in class. The subsequent discussion of technology-supported social constructivism for learning provides the framework for the design experiment, and precedes the description of our design experiment, data, analysis, and results.

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## 2. Background

Students are frequently engaged with learning tasks that take place outside of the classroom, commonly known as homework. It is the expectation that these activities help students understand and master the content of the course. Many research studies have explored the relationship between homework and achievement in courses [1,6,7]. Even though the contribution of each homework factor to achievement is not clear, there is a consensus that assigning homework helps students learn, especially at the secondary and post-secondary levels [1,6-8]. Time spent on homework has shown to be positively correlated with student achievement in a course [1,6]. Furthermore, grading can provide the instructor with a feeling of how the class is learning.

Studies that focus on the impact of online homework on achievement indicate that students who complete online homework assignments perform as well as students completing traditional assignments [2,3]. Moreover, students complete online homework at a higher rate than traditional paper and pencil assignments [4]. Most commercially available online homework systems collect students' entered answers and provide a basic quantitative analysis reporting the number of students with correct and incorrect attempts to each question. Such information helps instructors quickly identify common troublesome questions and could be used as formative assessment tool [9].

CLASS provides all these benefits and additional information. Similar to tools developed within the SAIL-M project [9], CLASS provides a visual representation of students' overall performances. However, different from tools in the SAIL-M project, each student's written solutions can be viewed and replayed through the CLASS digital work map (Figure 2).



**Figure 2.** Digital work map.

The CLASS performance map (Figure 3) allows instructors to examine success rates on a chosen set of related questions and the associated students' answers to see how students understand a concept [10]. For example, various questions in an assignment might focus on different aspects of a particular concept, such as symbolic manipulation, use of different representations, word problem. An instructor could create a CLASS performance map, sorting correct and incorrect answers to each of these different questions to see a possible development of a concept by different categories of students. See Figure 3 for an example based on function composition problems.
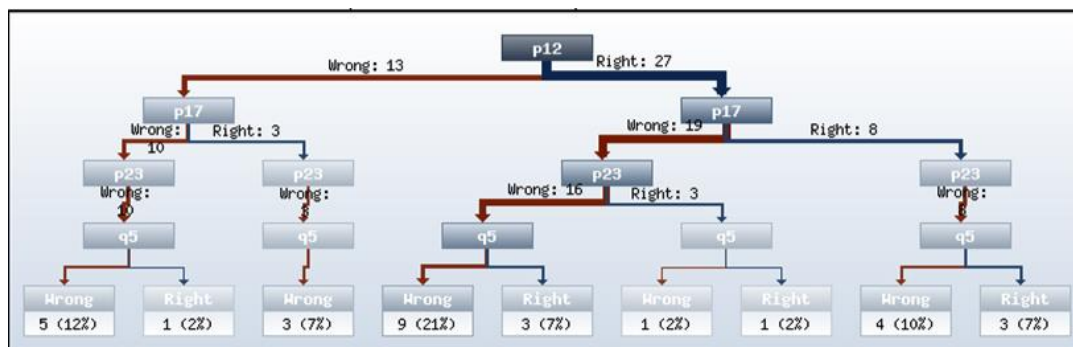


**Figure 3.** CLASS performance map.

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The CLASS performance map can be used to identify areas where the problem solving process breaks down for students and students' written solutions viewed in the CLASS digital map provides more details of this process. Such instances could be valuable to structure classroom discourse. Reform movements in mathematics education highlight the importance of communication about the process of problem solving and reasoning in classrooms. [11,12] Such shifts call for altered interactions between students and teachers in the classroom. Research has shown that students learn more effectively when they are actively engaged with the material [13-15]. It is critical to integrate tools such as CLASS that promote such learning environments and support student reasoning, communication, and problem solving skills.

Many researchers have suggested that the use of technology in classrooms should contribute to student learning. Graphing calculators and audience response systems (*clickers*) have been used to promote student thinking [16] and have students interact with the teacher, but the technology has not moved substantially beyond these capabilities. Donovan and Loch [14] used pen-enabled screens to provide opportunities for students to solve problems in groups, give and receive feedback on problems during class time, and become more active learners. Balacheff and Kaput [17] pointed out that the computer-based learning environments in mathematics are powerful not only in their impact on daily practices but also in their contribution to the deeper development of knowledge of individuals. Pea [18] suggested technologies should help students take control of their own learning by supporting the development of skills like problem posing, flexible strategies, creative thinking, summarization, and cooperative problem solving. We will illustrate how we leverage CLASS features to engage students with mathematics.

## 3. Theoretical Perspective

Our design experiment is structured around social constructivist ideas [19-21]. We are interested in the process of growth and change rather than just the product of development. An important tenet of this theoretical framework is that mental processes are mediated by tools and signs such as language, writing, technological tools, and so on. As students actively engage with tools in the classroom, their experience is internalized and executed on an inner, mental plane. Vygotsky [20, 21] states this incorporation of tools into human action presents new "functions" associated with the tools and alters mental processes. Hence, incorporating technological tools like computers in class affects the nature of the human activity, and can, for example, change the goal of the "learning" activity as well as contribute to the development of psychological tools (signs). Specifically regarding the effect of using computers in the classroom, Moreno-Armella & Sriraman [22] stated, "[A computer] is, simultaneously, a tool that can affect the human activity… and the cognition of the agent user (reorganizing of her ideas). In other words, the computer is externally oriented and at the same time internally oriented." (p.223) CLASS is the primary tool we are using to restructure the learning experience of the student both in and out of the classroom.

Collectively, the ideas from social constructivist perspectives suggest looking deeper into the construct of the *zone of proximal development* [21,p. 86], which requires re-evaluation of the role of imitation in learning. It is well documented that students prefer to learn from examples [23-27]. However, evidence suggests this strategy is not as successful as one may hope [23-27]. Silver and Marshall [26] indicated that while experts focus on structure, novices tend to focus on superficial features when attempting to solve word problems. As Boesen et al. [27] reported, most students (86%) successfully solve tasks using imitative reasoning when the new task shares similar surface features with the ones from textbooks or class examples. However, when these surface features are absent, this rate

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significantly drops (19%). Capitalizing on information available through CLASS, the researcher can efficiently identify patterns of example imitation on problems similar to in-class and textbook examples. More importantly, CLASS provides mechanisms to compare the in-class and out-of-class work of students on similar tasks which do or do not share superficial features.

## 4. Method

Our project is a design experiment of a "learning ecology" [5, p.9] as we aim to develop the multiple features of CLASS and study how they function together to support learning in the calculus classroom. By their very nature, design experiments require the collection of large, longitudinal data sets through successive iterations. The data includes products of learning (student work, tests, etc.) as well as data collected from classroom discourse, interviews, social interactions, inscriptions and notations, and other tools. Each iteration of data collection and analysis leads to revisions and testing in the subsequent cycles of the design experiment which make it suitable for testing classroom innovations. Our research team, including the instructor of the course, collaboratively collected and analyzed data, made revisions to CLASS and its use.

### *4.1 Design Experiment Cycles*

During the planning of our design experiment we enhanced the WeBWorK system to create WeBWorK CLASS. After the creation of CLASS, it was first implemented as an online assessment and research tool in a first semester calculus course during the Fall 2010 semester (iteration 1). Our focus in this particular course was the students' development of calculus concepts that use function composition. This narrow focus was intentional and could be supported by one of the goals of design experiment methodology: "design experiments are conducted to develop theories,... [T]hese theories are relatively humble in that they target domain-specific learning processes." [5, p.9]

Iteration 2 implemented CLASS in the classroom to capture detailed students' interactions with each other. In this iteration, the role of CLASS expanded from primarily out of class assessment into the classroom learning experience, and built upon the data analysis from iteration 1. The focus of data collection still revolved around concepts related to function composition.

### *4.2 Why Function Composition*

Historically, calculus is a gateway course for STEM majors. Without achieving success in calculus, STEM majors often move on to other disciplines. Success in calculus can be linked to students' understanding of the concept of function [28, 29]. The concept of function is known to be difficult for students and slow to develop [30]. While there is fairly extensive literature about student understanding of function, there is little that focuses explicitly on student understanding of function composition. Being able to work with functions in different contexts is particularly problematic for students, especially when dealing with function composition [31]. Having a robust understanding of function composition is necessary for success in calculus as this idea permeates the curriculum. One of the critical units where this concept is foundational is the chain rule and its applications. To understand students' concept development of function composition and related calculus concepts, we collected students' written solutions and answers to questions related to this topic throughout a first semester calculus course in both iterations.

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### 4.3 Data Collection

During iteration 1, data was collected from a first semester calculus course taught by one of the authors at an institution that provided tablet technology to all students. The collected data set included all 34 students' work on pretests[3] (Precalculus Concept Assessment instrument[4]- PCA, [28]), a sequence of seven CLASS quizzes, online assignments, four midterms, and the final exam throughout the 15-week semester. Students were informed about the data collection and given code numbers prior to any coding to keep their identity anonymous.

Each quiz in the sequence consisted of five questions, two of which involved an aspect of function composition. Students had up to three attempts to answer each quiz question correctly. Each student wrote their solution using digital ink on the whiteboard (Figure 1) integrated into CLASS. This system, further described in [10], recorded student actions and prevented a student's submission until they had provided enough work on the whiteboard. The whiteboard immediately provided the student with her previous work, enabling her to find and fix mistakes before resubmission. In this way, the digital ink system captured each student's attempts at identifying mistakes and how they thought the mistakes should be fixed. This aspect, distinct from other tools such as those described in [32,33] was intentionally built to capture authentic student work. Such student work enables instructors to see how students wanted to solve problems free from interference of hints or a suggested procedure which may be in conflict with the student's own approach to solving the problem. Furthermore, it helps researchers to explore a student's thinking process.

## 5. Results

To assist us with categorizing solutions, the researchers sorted students using the CLASS performance map feature of CLASS according to their performances on five function composition questions from the PCA (see Figure 3). For the following analysis of solutions to quiz questions, we chose cases from a group of students who correctly answered function composition questions from the PCA that involved finding an algebraic formula or using a table or graph, but failed to answer the fifth question that required them to identify function composition in a contextual problem. To illustrate, we share our results from the analysis of two students' solutions to one quiz question involving the chain rule, and hence function composition, given during week 7 of the semester.

CLASS coding features allow one to create keys and descriptions of student work. This feature was designed to allow researchers to apply open and axial coding procedures as described by Strauss and Corbin [34]. After sorting work according to a CLASS performance map [10], the tool provides researchers with collections of digital work maps representing each student's attempts at solving the problem. Figure 4 (also in Figure 2) shows the digital work map; this tells the instructor how long a student spent on a problem (purple lines are time spent writing), how many times they erased (pink lines), and how many different answers the student submitted (empty/filled circles). Clicking on any point in the work map shows the student's work up to that point in time.

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[3] There were 40 students at the beginning of the semester, as seen on CLASS performance map of pretest in Figure 3. Six students dropped the course after taking the pre-test and prior to other data collection.

[4] The researchers received permission to utilize the PCA instrument in Fall 2009.

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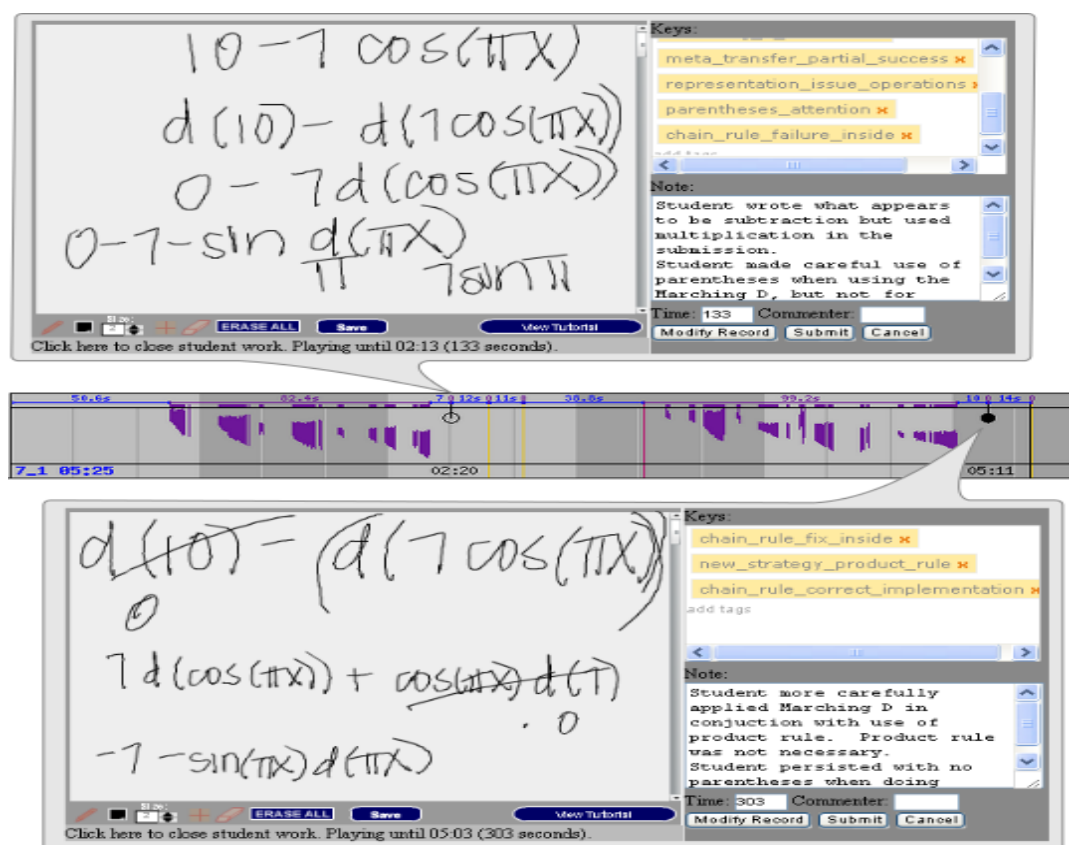
**Figure 4.** Work for Student 2 with annotations.

In addition, an annotation window is opened next to the student's work (Figure 4). The annotation window provides a place for researchers to assign key codes and comments which are then tied to that specific time marked on the work map. Researchers can instantly supply new key codes and their descriptions or select from a menu of all existing key codes using a text search mechanism. When finished with the annotations, the replay time, associated key codes, and comments are displayed with the researcher's initials below the student's work map in chronological order. The annotations are collaborative and editable. By clicking on the replay time, other researchers can instantly jump to the specific replay point and add additional keys or comments.

One problem we chose to analyze was: *Find the derivative of the function $f(x) = 12 - 4\cos(\pi x)$*. The problem involved two random parameters (12 and 4) that were different for each student. We selected two students with different answers to this quiz question; however they were on the same branch of the PCA CLASS performance map.

The first student was unsuccessful in his attempts to solve the problem (Figure 5). For this student's work, the codes of *chain_rule_failure_recognition*, *deriv_const_correct*, *chain_rule_failure_partial_recognition*, *chain_rule_incorrect_implementation*, *d_oversimplification*, *meta_transfer_failure*, and *deriv_trig_correct* were created. These codes are chosen to be descriptive of the errors made by the student. The first code, *chain_rule_failure_recognition* was used because the student did not attempt to use the chain rule at all in his first attempt at the problem. The student attempted to take the derivative of each piece that he came across in the expression. Hence, the student correctly took the derivative of 12 and obtained 0, coded as *deriv_constant_correct*. The student then attempted the derivative of $4\cos(\pi x)$ by taking the derivative of the constant 4 and getting 0 then proceeding with taking the derivative of $\cos(\pi x)$ as $\sin(\pi x)$. The student was neglecting the

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chain rule as well as the fact that they had just found the derivative of the constant in front of cosine to be 0.
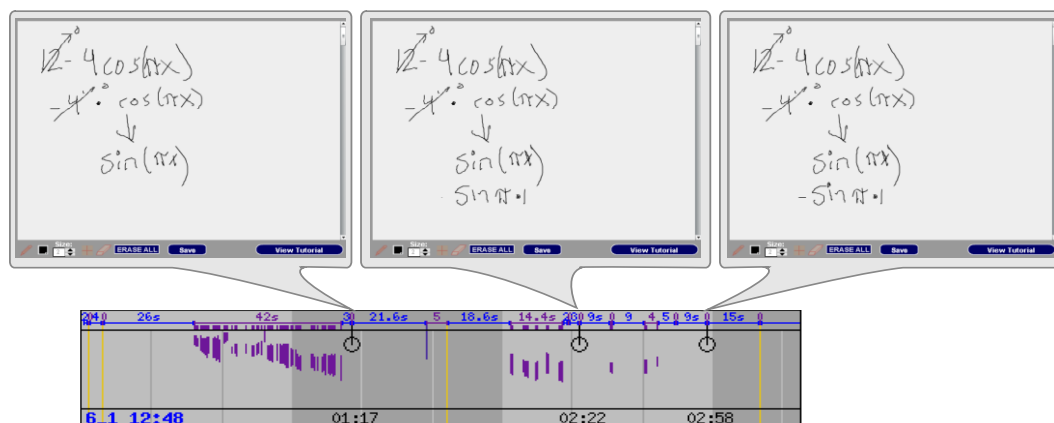


**Figure 5.** Work for Student 1.

Upon submitting his answer and learning that it was incorrect, the student made a second attempt at solving the problem. In the second attempt, we observe that the student now recognizes the need to use the chain rule, as evidenced by his use of a $\pi \cdot 1$ inside the sine function, but does not correctly apply it, as he does not multiply by $\pi$ outside the function. This was initially puzzling to two of the researchers, whose discussion and conjecture with the instructor led to a breakthrough for interpreting the student's solution process. The missing element was how differentiation had been introduced to the students.

The instructor (co-author and a member of the research team) introduced derivatives as the ratio of two differentials, such as $df$ and $dt$, which represent small changes in $f$ and $t$, respectively. This provides a mechanism which we refer to as the *Marching D* that replaces a single process for derivative rules with a two-step process ("zap with d" and "solve for the appropriate derivative") incorporating the chain rule, and error-checking mechanisms. For example, when differentiating $f(x) = 12 - 4\cos(\pi x)$, zapping with $d$ produces $df(x) = d(12) - d(4\cos(\pi x))$ or $df(x) = 0 - 4(-\sin(\pi x))d(\pi x)$.[5] The $d(\pi x)$ in the final term indicates the need for an additional differential rule, reducing the expression to $df(x) = -4(-\sin(\pi x))\pi dx$. Notice each term contains a differential, making it possible to solve for the derivative $df/dx = 4\pi \sin(\pi x)$.

The code *d_oversimplification* was created in reference to the *Marching D* as a result of the discussion with the instructor about how he taught the derivative rules. Upon discovering what had been presented in class, it became evident to us that the student was attempting to use the procedure without understanding what he was doing. The student was allowing the $d$ to march through and take derivatives from left to right with no regard for whether there were inside functions. The student did identify the correct derivative of cosine in the final attempt at the problem.

The second student, whose work (Figure 4) we chose to code, successfully solved the problem on the second attempt. This student had $f(x) = 10 - 7\cos(\pi x)$ as his problem. Upon opening the student's work, we immediately saw very clear evidence of the *Marching*

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[5] The term *d(4cos(πx))* can also be expanded with the product rule, giving *cos(πx) d(4) + 4d(cos(πx))*, but *d(4) = 0* as four is a constant and does not change. This use of the product rule is evident in second attempt of Student 2 which produced a correct answer.

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*D* as described above. We coded the work using the same codes from the previous coding sessions and created new codes of *meta_transfer_partial_success*, *representation_issue_operations*, *parentheses_attention*, *chain_rule_failure_inside*, *chain_rule_fix_inside, new_strategy_product_rule*, and *chain_rule_correct_implementation*.

This student was successful in identifying his errors and making adjustments, the most significant of which were fixing the chain rule and choosing to use a different strategy. This student chose to view the second part of the problem as a product of two functions. While not necessary to get the right answer, it did allow him to address an error. The student also provided us with something unexpected which led us to create the code *representation_issue_operations*. The student was able to enter a correct answer, but examination of the written work showed that his notation was lacking. The student wrote $-7 - \sin(\pi x)\, d(\pi x)$ on both attempts, but in typing in his answer he put in a multiplication between the $-7$ and the $-sin$. This provided evidence that the student was thinking about the operation as multiplication but the written work did not indicate that and communicated a subtraction operation. As a result, student 2 did not accurately communicate how he was thinking about the problem. Such a problem with the associated work is valuable for a teacher to facilitate a classroom discussion about the importance of how one communicates mathematical ideas both in writing and verbally.

After completing the data coding for student 2, the research team examined the codes for student 1 and fine-tuned both the keys and comments. We analyzed other students' work on similar problems using these existing keys and new codes were added to the list. As a result of this first analysis, we were able to identify misconceptions about the *Marching D*, which can be used to inform future iterations of the design experiment.

## 6. Discussion

This brief analysis of two students' solutions to the same problem demonstrates the use of CLASS to highlight important aspects of students' learning and discover how students interpret what was presented in lecture. Even though both students had similar scores on the PCA pretest, their attempts and solutions showed different characteristics in how they solved the problem. Analysis illuminates the ways in which both students were trying to imitate the in-class presentation of the *Marching D* idea. Student 1 did not produce a correct answer, whereas Student 2 was able to fix his mistake leading to a correct answer.

One significant difference between Student 1 and Student 2 is related to students' abilities to re-assess and re-evaluate their solution process once informed their original answer was incorrect. Student 2 employed a more thorough application of the *Marching D* idea, and was able to produce a correct answer and solution, while Student 1 failed to recognize the role function composition played in the problem. In fact, this was an issue across students in iteration 1 – told their answer was incorrect, many students were unable to assess their work for mistakes. Many repeated their original solution process, adjusted them in insignificant ways, or changed the form in which they represented their answer without changing the value (incorrectness) of their answer. Iteration 2 was designed to help students analyze their and others' solutions (both correct and incorrect), discuss possible ways to improve these solutions, and assess different solution strategies. This necessitated introducing CLASS into classroom teaching to create active learning environments through authentic students' work. Furthermore, incorporation of CLASS in the classroom provided students' with a coherent learning experience where the same tool is being used both in and out of classroom. There are early indicators from iteration 2 that students are becoming better at reflecting on incorrect solutions and moving beyond mere imitation of procedures.

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## 7. Conclusion

The purpose of this article was to demonstrate how an enhanced online assignment system, WeBWorK CLASS, could be used as a dynamic teaching and research tool in a design experiment project. CLASS assists in capturing, organizing, and analyzing large longitudinal data sets, which are important assets for understanding students' concept development. The annotation tool allows researchers to quickly find groups of students, with the help of CLASS performance maps, who would be expected to have similar reasoning patterns. Once a group is identified, the annotation tool helps to classify the student's reasoning skills and problem solving strategies. These classifications help researchers to identify what patterns are emerging in students' concept development. Similarly, an instructor could use different students' strategies in class to deepen students' conceptual understanding through discussions. Our future work will focus more on the use of CLASS in the classroom by analysing data collected in iteration 2 of our design experiment.

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# Teaching First-Year Business Statistics Three Ways

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**Keywords:** mathematics; business statistics; in-class response systems; assessment mechanisms; open book assessments

Teaching first-year service courses in mathematics and statistics is often a challenge. Such units are usually taught to large classes, most with poor mathematics backgrounds. Since these are often compulsory units, most students take them unwillingly. Attendance tends to drop off very fast and many students do not complete set work. This paper reports an innovative way to teach first-year Business Statistics that evolved over four semesters. Changes were made each semester to how the unit was delivered. Student performance is compared for the four methods, giving some perhaps surprising results. Some insight is discussed regarding the lessons learnt.

## 1. Introduction

Teaching first-year service units in mathematics and statistics is a challenging task. Most students enrol in these units not out of choice but out of compulsion and such units often have students with a wide range of mathematical preparation and backgrounds. Consequently, engaging students and maintaining their interest for long is difficult. Experience and data show that class attendance drops off very quickly as the semester progresses. For such large first-year classes attendance ranges from 70% [1, 2] to as low as 7% [2], and is typically around 30% [3]. Reasons for missing classes include lecture quality, lecturer quality, work commitments, ease of access to university and perceived difficulty of subject [4, 5, 6]. Several researchers have investigated the relationship between attending classes and student performance [1, 6, 7, 8, 9, 10]. Nyamapfene [8] reported that class attendance "is the key determinant for academic performance". His study was for courses with online lecture notes, which is almost universal these days. Purcell [1] found that for civil engineering students a 10% increase in lecture attendance resulted in a 3% increase in examination performance. Obeidat et al. [9] found that for industrial engineering students lecture attendance was strongly related to performance. Similarly, Rico et al. [10] reported that less well attended classes had higher failure rates.

This paper presents a method for teaching business statistics that evolved over four semesters. The unit is a requirement for all majors in Business, with around 1,000 students each year over two semesters. Students who do not have the appropriate mathematics pre-requisites from high school are first required to obtain a pass in a bridging unit. Consequently, in general the first semester class has a better mathematics background.

The unit content is typical of a first course in statistics, and the topics covered are, in order, exploratory data analysis, probability, random variables, normal, binomial and Poisson distributions, sampling theory, estimation, one-sample tests for means and proportions, two-sample tests for means, ANOVA, simple and multiple regression, and chi square tests of independence for contingency tables.

I had previously taught this unit for several years in the traditional way – three lectures per week, a one hour tutorial and a one hour laboratory class. I again taught the unit in semester 2 of 2011 after a three year break. This paper is on the evolution of teaching and learning over four semesters, based on lessons learnt from my previous experience and the facilities afforded by changing technology.

The rest of this paper is organised as follows. In the next section I outline the issues associated with the traditional teaching methods in my experience. In Section 3, the details of the changes are presented. Section 4 contains data analysis of student performance and Section 5 contains discussion and conclusions.

## 2. Previous Teaching Methods

### 2.1 Teaching activities

The traditional teaching method for business statistics is based on lectures. Supplementing these may be tutorials and laboratory classes. A survey was conducted on the teaching methods for first-year business statistics in Australian universities by email and also from course websites. Table 1 summarises the findings. Of the 25 universities (not including mine) for which information was obtained, sixteen had two hours of lectures per week (most having these as a single session and others as two separate one-hour sessions), eight had three (one-hour) lectures per week and one had four (one-hour) lectures per week. Seven did not have any tutorials or laboratory classes. Eleven had a one-hour tutorial per week but no laboratory classes and one had both a one-hour tutorial and a separate one-hour laboratory class per week. Three had a two-hour tutorial per week, while another three had a one-hour combined tutorial-laboratory session. One university had a one-hour weekly problem solving session. Eleven of the universities had four or more contact hours per week. The teaching activities of the majority of universities are based on the traditional lecture-tutorial model.

Our previous teaching regime was lecture-based supplemented by a one-hour tutorial session and a separate one-hour laboratory session. These sessions were not chronological, so a student could have the tutorial before the laboratory or vice-versa. Tutorials focused on theoretical concepts and problem solving using a calculator for numerical calculations, whereas in the laboratory session data was analysed using Excel and the results were interpreted. For example, in tutorials a data-based question would be asked with summary statistics provided in order to reduce calculations. The same question would be asked in laboratory sessions where students would analyse the original data using Excel and interpret and report the results. This meant that the principles underlying the question had to be covered twice, once in the tutorial and then again in the laboratory. The laboratory session was held in a large computer laboratory with 128 computers and two staff. An attendance mark was given for each tutorial to encourage attendance but otherwise there were no other assessments in the tutorials. There were no marks for laboratory attendance but there were two Excel-based laboratory tests based on this work.

### 2.2 Issues with the Teaching mode

There were several issues with student participation and performance, particularly in laboratory and tutorial classes.

1. Lecture attendance declined after a few weeks to about a third of the class by mid-semester.

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2. Laboratory sessions were poorly attended. Student feedback indicated that this was mainly due to inadequate staffing.
3. Students attended the tutorials classes simply to get the marks. There were reports from tutors that participation in the tutorials was low and students did not attempt the questions beforehand.
4. Student engagement was low and many students did not attempt semester assessments. Several students who failed by only a few marks would have passed if they had attempted a few more of the semester assessments, and in particular if they had attended a few more tutorials.
5. A lot of synergy existed between tutorial and laboratory questions, but this was lost since these teaching activities were separated.

## 3. Changes to the Teaching Activities

The changes in teaching regime were motivated in order to address the issues raised in the last section and to improve student engagement and participation as well as enhance understanding of course material. More specifically, the following were the aims of the changes.

1. To improve student participation in teaching and learning activities.
2. To improve student engagement and improve participation in assessment tasks.
3. To increase student attendance in tutorial and laboratory classes.
4. To provide more learning opportunities in laboratory classes.

The changes were undertaken over four semesters as ideas evolved. Student feedback was also used in formulating the teaching and learning activities. The details of the changes are given in the following subsections.

**Table 1.** A summary of teaching activities for business statistics units in surveyed Australian universities. Table entries are the number of institutions with the corresponding teaching regime of lectures, tutorials and laboratories (hours per week).

| | | Lectures | | | |
|---|---|---|---|---|---|
| Tutorials | Laboratory | 2 | 3 | 4 | Totals |
| 0 | 0 | 6 | 1 | 0 | 7 |
| 1 | 0 | 6 | 5 | 0 | 11 |
| | 1 | 1 | 0 | 0 | 1 |
| 2 | 0 | 3 | 0 | 0 | 3 |
| Tut/Lab | 1 | 0 | 2 | 1 | 3 |
| | Totals | 16 | 8 | 1 | 25 |

### 3.1 Semester 2, 2011

The first change was to merge the tutorial and laboratory classes into a combined two-hour session with one tutor. The class sizes were capped at a maximum of 22, requiring around 35 sessions for a total of 745 students. This session included tutorial type problems as well as Excel-based data analysis. The first issue was a logistic one. These sessions now had to run in smaller computer laboratories with facilities to run a tutorial type class as well. Finding a sufficient number of such venues that were available posed a significant challenge, but this was resolved with help from support staff. The second issue was merging the existing tutorial and laboratory material in a seamless manner so that the combined session could flow smoothly, and tutors could switch from a tutorial type activity to a laboratory activity without disruption. For example, questions in tutorials and laboratories based on the same data had to be re-written to remove

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duplication while at the same time including aspects of both questions. This required around a month of work with the help of an assistant. A final challenge was finding sufficient staff of appropriate quality. We opted to complement our tutors with senior undergraduate students for two reasons — they were younger and would be able to relate to the class more easily in such a setting, and they were more cost-effective. The second reason was important since the administration had to be convinced that the changes would not cost more.

Another change introduced was that in each tutorial-laboratory class, students were required to submit the solution to one question to their tutor for assessment, worth 1% each. Some of the submitted work involved laboratory work, where the student would show a computer output to the tutor on the screen. Often the tutor could mark assessed work in the session itself, so the marking task was not onerous.

An addition to the teaching activities was a one-hour problem solving session per week, where the facilitator would solve interactively with the class some extra examination style problems. The rationale for this was that more time was spent in lectures re-enforcing concepts, but this left less time for examples. Thus in total there were now six contact hours per week compared to five previously, the extra hour being the problem solving session.

All unit material—lecture notes, tutorial problems and solutions, problem solving questions and solutions—were placed online. However, it was a conscious decision that lectures were not recorded.

A few differences were immediately observed during the semester.

1. Records showed an immediate increase in attendance at tutorial-laboratory classes compared with the previous semesters. High attendance levels were maintained throughout the semester.
2. Tutors with previous experience reported that students seemed more engaged and focused. This may be because there was an assessment at the end of the class.
3. The level of assistance with laboratory type work was markedly increased as there was now one tutor per a maximum of twenty two students in a dedicated facility, as compared with two staff for 128 students. This may also be a reason for the higher attendance.
4. Tutors with previous experience reported that exploiting the synergies between laboratory and tutorial type activities allowed the connections between the theory and data analysis to be made more easily. They also reported that the classes were easier to conduct.

A final change related to assessments. There were two short tests (worth 5% each) and a mid-semester examination (worth 15%) during the semester. The material tested in these was then not re-tested directly in the final examination, but knowledge of this was still required since the later material depended on this earlier work. This meant that the final examination was only on the inference and modelling sections of the course, from one-sample tests onwards.

### 3.2 Semester 1, 2012

Following the changes made in the previous semester, an online survey of the class was conducted at the end of the semester to determine the student perception of the changes. In particular, students were asked to indicate if they had taken this unit previously— their feedback and comments received special attention as they had experienced both forms of delivery. A total of 400 responses were received, of which thirty were from

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repeat students (from a total of sixty repeat students). Besides the online survey, informal feedback was obtained in conversations with fifty students selected at random from the class. Overall the response was very positive and encouraged us to take the changes further, relying on student and tutor feedback for ideas. Based on this feedback two changes were made.

The first aspect related to lecture recordings. Lectures were not recorded by choice the previous semester. This semester lectures were recorded and made available to the class. There are many arguments for and against recording lectures [11, 12, 13]. Bell et al. [13] consider various advantages of lecture recording, including flexibility for students that results in a broader student base, timetable clashes, venue capacity, availability of multiple viewing by students, and freedom of place and time for viewing the lecture. A further advantage not mentioned in literature is that the lecturer can also view the recording with the aim of evaluating and improving his/her performance. A major (perceived) disadvantage is that lecture attendance is much lower when lectures are recorded. While several reasons have been forwarded for this decline [3, 4, 5], a major reason is the availability of recorded lectures [3]. (There is evidence based on my as yet unpublished research that recorded lectures are only of benefit to students who attend lectures.) I also experienced a reduction in lecture attendance compared with the previous semester.

A second aspect was that most business units provided a unit reader containing all the unit material. In the previous semester lectures were based on partially blank slides that were made available online prior to the lecture, and the lecturer would fill these in during the lecture. Later, the full slides were made available online. So it was a simple matter to organise the blank slides into a unit reader that was made available for purchase in the bookshop.

### 3.3 Semester 2, 2012

The changes in the previous semesters had improved student participation, but reduced lecture attendance. My focus for this semester was to increase lecture attendance. I decided to trial some ideas by Professor Eric Mazur on peer instruction [14, 15, 16]. As such, I adopted the following.

1. I provided full lecture notes as a unit reader. Students were required to read the lecture notes for each lecture before the lecture.
2. I required the class to complete a concept-based quiz before the lecture based on their readings. The quizzes over each week were awarded a mark of 1% as an incentive.
3. Lecture material could now be covered faster as there was very little writing. The lecturer would cover the more tricky aspects of the material and concentrate on stressing important points. In addition, based on the online quiz results, student misconceptions could be identified and addressed.
4. A mobile-phone based in-class response system (provided by a publisher) was used to test and reinforce student understanding of concepts. Students responded to quiz questions during the lecture at appropriate times. They were then allowed time to discuss their response in groups (peer-instruction) before re-submitting their answer.
5. All assessments were made *open book*, and students could take with them hard copies of any material, including past assessments and solutions.

As a result of these changes, lecture attendances were higher than the previous semester, with a decline only in the last three weeks. Student participation in increased and there

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was clearly an interest and eagerness in attending lectures. The class was now an active participant rather than a mostly passive audience. Further, despite my early doubts, students quickly returned their attention to the lecture once they had responded to the mobile phone-based quiz questions.

### 3.4 Semester 1, 2013

At the end of semester 2 2012 another web-based survey of the class was taken. A total of 300 responses were received. There were three main findings from the survey. Firstly, students overwhelmingly (82%) preferred the open book assessments. This is similar to findings by other researchers [17, 18, 19 20]. Gharib et al. [19] report that students had less anxiety during open book examinations.

   Secondly, mobile phone based in-class quizzes received a very favourable rating by students. Similar to findings by Dunn et al. [21], the students preferred the anonymous response afforded by such a system. They also appreciated the chance to discuss problems and concepts with fellow students and to consolidate the lecture material immediately.

   Finally, daily online quizzes were not very popular and were considered too onerous. Consequently, for this semester the only change was having weekly quizzes based on the previous week's lectures.

### 4. Statistical Analyses of Student Performance

All data analysis was performed using the R statistical software [22]. Data over the four semesters were analysed to determine any differences in student performance. The final marks and the examination marks were used as responses. A summary of the marks is presented in Table 2. Boxplots of the marks are presented in Figure 1. Semester 1 2012 has the highest median final mark, while semester 2 2012 has the lowest median final mark. The highest mean exam mark was obtained in semester 2 2013, while the lowest mean examination mark was in semester 2 2011. It is interesting to note that the standard deviations of the marks were lowest for semester 1 2013.

**Table 2.** A summary of student performance over the four semesters.

|  |  | Class Size | Median | Mean (SE) | SD |
|---|---|---|---|---|---|
| Final Mark (%) | Sem 2 2011 | 745 | 63 | 62.9 (0.65) | 17.7 |
|  | Sem 1 2012 | 342 | 65 | 62.8 (0.98) | 18.2 |
|  | Sem 2 2012 | 580 | 62 | 60.7 (0.74) | 17.9 |
|  | Sem 1 2013 | 580 | 63 | 62.5 (0.74) | 14.9 |
| Exam Mark (%) | Sem 2 2011 | 745 | 45 | 45.3 (0.85) | 23.1 |
|  | Sem 1 2012 | 342 | 60 | 55.5 (1.26) | 23.3 |
|  | Sem 2 2012 | 580 | 56 | 53.6 (0.82) | 19.7 |
|  | Sem 1 2013 | 580 | 63 | 61.1 (0.74) | 16.6 |

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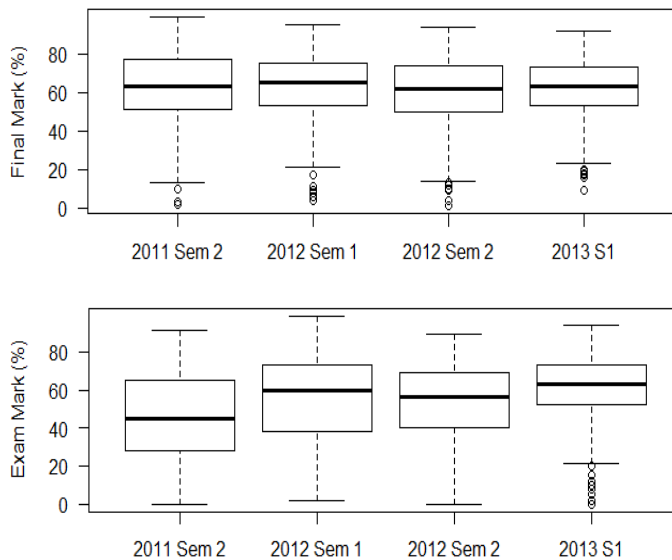
**Figure 1.** Boxplots of the overall marks and exam marks.

Statistical analyses were conducted as below.

1. A one-way Anova [23] of the final marks revealed no significant difference in the mean final marks over the four semesters (p-value = 0.109).

2. A one-way Anova of the exam marks showed that the mean exam marks for the four semesters were not all equal (p-value « 0.001). A multiple comparison using a Bonferroni correction revealed that all the means were different from each other. Thus semester 2 2011 has the lowest mean, while semester 1 2013 has the highest mean exam mark.

3. A Kruskal-Wallis test [24] revealed that there were no differences in the medians of the final marks (p-value = 0.20). For the exam marks, a similar analysis revealed that the median exams marks were significantly different. Further pairwise Kruskal-Wallis tests showed that the median exam mark was lowest for the semester 2 2011 and highest for semester 1 2013, with no significant difference between the other two.

## 5. Discussion and Conclusions

In this paper we described an evolution of teaching over four semesters. There is no difference in the median or mean final marks over the four semesters under consideration. However, traditional teaching methods (semester 2 2011) gave the lowest mean and median exam marks. The higher mean and median exam mark in semester 1 2012 compared to the previous semester may be the availability of a unit reader. The most plausible explanation for this is that the unit reader provided a more organised information source for the students, and this facilitated learning and revision. In the next semester open book assessments were introduced. While there was no significant change in the median exam mark, the mean exam mark was significantly lower. A reason for this may be that the open book assessment style was different to the closed book one. Indeed, the open book exam was not the traditional type, but was more concept-based and required more interpretation and synthesis of ideas than in previous exams. In the final semester the mean exam mark was significantly higher than in previous semesters. This is largely since the students had seen the open book exam from

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the previous semester and were better prepared for this style of exam. Other researchers have reported higher mean marks for open book examinations [18, 19], but Loi and Teo [20] found no difference in student achievement. Interestingly, the pass rates in the four semesters were respectively 79%, 82%, 74% and 81%. The open book assessment regime seems to have advantaged only the better students, leading to a higher mean mark but not a higher pass rate. There is evidence of this from Figure 1. In semester 1 2013, the examination marks were higher on average, but the long lower tail indicates that the performance of the lower end of the class had not improved. This is also supported by the fact that the median marks were not significantly different, so the relative positions of the upper half and lower half of the classes was maintained.

My earlier trepidation that a mobile phone-based in-class response system may be distracting was unfounded. Students had no difficulty returning attention to the class after responding to the quiz questions. Similar experiences have been reported by other researchers. Dunn et al. [21] reported that students were not distracted by using mobile phones, nor were they distracted by other students using mobile phones. The interactive nature of the lecture was far more engaging for the students. In addition, students felt far more comfortable in responding in an anonymous way and the immediate feedback for the lecturer and students was very valuable for both. Peer instruction was also very useful and greatly reduced my out-of-class consultation hours. However, Mazur's [15] idea of having students read lecture notes beforehand and attempt an online quiz per lecture was not very successful. The reason for this I believe is that my students were not as motivated as those of Professor Eric Mazur.

Recorded lectures did reduce lecture attendance. Online student surveys at the end of semester 2 2012 and semester 1 2013 revealed that a major reason for missing lectures was that it was recorded (23% and 25.8% respectively in each semester).

Changing technology in the classroom has made it imperative that teaching methods also change. It is common for universities to put all learning materials online, including recorded lectures. In such an environment missing lectures becomes more attractive. Research and experience shows however, that attending classes is important for engagement with course material and student achievement [7, 8, 9, 10].

Traditional methods of assessment have also been challenged. An open book assessment scheme for a statistics unit is not the norm and would not be considered by many. In this case having open book assessments did not seem to inflate the overall marks on average. Research shows that students overwhelmingly prefer open book assessments or cheat sheets [17, 18, 19, 20].

Students do not engage in the way that traditional teaching methods assume. Learning styles have changed in leaps and bounds over the last five years. Our teaching methods need to match student learning styles. This is especially crucial in lower level and service teaching, as it is a priori difficult to engage such classes. Our experience in this investigation is that it is possible to engage such classes by using appropriate teaching methods and appropriate use of technology.

**Acknowledgements**

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# Perceptions of feedback in mathematics – results from a preliminary investigation at three Australian universities

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Feedback on learning is recognised as so important that it features on student evaluation of teaching surveys and on Australia's national Course Experience Questionnaire. Ideally, the student responses are then used to improve on practices. However, we argue that this process is flawed in first year mathematics. In this pilot study, we surveyed students enrolled in first year mathematics subjects at three Australian universities about their perceptions of feedback. Students were asked what they considered to be feedback in mathematics and what feedback they had received in their mathematics subject. In this study we compare these answers to the lecturers' views of what types of feedback were provided. We come to the conclusion that students enrolled in first year mathematics subjects perceive feedback very differently to their lecturers. This devalues the usefulness of questions about feedback on end-of-semester surveys on the quality of teaching. We also question whether students may be missing out on accessing feedback that is intentionally provided to improve their learning.

## Introduction

The place of feedback in a student's learning is pivotal. Used well, it helps students to improve their work, feel supported and secure as they grapple with new concepts and material. Feedback contributes to them achieving their potential. Understandably, feedback is an item students comment on both in the Australian Course[1] Experience Questionnaire (CEQ) and in the university student evaluations of teaching. The feedback we give as instructors is often modelled on the type of feedback we received as students and what we perceived as useful. Are these tried and true feedback attempts still relevant and effective to the current generation of students?

The term "feedback" implies that a message is given and that a message is received. The message could be verbal or written, specific or general, but in all cases needs to be received to be of any value. Our study of feedback came about through discussions between the authors at the last Delta conference, who all exhibited various degrees of frustration at a mismatch between the types of feedback they thought they were providing to their students, and the views that their students gave on the types of feedback they had received. It seemed that this was a widespread frustration, at least for mathematicians. We therefore ask the following questions:

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[1] Here "course" means "degree".

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- What do students consider to be feedback in mathematics?
- What feedback have students received in their mathematics subject[2]?

To find answers to these questions, we ran a survey across three Australian universities: A research intensive university, a metropolitan technical university (both located in Melbourne), and a multi-campus university that provides higher education to the greater Western Sydney region. The survey was held in class in first year mathematics subjects. Students were not guided or prompted to answer the questions in a particular direction. The two questions asked that were relevant to the research questions above were: "What do you consider to be feedback?" and "What, if any, feedback have you received so far in [subject name]?"

In this paper, we discuss the students' responses in the context of the feedback the lecturers have provided in the subjects. This study forms a preliminary stage of our research into feedback in mathematics. Experiences from this pilot study will be used to redesign the questionnaire.

This paper is organised as follows. We first provide an overview of the literature on feedback, and in particular on feedback in mathematics learning. We then provide the contexts of the three mathematics subjects that have been surveyed, and detail the methods used in this study. We analyse and discuss the student responses to the survey in light of what feedback the lecturers said was available, to provide answers to the questions we have raised. We conclude with suggestions for future directions.

## Literature review

We will give a brief review of the general role and importance of feedback in higher education before we focus on feedback in tertiary mathematics education.

### *Feedback in tertiary education*

Sadler [1] sees feedback as a key element in formative assessment, and lists three necessary conditions for effective feedback: understanding the required level of performance, comparing your own performance to this required level, and engaging in action to lead to a reduction of this gap. Gibbs and Simpson [2] provide ten "conditions under which assessment supports learning", seven of which relate to feedback [3], with emphasis on timing, content/quality, and student engagement. This highlights the importance of feedback as part of the assessment process.

Walker [4, p.68] writes that feedback could be information about the gap between performance and a reference level, or it could be information about a gap that can be used by the student to alter that gap. However, information that there is a gap could be considered to be a form of feedback. Something as simple as "good work" which one would usually take to mean that the required standard has been reached could also be considered as feedback. Walker [4, p.69] notes that comments which praise students' work might be appreciated and so be important, yet Hattie and Timperley [5, p.96] claim that praise is ineffective. They see feedback as "information provided by an agent (e.g., teacher, peer, book, parent, self, experience) regarding aspects of one's performance or understanding." [5, p.81].

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[2] For simplicity, in this paper we will refer to individual units, courses, subjects, such as "Mathematics 1", as **subjects**. We will refer to programs, courses, degrees, such as "Bachelor of Engineering", as **degrees**.

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Walker [4, p.73] found that 67% of comments on assignments were useful to students for subsequent assignments. When asked what students wanted,

> Two themes emerged strongly. One was that they wished to be told what they had got wrong, and why, and how to do better. … The other was that they would appreciate being given things to work on or watch out for in future assignments, or just receiving general suggestions for their future assignments.

Weaver [6, p.379] writes "Focus has been primarily on content analysis of feedback, and providing advice on writing effective feedback". This is not where our research lies; it lies in a less researched area, student perceptions, as Weaver continues: "the topic of feedback to students is an under-researched area, and there has been little empirical research published which focuses on student perceptions." [6, p.379]. Restricting to technical subjects, such as mathematics, where the assessments are not essays, one finds very little research. Some universities ask about feedback in student evaluation of teaching surveys, however, if staff do not understand what students perceive to be feedback then such survey questions are of little use.

Students' perceptions of the feedback they receive are important. James et al. [7], on the first year experience, write "Feedback continues to be an issue. One-third of students do not believe they receive helpful feedback on their progress. Only 26 per cent of first year students believe staff take an interest in their progress." Ramsden [8, p.107] comments that on the CEQ, the statement "Teaching staff here normally give helpful feedback on how you are going" finds student agreement on the "good courses[3]", and disagreement on the "bad courses[3]". He claims that "this item most clearly differentiated the best and worst courses[3]".

In response to the statements from Ramsden above, Gibbs and Simpson [2, p.10] comment that "it means that whether or not they give helpful feedback makes more difference than anything else they do." Hence those responsible for first year teaching should care about students' perceptions of feedback. For those who focus primarily on the CEQ results, and in many places the student experience is considered to be of prime importance, feedback could therefore be less about a gap in knowledge and more about student happiness. The issue of feedback is not straightforward.

### *Feedback in tertiary mathematics education*

The following quote from a UK study on effective feedback in mathematics [3] shows the length to which some staff will go to improve their feedback scores:

> We don't mark work anymore; we give feedback. We don't talk to students; we give feedback. We don't give out model solutions; we give them feedback sheets. I don't have office hours; I have feedback hours. Basically it's a big rebranding exercise.

Such a "rebranding exercise" may improve awareness among students of what the university wants them to think feedback is. It does not, however, improve the quality of feedback provided or the effectiveness of that feedback.

On the other hand, James et al. [7] find that "overall, the mature-age students aged 25 years and over emerge as a highly satisfied group. They express strong satisfaction with their courses and strongly believe they are receiving helpful feedback from their teachers." We

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[3] We interpret these as *subjects*

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argue that this is in contrast to first year mathematics subjects where the majority of students transitioned straight from secondary school. It appears that first year students may not have the maturity to use feedback to benefit their learning, to identify where they should be, where they are, and how to get to where they should be (rephrasing Sadler [1]). These students have not yet developed skills (and some may never do so) to self-regulate their own performance, they don't actively engage in processing learning resources such as feedback [9], and may therefore not become effective learners.

Robinson's study [3] is motivated by similar reasons to ours, that is, that students in the UK also score questions relating to feedback poorly on the National Student Survey. To identify what actually constitutes effective feedback in mathematics, Robinson compares current typical practices from a number of UK mathematics departments, including innovations in the type and provision of feedback. He finds that there are two aspects that much mathematical feedback focuses on, namely "providing exemplars, typically in the form of model solutions, and identifying student errors". At the same time, he acknowledges that students may also need different types of feedback, and in particular feedback that prompts them to take action. In fact, Robinson establishes that "effective feedback needs to be timely, good quality, and crucially [...] it should provoke some action from students to promote further learning." Robinson asked questions such as what students considered as "feedback", to what extent they used the feedback, or found it helpful, and what students actually do with the feedback they receive. For the interested reader, Robinson combines examples of the role of feedback from Gibbs and Simpson [2] with the principles of good feedback practice by Nicol and Macfarlane-Dick [9] to a list of ten possible goals of feedback [3].

The purpose of this paper is to compare student and staff views of feedback provided in three mathematics subjects. The intention is not to determine whether feedback in the subjects under consideration is effective in improving learning. Indeed, the issue of feedback can be very complicated and problematic; for example, Lizzio and Wilson [10, p.263] state that "in over one-third of cases feedback interventions were found to actually decrease performance."

## Contexts and methods

We first outline the contexts of the three subjects surveyed in this study, and then detail the methods used. At each of the three participating institutions, the particular lecture stream of the subject that was surveyed was lectured by one of the authors. For each, we present the type of subject that was surveyed including student background and size of enrolment.

### *The University of Melbourne*

At the University of Melbourne (UM), the survey was conducted in Calculus 1. The students in this subject are primarily first year students in their first semester of university study. The prerequisite for entry to the subject is the subject Maths Methods 3/4 (a calculus based intermediate level course taken in the final year of secondary school). The enrolment in the subject was 777, with 509 students from the Faculty of Science (including engineering pathways), 128 students from the Faculty of Environments (also including some engineering pathways), 104 students from the Faculty of Commerce, with the remaining 36 students from the Faculties of Arts, Music and Biomedicine. The lecture stream surveyed had approximately 300 students enrolled.

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### Swinburne University of Technology

At Swinburne University of Technology (SUT) the survey was conducted in Engineering Mathematics 1. As with the UM subject Calculus 1, the students enrolled in this subject were primarily first year students in their first semester of study. The prerequisite for the subject is also Maths Methods 3/4. The enrolment was approximately 500 students. The majority of students enrolled were engineering students with some science and IT students. There were approximately 70 students enrolled in the lecture stream that was surveyed.

### University of Western Sydney

At the University of Western Sydney (UWS) students in the subject Fundamentals of Mathematics were surveyed. The enrolment of the subject was 117 and there was only one lecture stream. The subject is a basic first year, service mathematics subject. The vast majority of students are studying industrial design (90%); for these students the subject is compulsory. Students in this subject might not have completed any mathematics subjects in their final year of secondary school.

### Methods

The survey was first offered at UWS. Following analysis of the responses in which it was found that few students gave meaningful answers to more specific questions on feedback, these questions were removed and the shortened survey was then run at UM and SUT. The paper-based surveys consisted of open ended questions and were handed out in lectures in the three first year mathematics subjects listed above: Calculus 1, Engineering Mathematics 1, and Fundamentals of Mathematics. We received 58 responses from the 300 UM students, 39 from the 70 SUT students, and 53 responses from the 117 UWS students.

A thematic analysis of the data was undertaken with at least two of the authors reading all answers to the two questions from each university. This analysis focused on answering the research questions:

- What do students consider to be feedback in mathematics?
- What feedback have students received in their mathematics subject?

We interpret the second question in two ways: What students think they have received, and what lecturers have provided. We used qualitative research methods to investigate students' perceptions of feedback. In the results below, we take a three step approach to answer the two research questions.

### Results

We first list the types of feedback we as teaching staff thought we had provided to students at the three universities. In step two, we show and discuss the themes we extracted from the student answers to the open-ended questions: "What do you consider to be feedback", and "what, if any, feedback have you received so far in [*subject name*]". At least two of the authors read the responses from each university. All three authors discussed possible themes, searching for commonality across universities. The themes for SUT are necessarily different to those for UM and UWS, since the assignments in the SUT subject were marked electronically without individual marker feedback. There is some overlap in the themes, for example, "assessments marked" and "ways to improve"; this is a result of the open ended questions. Note that some themes extracted at individual universities are described in the text rather than being visualised in diagrams. In step three, we compare the two perspectives and discuss the results from the lecturer perspective. We report on this separately for the three universities.

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### The University of Melbourne

#### 4.1.1   Lecturer view of feedback provided

During the semester a variety of feedback processes were employed:

- Ten weekly assignments were marked and returned to students with minimal comments given on each. Fully worked solutions were made available.
- During weekly tutorials students worked in groups interacting with each other and with tutors. Fully worked solutions for all tutorial questions were given to students at the end of each class.
- The lectures were interactive, providing opportunities for students to engage with the material and do a variety of problems themselves during each lecture. The lecturer walked around the lecture theatre fielding questions.
- Students had the opportunity to question the lecturer for five minutes or so at the end of most lectures.
- Three hours of consultation per week with the lecturer available to students with many additional consultation hours provided by tutors.

Note that the student evaluations of teaching included the question "I received valuable feedback on my progress". For this subject, the score on this question was 4 out of 5, where 3 means neutral, 4 agree, and 5 strongly agree. This was significantly higher than the average on this question across the faculty.

#### 4.1.2   Student view of what feedback is, and what feedback they received

Figure 1 illustrates the themes that emerged from the data, and the number of students who were coded with this response. Fewer than half the students (40%) considered that assignments marked constituted feedback, whilst around 26% considered written comments on assignments as feedback. A larger group of students (31%) claimed that comments on their progress or comments in tutorials is feedback, with 46% of students explicitly commenting that feedback is about ways to improve or constructive criticism.

Many students explicitly stated that useful feedback not only involves highlighting errors or deficiencies in their work, but also needs to contain clear instructions about how to do things correctly. Comments included, *"how you went in them* [assignments, sic]*, explanations of answers/how to do them"* and *"what I did wrong and how to fix it/do it correctly"*. Most students seemed only to consider comments directly related to their work as feedback. So whilst the lectures in this subject were interactive and students asked questions at the end of most lectures, only 7% of students cited this as feedback.

Of those students who answered the survey, 10% interpreted the question "What is feedback" broadly, answering that feedback is *"one of the key things of the effective communication process"*, or simply *"my opinions"*. Only 7% of students said they had received constructive criticism on their work, with 17% stating that they had received comments on their progress in tutorials or elsewhere. Around 15% of students claimed to have received no feedback at all.

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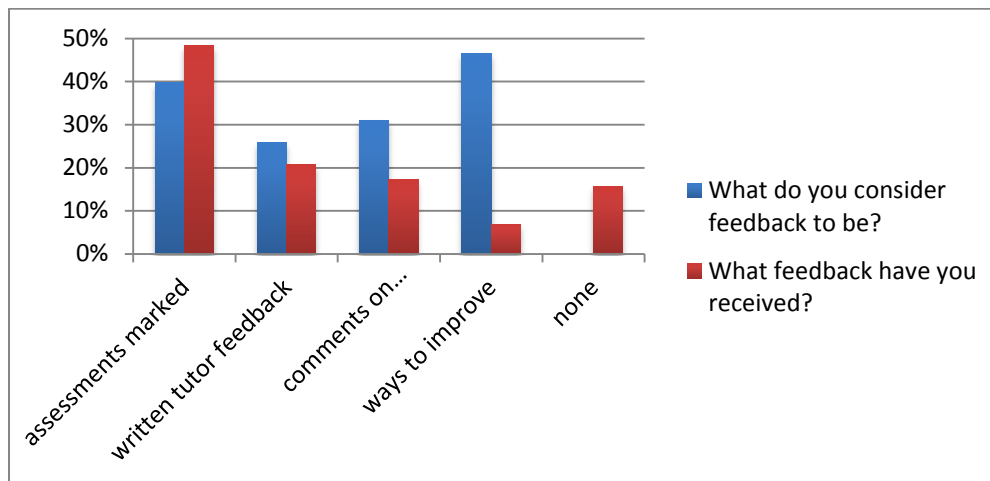
**Figure 1.** Major themes from student responses to the two questions at UM

### 4.1.3   Comparison between lecturer and student views

The lecturer thought that the most valuable feedback was provided via interaction with tutors in the tutorials coupled with the weekly assignment marks and comments. This gave the student an indication of their progress after working through the material in the tutorial and they could incorporate the tutor's feedback.

Given the weekly interaction with tutors, the small number of students who said they had received feedback during tutorials was surprising. This coupled with the even smaller number of students who stated that interaction in lectures was feedback, seems to suggest that comments that are not directly related to a piece of assessment are not considered to be feedback.

It is also interesting that the students who said that "ways to improve" was valuable feedback, seemed to not think that they had received this. For example, fully worked solutions to all assignment questions were provided so students either did not access these or thought these did not show them a way to improve.

### Swinburne University of Technology

### 4.2.1   Lecturer view of feedback provided

The students were provided with the following feedback opportunities: Students
*   were given the opportunity to view their marked diagnostic test (held in the first tutorial, shown in the second tutorial)
*   were given the opportunity to review their two one-hour tests in lectures
*   received immediate notification about which questions were correctly or incorrectly done on the 12 weekly online assignments. The best of three attempts counted towards their grade. Note that no full solutions were made available.
*   received an individual study plan after each assignment on most topics, detailing which topics to revise (generated by the online assessment system)
*   could visit the mathematics help centre for drop in support
*   were able to ask the lecturer questions for 15 minutes following each lecture.  The lecturer was also available for consultation in the mathematics help centre.

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Note that the student evaluation of teaching survey at SUT does not include a question on "feedback".

### 4.2.2   Student view of what feedback is, and what feedback they received

Six of the 39 respondents at SUT thought "feedback" related to student feedback on the quality of teaching – they did not refer to their own learning in their comments. Figure 2 shows themes emerging from the responses of the remaining 33 students.

*Ways to improve* was a recurring theme when students defined feedback, with 45% of the students commenting that this is a role of feedback. However, only 12% reported having received such feedback, with most specifically mentioning the study plan. Nearly half of the students (48%) mentioned that *evaluation of performance* and *marks* are feedback, and just over half of all students said they had received this type of feedback.
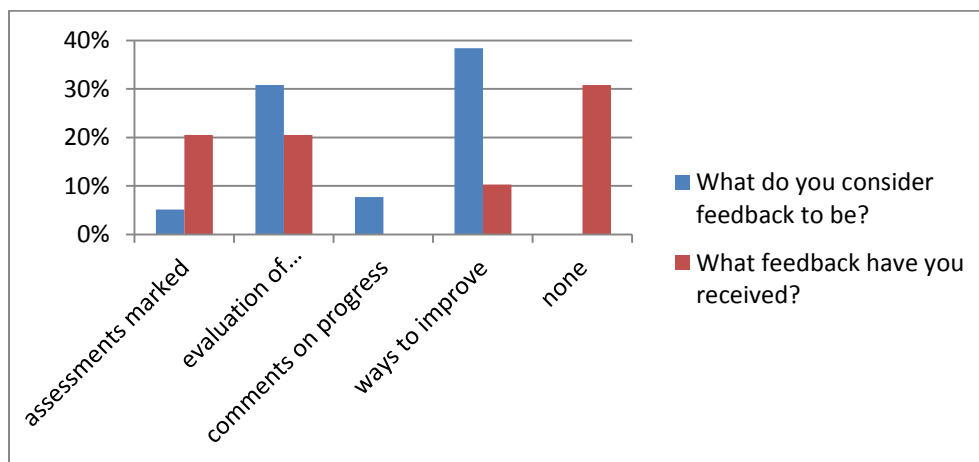


**Figure 2.** Major themes from student responses to the two questions, at SUT

Although 24% of students mentioned a word such as *right, wrong, correct, good, bad, mistake* or *error* in their definition of feedback, none repeated these words in their description of feedback received. Finally, more than a third of students claimed not to have received any feedback.

### 4.2.3   Comparison between lecturer and student views

There appears to be a clear mismatch between student and lecturer views of what feedback is. Students also commented on feedback they had not received, or what they didn't like about the feedback they had received. The policy in this subject is not to return tests to students, but to allow students to view their tests in a lecture. The students appeared to be unhappy that they were not able to spend more time looking through their tests in order to identify where they had gone wrong. Also, students complained that the electronic assignment system was lacking a "human element", as the following student comment shows:

> *Feedback to me is communication about my efforts towards the subject. We get some viewing our corrected test papers. Though with the online assignments the process is indirect and has no human element, so feedback is difficult.*

Students, in particular, wanted to see detailed, worked solutions for the questions on the online assignments. Unfortunately the commercial software used does not provide

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worked solutions. What we can learn from these student responses is that the study plan is seen as very useful, but also that consideration may need to be given to the provision of full worked solutions in the future.

### *The University of Western Sydney*

#### *4.3.1   Lecturer view of feedback provided*

Opportunities for feedback in this subject were:

- Three class tests, marked and returned to students. At the time of taking the survey, the first test had been marked and returned to them.
- Feedback from tutors in weekly tutorials.
- Special support workshops run by mathematics support staff.
- The lecturer was available for consultation each week, and also available for questions during the break in the lecture and after the lecture.
- Online mathematics revision on most topics with immediate marking.

At UWS, the end of semester feedback on teaching survey contains the question "I was able to learn from feedback I received in this unit[4]". The score for this question was 3.4 out of 5, where 5 means "strongly agree" and 3 means "neutral".

#### *4.3.2   Student view of what feedback is, and what feedback they received*

In the UWS survey, 15% of students considered marked assessment as feedback, with 5% mentioning written comments on assignments as feedback, see Figure 3. Receiving verbal comments on their progress or comments in tutorials was listed as feedback by 4% of students, with many more students (21%) explicitly naming "ways to improve" or "constructive criticism".

Of the students surveyed, 8% stated they had received constructive criticism on their work with 9% saying they had received comments on their progress in tutorials or elsewhere. Of the students who answered the question on whether they had received feedback, 36% claimed to have received no feedback. Some students (16%) interpreted the question "What is feedback" broadly or gave their own feedback as a response. For example, "*An evaluation of services offered*".
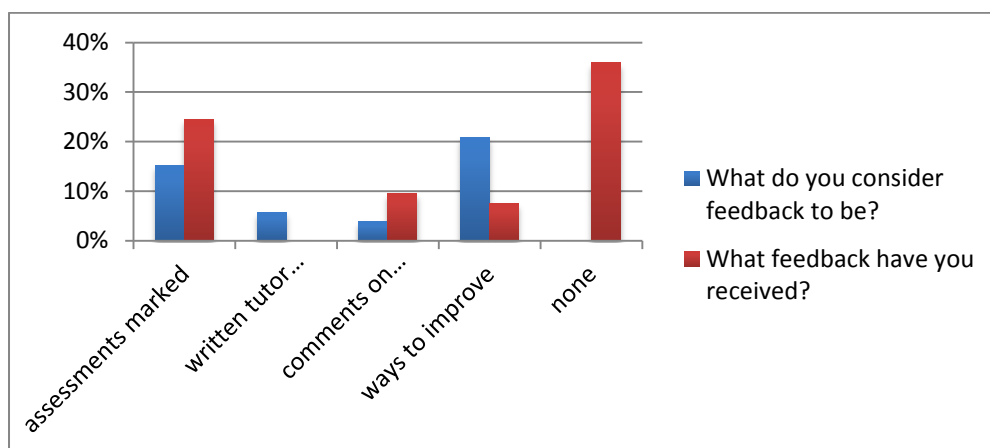


**Figure 3.** Major themes from student responses to the two questions, at UWS

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[4] Unit means subject in this context

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### 4.3.3 Comparison between lecturer and student views

At the time of the survey, Test 1 results were available but many students had not collected their marked tests and so did not know what they had done well and where they needed to improve. Despite this, we note that it is this type of feedback that was recognised most in the survey responses.

The online revision, with its many questions, was not mentioned at all in survey responses. This raises the question if it is used by many students. Also, while there were many opportunities for feedback, most were poorly accessed and the only compulsory activity through which feedback was provided, the test, was the one that was most often mentioned. "Improvement" was the most common theme in what students wanted from feedback, yet most opportunities for improvement were not utilised. This could illustrate the unwillingness of some non-mathematics major students who are required to do mathematics as part of their degree to engage in any non-compulsory activity.

## Discussion and conclusion

Sadler's [1] three conditions for effective feedback, understanding the required level of performance, comparing your own performance to this required level, and engaging in action to lead to a reduction of this gap, were visible in students' responses. Though few mentioned *pass*, many wrote *results* or *marks* from assessments. Some students aim for the 50% pass mark, as one comment clearly illustrated: "*Helpful information to help me pass the unit*". Some students, perhaps many students, expect in feedback information on how their performance compares with the pass mark. Other students aspire to high marks and value feedback as a tool to help them achieve their goals. However, no matter what the goal, feedback appears to be associated with *marks* rather than *learning*.

Improvement was a common thread in responses, and we can only hope that once armed with such information students will act on it and take part in Sadler's third condition: engaging in action to improve. Engagement in such actions was not measured in this study. Future work will be to follow up on the use of feedback by first year mathematics students and to measure how, and if, students actively engage in the various activities that we see as providing feedback. This may also indicate to what level our students are actually self-regulated learners.

When we wrote the survey questions we expected students to have at least some sort of idea what feedback is, and that they would be able to match this with what they had actually received. Through the open ended nature of the survey questions we tried to extract this information from the students. What we did not realise is that with no guidance students struggled with the definition of feedback and with the description of feedback received. This appears to have led to a number of students not interpreting the questions the way we anticipated.

Finally, teaching staff see a much broader range of student-staff and student-resource interactions as providing feedback than students. Evidence for the difference between student and staff perceptions of what constitutes feedback may be found in the following two quotes in response to the question on what feedback had been received: "*No direct feedback; I know from my marks I need to improve though*", and "*I've had no direct feedback. There was a little indirect feedback on my first test paper*". This supports our contention that student responses on feedback on teaching evaluation questionnaires should be questioned. At the same time, we should ask ourselves how we can better align our perceptions of feedback with those of our students. Let's hope this is possible without the rebranding exercise from [3].

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# From innovation to implementation: Multi-institution pedagogical reform in undergraduate mathematics

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The use of student-centered teaching approaches improves student learning and persistence in undergraduate science, engineering and mathematics, but most prior studies have investigated these reforms on small scales and in well-understood conditions. A study of inquiry-based learning (IBL) as applied in undergraduate mathematics at four U.S. research universities demonstrates that such reforms are effective when applied on a multi-course, multi-institution scale that can make a real impact on student outcomes. Here we highlight three key points relevant to research and practice. First, despite variation in the nature and quality of IBL implementation across some 40 courses and 100 course sections studied, positive student outcomes are detected relative to traditionally taught courses. The use of IBL methods benefits women, first-year and initially lower-achieving students in particular ways. Second, the research design appropriate for such studies will necessarily differ from those possible in single courses, requiring general measures that can be used with multiple courses and audiences. Finally, sizable investment in course-specific reforms does not assure that they will remain in place, but professional development of early-career instructors is a powerful byproduct that spreads the method to new courses and institutions and thus helps to broaden faculty use of research-based reforms.

## 1    Introduction

Mounting and persuasive evidence shows that the use of student-centered "active learning" approaches to science and mathematics instruction improves students' learning and persistence [1-4]. Yet most undergraduates do not experience these proven, "high-impact" educational practices, and those most likely to benefit—minority, poor, and first-generation university students—are least likely to do so [5]. Today, the bottleneck in improving undergraduate education in the so-called STEM (science, technology, engineering and mathematics) fields is not lack of evidence that student-centered approaches work, but lack of uptake: adoption by many faculty at all types of institutions, and institutional commitment to sustaining these approaches [6-9]. That is, "The problem in STEM education lies less in not knowing what works and more in getting people to use proven techniques" [6, p. 28].

Taking such a view of STEM higher education, I argue, shifts the task of both practitioners and researchers. This is analogous to the process for approval of new drugs or medical devices in many countries: researchers first focus on the safety and efficacy of the intervention, but once this is satisfactorily demonstrated, they turn to large clinical trials to understand how well treatments work in different sub-populations, identify less common or long-term effects, and compare treatment regimes. So too must educators' focus shift from demonstrating proof of concept in known conditions to

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applying and understanding the outcomes of educational interventions on a large scale, for students of diverse backgrounds and under conditions of real-world variability.

Our team has studied an example of reform on this larger scale of multiple courses and multiple institutions, the application of inquiry-based learning (IBL) to undergraduate mathematics at four U.S. research universities with privately funded "IBL Math Centers." Similar to other inductive teaching approaches [10], IBL approaches in mathematics involve students in working out ill-structured but meaningful challenges. Students construct, analyze and critique mathematical arguments, building toward major concepts through a carefully designed sequence of problems rather than a textbook. Thus students' ideas and explanations define and drive progress through the curriculum. Class time is used not for introducing ideas, but for working them through, as students present and discuss problem solutions individually or during structured small group work, while instructors guide and monitor this process.

In U.S. mathematics, IBL approaches have developed mainly through collegial sharing and adaptation of the Socratic methods of renowned topologist and teacher R. L. Moore (1882-1974). In Moore's courses, students worked entirely independently, given axioms but no lectures or textbook, and presented their proofs in class; a student who had not solved a proof was to leave class rather than listen to the solution. Modern incarnations are importantly modified to incorporate peer interaction, foster a positive learning environment, and welcome a broader talent pool. Today the name IBL describes a set of approaches that emphasize learning by participating in authentic mathematical practice and that incorporate well-structured collaborative work as well as individual work on meaningful mathematical tasks [11]. These are consistent with but not historically grounded in cognitive science or learning theory.

In 2004, the IBL Math Centers were established to promote IBL methods through course development, dissemination, and professional development of graduate students and postdoctoral scholars. The Centers were located at U.S. research universities (three public, one private) with selective undergraduate enrollment and highly ranked graduate programs in mathematics. Faculty have offered over 40 IBL courses across multiple topics (analysis, number theory, discrete math, cryptology, multivariable calculus, differential equations, foundations of school mathematics) to varied audiences (first-year honors students, upper-division math majors, pre-service primary and secondary teachers). Each university independently selected courses and prepared its instructors. The resulting suite of courses shared a general philosophy and drew from a common set of teaching approaches, but were diverse in structure, content, and implementation. While posing challenges for study design, these highly variable conditions are typical of any real-world case of educational reform.

## 2   Study Methods

Our study examined student outcomes of the IBL courses, their variation among student subgroups, and teaching and learning processes in the courses. Because neither students nor instructors could be assigned to IBL methods, the study used a quasi-experimental design. Given the large range of courses and audiences, as well as variability in how faculty implemented IBL, we applied general measures, rather than relying on course-specific learning assessments, to examine student outcomes. These measures included:
- pre/post surveys of attitudes, beliefs and approaches to learning mathematics
- post-course surveys of learning gains, via the SALG-M, based on the Student Assessment of their Learning Gains (salgsite.org) [12]

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- course grades and course completions subsequent to the IBL or comparative course, from academic records
- mathematics content tests applied to small subsets of specific courses
- interviews of students, faculty and graduate teaching assistants about their learning and teaching experiences, outcomes and processes
- classroom observations of the use of class time and the learning environment.

Each of these methods is described in detail in [13]. In all, the data include some 300 hours of classroom observation, 1100 surveys, 220 tests, 3200 student transcripts, and 110 interviews, gathered from over 100 course sections at four campuses over two academic years. Comparative data for "non-IBL" sections of the same course were available only for "math-track" courses, those for students seeking a mathematics degree or a science/engineering degree with high math requirements. Pre-measures were applied to control for differences in IBL and non-IBL student samples. Data from IBL math courses targeted to pre-service schoolteachers were analysed separately due to the differing mathematics backgrounds, goals and attitudes of these students [14].

## 3    What is IBL?  Characterizing the Educational Intervention

To argue that the Centers' work is a multi-institution "reform," we must establish that the instruction was both student-centered and distinct in IBL vs. non-IBL courses. We accepted the Centers' labelling of course sections as IBL or non-IBL, but used classroom observation to describe what actually took place, corroborated by student surveys and interviews and by instructors' descriptions of their practice and rationale.

These data showed that, during IBL classes, students gave and listened to student presentations, worked in small groups, used computers for simulation and modelling, and discussed ideas that arose from these experiences. On average over 60% of IBL class time was spent on such student-centered activities. In contrast, students in non-IBL courses spent 87% of class time listening to their instructors talk. IBL courses also featured greater degrees of student leadership, more variety in classroom activities, and higher frequencies of student question-asking. Observers rated IBL courses higher for a supportive classroom environment, students' intellectual contributions, and in-class feedback to students on their work. Interestingly, ratings of instructor-centered behaviors did not notably differ, suggesting that the main distinction between IBL and non-IBL classrooms lies in instructors' choice of instructional activities rather than in their intent as teachers or interest in student learning.

Among IBL courses, there was substantial variation in the extent and nature of student-centered activity (Figure 1) as well as in instructor skill at executing these methods.  Nonetheless, IBL courses commonly featured:

- learning goals focused on problem-solving and communication
- a curriculum driven by a carefully constructed sequence of problems or proofs, driving toward a small number of "big ideas"
- course pace set by students' progress through this sequence
- class time used for a mix of active and collaborative problem-solving tasks
- instructors who guided student work instead of delivering information.

Outside class, interviews reveal that much of students' substantial work time was spent in preparing for class: solving problems or deriving proofs to present or discuss. Because work was due nearly every class, the workload was steady rather than test-driven. Instructors invested their own time in constructing the "script," or problem sequence, or in adapting scripts shared by other instructors.  Checking homework also

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took on more importance for IBL courses, because students' work improved most rapidly when they got timely feedback. Students made much use of their instructors' office hours outside class, because timely help could be important to making progress.

In addition to detecting clear differences in instructional practice between IBL and non-IBL courses, we also saw patterns in preferred campus styles of IBL, and in courses taught for certain audiences. For example, structured small group work was more common in courses for first-year students and for pre-service teachers, while individual student presentations were more common in advanced courses. These patterns reflect instructors' beliefs, shared in interviews, about the balance of support and independence appropriate for students at different developmental levels.
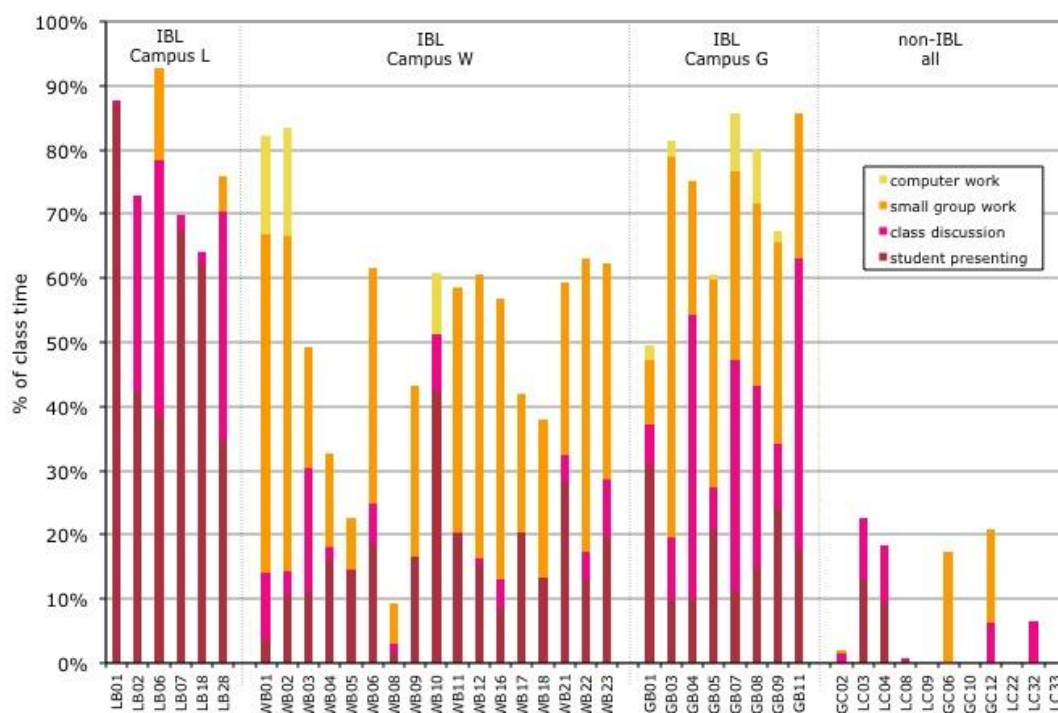


**Figure 1:** Distribution of classroom learning activities for individual courses, categorized by an external observer, as percentage of all class time observed. Eleven non-IBL and 31 IBL course sections on three campuses were observed for multiple sessions totalling (on average) 313 minutes each; course sections are denoted by codes. Four student-centered categories of learning activity are shown; the remaining time was categorized in two instructor-centered categories or as non-instructional business.

## 4   Results:  Student Outcomes

Knowing that there are real and meaningful differences in what students experience in the classroom, we can then examine student outcomes for differences that may be attributed to IBL instructional approaches. Focusing mainly on "math-track" courses, we present results for each type of measurement in the study. We report differences among IBL and non-IBL students, then consider important sub-groups of students. Further details can be found in our comprehensive report [13].

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### 4.1 Learning Gains

On the SALG-M, IBL math-track students reported greater learning gains than their non-IBL peers, on every measure, including cognitive gains in mathematical thinking and understanding; affective gains in confidence, persistence, and positive attitude about math; and collaborative gains in working with others, seeking help, and appreciating different perspectives (Figure 2). Students' comments corroborate these gains, especially the deep and lasting learning that came from working through ideas for themselves. They saw gains in thinking and problem-solving skills as transferable to other courses and to life in general. Changes in learning included personal learning changes such as self-awareness, persistence and independence, and greater appreciation for the benefits of collaborative work. Instructors' observations concurred, but they could better spot gains that reflected students' growth as budding mathematicians, such as communication skills and understanding the nature of mathematics.
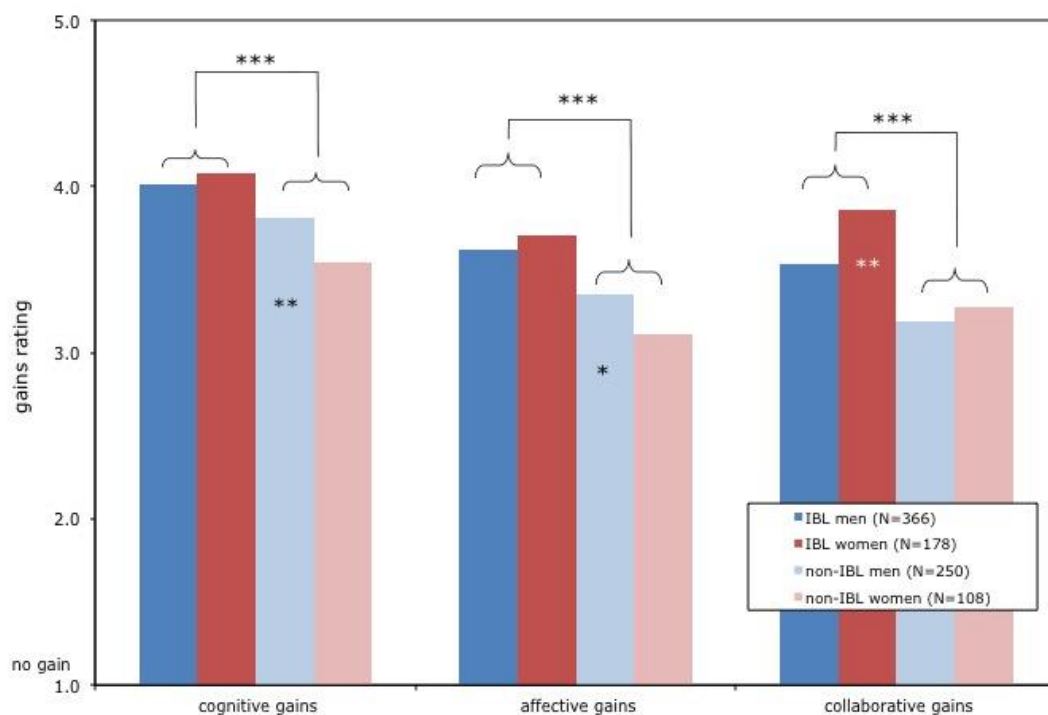


**Figure 2:** Mean learning gains for men and women students from IBL and non-IBL courses. The mean for cognitive gains represents a composite of 4 items (math concepts, math thinking, application, and teaching), and for affective gains a composite of 3 items (confidence, positive attitude, persistence). Collaborative gains represents 1 item (working with others). Asterisks denote statistical significance: \*\*\* $p<0.001$; \*\* $p<0.01$; \* $p<0.05$. In-group comparisons (e.g. men vs. women in IBL courses) are marked on the bars; between-group comparisons (IBL vs. non-IBL) above the bars.

The gains are more dramatic when separated by gender (Figure 2). Women in non-IBL courses reported substantially lower cognitive and affective gains than did their male classmates. But in IBL courses, women's gains were statistically identical to those of men. To more rigorously model this result, we first tested for differences in the IBL and non-IBL groups' initial characteristics using variables such as academic background and prior mathematics experience. Using propensity analysis [15], we

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created a combined variable which, when used as a covariate, held these characteristics constant in our Analysis of Variance (ANOVA). In the same model, we also used three attitudinal covariates derived from factor analysis: Preference for Collaborative Learning, Preference for Independent Learning, and Interest and Motivation to Pursue Math. With these statistical controls in place, the ANOVA results showed significant main effects for IBL status and gender, with IBL students and men scoring higher. We also found a gender by IBL group interaction with no difference between women and men in the IBL group, but men scoring higher in the non-IBL group [16].

Finally, among IBL math-track students, first-year students reported higher gains than did later-stage students in mathematical thinking, persistence, and collaboration. Their gains also surpassed their non-IBL peers in these areas as well as confidence and positive attitude about mathematics. A similar pattern was found when data were differentiated by the number of prior college mathematics courses taken, with less experienced IBL students reporting higher gains than more experienced students—a pattern not seen for non-IBL peers. Finally, this pattern was echoed in interviews, where first-year students reported more gains overall than advanced students, especially cognitive gains, changes in their understanding of the nature of mathematics, and affective gains including confidence and enjoyment.

### 4.2    Attitudes and Beliefs

On the attitudinal survey, among IBL students, most interest and confidence measures remained flat or increased modestly from pre- to post-course, while the confidence of non-IBL students declined (Figure 3). In general, IBL students reported attitudes and beliefs that were more supportive of learning mathematics.
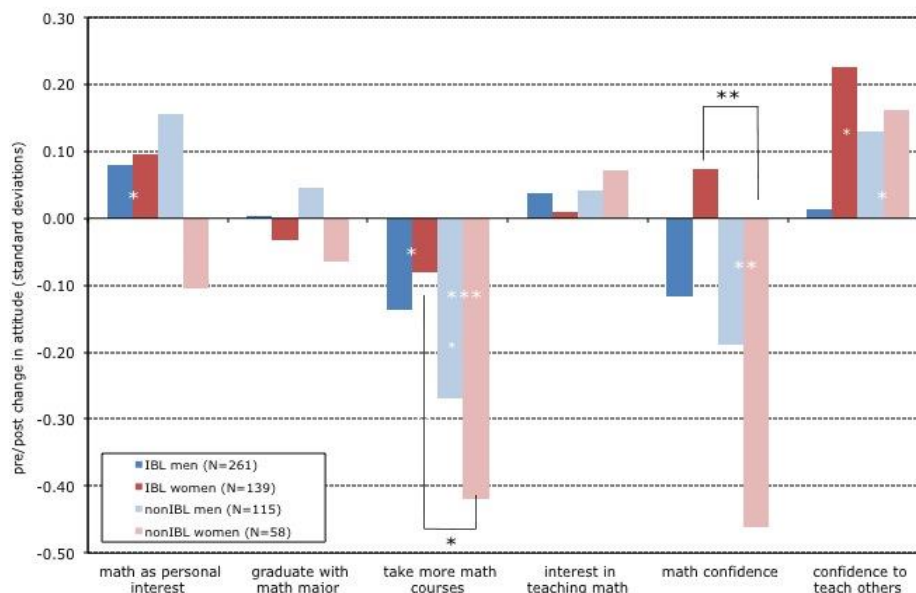


**Figure 3:**  Mean pre/post changes in attitude, for men and women in IBL and non-IBL courses.  Items used a 7-point Likert-style scale; the change in standard deviations of pre-survey response is graphed. Asterisks denote statistical significance:  \*\*\* $p<0.001$; \*\* $p<0.01$; \* $p<0.05$.  The pre/post comparison (for any student sub-group) is marked on the bars; between-group comparisons are bracketed and marked outside the bars.

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Like the learning gains, pre/post-course changes in students' math confidence and interest differed by gender (Figure 3). Both non-IBL men and women reported losses in their confidence and intent to pursue more math, with women's more pronounced. But after IBL courses, women reported increased confidence to do and to teach math, and greater personal interest in math.

### 4.3 Grades and Course-taking Patterns

The academic records study examined students' grades and course-taking patterns [17]. The study did not compare grades in IBL and non-IBL courses directly, where assessment tasks and standards may differ, but in later courses, where students who did and did not experience the intervention all enrolled together. These data show that IBL students earned as good or better math grades than their non-IBL peers, and took as many or more later math courses. This was equally true for men and women.

Some learning outcomes also differed among students by prior achievement level [17]. In particular, IBL students with low prior mathematics grades (GPA<2.5) earned the same or better grades in subsequent courses of several types, typically increasing by 0.3-0.4 grade points (on a 4-point scale) (Figure 4). Such improvement was not seen for their low-performing peers in non-IBL courses, nor for their higher-achieving peers, whose grades generally dropped as courses became more difficult.
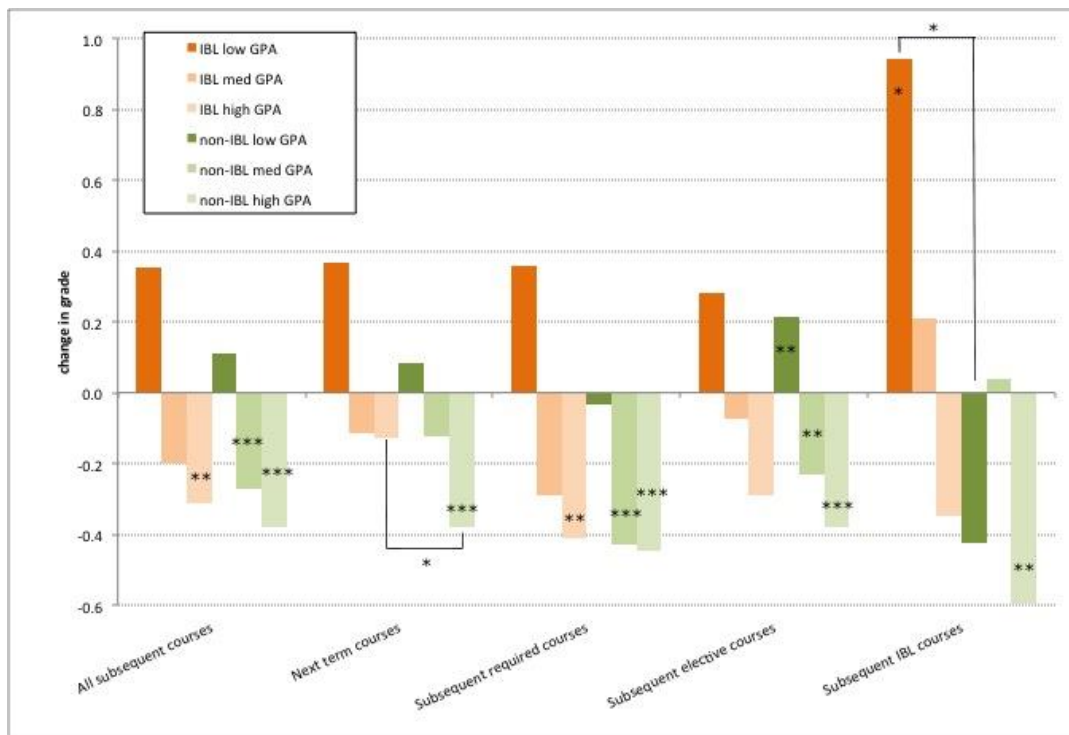


**Figure 4:** Marginal mean change in average mathematics grade for students who took IBL and non-IBL sections of course L1 in 2001-2008, by prior achievement level. Grades subsequent to course L1, for courses of particular types, are compared with prior math grades using a General Linear Model controlling for the number of prior math courses; see [17] for details. Students are divided by prior grade point average (GPA) on a 4-point scale: Low = GPA ≤ 2.5; Medium = 2.5 < GPA ≤ 3.4; High = GPA > 3.4. Asterisks denote statistical significance: * p<0.05; ** p <0.01; *** p<0.001. Significance of pre-to-post change within a student group is computed from paired

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samples test on mean without GLM controls and marked on the bar. Between-group comparisons are based on the GLM analysis and marked by brackets.

## 4.4 Tests

In two cases, we are able to administer an appropriate content assessment to subsets of students. For pre-service teachers, we used a well-validated external test of learning mathematics for teaching (LMT) [18]. Improved scores on this test have been connected to positive effects on teachers' instruction [19]. Changes in students' pre-to-post LMT scores reflected real gains in understanding, suggesting that IBL courses benefited students preparing to teach in ways that will benefit their future work as teachers [14]. Moreover, test score gains were anti-correlated with initial score: that is, students who had the lowest scores on the pre-test improved most on the post-test. This finding mirrors students' self-report of learning gains, where IBL pre-service teachers with lower overall GPAs reported higher gains than their classmates with higher grades.

A voluntary sample of math-track students took a test of their ability to evaluate mathematical arguments and their reasons for judging arguments to be proofs or not [20]. Both groups performed well; there were no general differences in their scores in total or on specific problems. However, there was some evidence that IBL students were more skilled in recognizing valid and invalid arguments and that they applied more expert-like reasoning in making such evaluations. The small sample of academically strong students limits interpretation of these findings. The test was not appropriate for many courses, thus limiting the utility of content-specific assessments in this setting.

## 5    Discussion

Several lines of evidence suggest that students who had a college IBL course grew as mathematicians and as learners in ways that their peers taking non-IBL courses did not. The nature and types of the observed cognitive, affective and social gains were very consistent across multiple data sources, and there is evidence that these gains carry over to benefit students' work in later courses. Thus, despite high variability in the student samples and in the nature of the intervention they experienced, the study detected real differences in student outcomes that align with the literature on the benefits of student-centered approaches, and showed particular benefit for several sub-groups of students.

## 5.1 Levelling the Playing Field for Women

In the U.S., women's share of undergraduate degrees in mathematics has declined in the past two decades, unlike most other STEM fields [21]. Thus we examined differences in student outcomes by gender. On the SALG-M, women in IBL classes reported learning gains as high or higher than their male classmates across all cognitive, affective and social gains areas, but women in non-IBL classes reported statistically much *lower* gains than male classmates. Thus, while both men and women benefited from IBL courses, traditional teaching approaches did specific disservice to women, inhibiting their learning and reducing their confidence.

The grades data show that, while women reported less mastery and lower confidence at the end of a non-IBL course, their later success was in fact equal to their male peers. This coupled with the sharp attitudinal declines of non-IBL women suggests that the problem with non-IBL courses is primarily affective, not an objective gap in performance. In contrast, IBL courses provided a learning environment that was effective for both men and women in both the short and longer term. Moreover, both

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men and women persist slightly better after an IBL course [17]—suggesting that better affective experiences may also enhance students' further pursuit of mathematical study.

Student interviews revealed no gender differences in reported gains, and few gender-based differences in their experiences. That gender issues are largely absent in the interview data in fact explains the survey data well: from students' perspective, IBL classrooms offered equitable environments where all could succeed. Indeed, several elements common in IBL courses have been noted as effective for women, including collaborative work [1], problem-solving and communication [22]. Public sharing and critique of mathematical work may foster self-efficacy and link effort, rather than innate talent, to success—enhancing women's sense of belonging to the discipline [23].

## 5.2 Narrowing the Gap for Lower-Achieving Students

Instructors commonly hypothesized that IBL experiences were most beneficial for students below the top of the class. Thus we examined the data for differences in student outcomes by prior achievement. Two lines of evidence suggest particular benefit for low achievers. First, the LMT test scores for pre-service teachers showed a clear trend that initially low-scoring students improved most on the post-test. Second, analysis of students' grades subsequent to an IBL course showed that, among math-track students, taking an IBL course flattened achievement differences among students. In non-IBL courses, previously low-achieving students gained no ground, but following an IBL course, low achievers' grades were boosted relative both to their own previous performance and to non-IBL peers. No other student groups made such gains.

We surmise that the benefits of IBL to low achievers arise from transferable changes to their problem-solving strategies and study habits. Particularly among students who had not previously developed these skills, this is a powerful effect [24]—yet one without harm to others. Indeed, high achievers were encouraged by an IBL experience to take more mathematics, especially more IBL courses [17]—consistent with instructor observations that strong students found the IBL approach stimulating.

## 5.3 Jump-Starting First-Year Students

In focus groups, first-year students were particularly enthusiastic about how IBL courses had enhanced their learning in other courses. Thus we examined differences in student outcomes for students taking IBL courses earlier or later in their college careers. Data from surveys and interviews were consistent in suggesting that IBL experiences were more powerful for students earlier in their college career. Positive early experiences may be tied to persistence: after a first-year IBL honors course, students took more mathematics courses than did a matched sample of non-IBL peers [17]. An early experience of IBL may contrast strikingly with students' high school work, and changes in students' approaches to learning or studying mathematics [13] may carry forward to influence their choices and success in later courses.

## 5.4 Lessons for Studying Change at Scale

So far we have focused on the study results—but there are larger lessons to be learned. One such lesson is that meaningful outcomes of student-centered learning can be measured on a multi-course, multi-institution scale—but evaluating these outcomes is not simple. The setting is inherently messy and the researcher has little control over implementation decisions. Our quasi-experimental research design emphasized general measures, such as self-report and grades, that apply across multiple courses, institutions, and student audiences. Such data provided sufficient statistical power to disaggregate

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results by student sub-group and to relate student outcomes to demographic and attitudinal covariates; the use of multiple research methods further strengthened interpretation of the results. But such studies are needed to scale up from "proof-of-concept" studies that examine the effectiveness of these instructional approaches in highly controlled but educationally unrealistic conditions.

Second, the study raises questions about whether initiatives like this make any difference, and what they tell us about how to carry out such change. This initiative approaches a scale that can make a substantive difference in student mathematical outcomes and, potentially, their retention in the discipline and workforce. At the four Centers, an estimated 40% of math majors experienced an IBL course during the study period, as did nearly all pre-service K-12 teachers at two sites. Moreover, the study results suggest that, while teachers' skill in implementing IBL does matter, very high fidelity of implementation is not essential to achieving improved student outcomes.

Yet challenges remain for sustaining these efforts. The Centers have made less progress in institutionalizing IBL courses than in changing instructors. Strategies that have worked in some contexts include informing and engaging departmental colleagues and leaders, even those who do not participate in IBL teaching; crafting instructor support through mentoring and team teaching; and using IBL to build a larger community of practice around teaching. Most effective have been efforts to draw in early-career instructors—postdoctoral scholars and graduate students—who come to view IBL as a flexible tool kit for teaching that they can carry along when they move on to new institutions [13]. In future work, we will examine the processes of change that have succeed or failed at the Centers and in the broader IBL movement in mathematics.

## Acknowledgements

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# StatsCasts: supporting student learning of introductory statistics

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With the diversity in the backgrounds of students currently entering universities, many students lack the preparedness to undertake studies that include quantitative content. At the same time, many of these students are required to enrol in an introductory statistics course at some stage during their undergraduate program of study. This, along with an associated prevalence of statistics anxiety, means that many students require additional assistance to progress and stay engaged. A variety of support mechanisms may need to be enacted to assist students to succeed. This paper introduces and discusses the development of StatsCasts: short, focused screencasts on topics students have struggled with in the past, for anywhere, anytime support in learning statistical concepts. An evaluation of the pilot stage of a larger research project is presented, to show how students at one of the three collaborating universities perceive these multi-media support resources, and to what level students access them. Initial findings indicate that most students found the StatsCasts beneficial to their learning and understanding of the relevant concepts.

## 1. Introduction and background

As the need for statistically literate citizens is becoming more widely recognised, greater numbers of university students are required to enrol in statistics courses as a necessary part of their tertiary studies. Moreover, these students are primarily non-specialists from a diverse range of disciplines which include medicine, health sciences, business, psychology, social sciences, education, and engineering, to name just a few. In their review of statistical education, Tishkovskaya and Lancaster acknowledge that "teaching statistical courses is challenging because they serve students with varying backgrounds and abilities, many of whom have had negative experiences with statistics and mathematics" [1,p.2]. They further note that even though there is a critical need for statistically educated citizens, students at all levels lack interest when taking introductory statistics courses.

In Australia many students entering university are required to study statistics in their very first semester at university, yet they often come with very little or insufficient exposure to quantitative (including statistical and mathematical) concepts. This lack of requisite quantitative skills needed to be successful in such courses has been discussed elsewhere [2– 6]. With this in mind, Rylands and Coady [4] recommend that the impact of this diversity in student preparedness to study quantitative content at university needs to be recognised and extra assistance provided.

Taking these circumstances into consideration it is unsurprising that many students experience statistical anxiety. In developing a model for predicting statistics achievement, Onwuegbuzie noted that "anxiety interferes with performance by impeding students' ability to receive, to concentrate on, and to encode statistical terminology, language, formulas, and concepts" making it difficult for them to solve statistical problems [7,p.1033]. Learning statistics can be likened to learning a foreign language, with the fear of statistical language being an important component of statistics anxiety [8,9]. Zeidner [10] found that statistics anxiety correlates with past history of negative experiences and poor performance in mathematics and a low level of mathematics self-efficacy.

In response to this lack of preparedness and prevalence of anxiety surrounding mathematics and statistics, most Australian universities provide students with avenues for upgrading their quantitative skills [11], such as one-on-one appointments, drop-in support, course-dedicated support classes, or university-wide generic skills programs [12]. While 32 of Australia's 39 universities had formal dedicated mathematics or statistics learning support centres in 2007, many students still do not have easy access to such support [11], particularly those who study at a distance. With this in mind, the authors commenced a collaborative project between Swinburne University of Technology (SUT), the University of the Sunshine Coast (USC) and the University of Southern Queensland (USQ) to produce multi-media resources to address this shortfall.

One impetus for this project was the international research collaboration between SUT (Australia), Loughborough University (United Kingdom) and Limerick University (Ireland), leading to the production of the increasingly-successful resource collection MathsCasts [13,14] and the aspiration of doing likewise for students of statistics. MathsCasts are short screencasts of a tutor explaining in handwriting a mathematical concept, or how to solve a mathematical problem. They are produced with the aim to provide anytime support to students outside potential face-to-face support hours, but also to allow students to access online resources when they are off-campus. They are designed to complement existing face-to-face support rather than replace it. Further to this motivation was the perceived need for this type of flexible learning support for our students of statistics. Statistics and mathematics support at SUT is currently provided by a support centre where tutors are available at specified times for one-to-one but also for group support for mathematics and statistics. In contrast there is no dedicated support centre available to students at USC. The situation at USQ is somewhere between these two—a dedicated support centre exists but it provides support across all aspects of academic studies, with statistics support being just one component.

In addition to this diversity in the level of support available, the student cohorts taking statistics courses at the three universities for whom the early StatsCasts were originally developed also have a wide variety of backgrounds and levels of statistics covered in their courses. Engineering students are the focus at SUT; psychology, business, and science students at USQ; and health and science students at USC. Furthermore, the majority of USQ students are enrolled in distance/online mode, while both SUT[1] and USC offer teaching to their cohorts entirely in traditional face-to-face

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[1] SUT has a large number of students studying statistics online via Open Universities Australia and Swinburne Online, however StatsCasts currently focus on engineering students who are taught statistics entirely face-to-face.

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mode. This diversity highlighted a number of challenges facing the research team – which concepts to present in the StatsCasts, how to present the concepts and what terminology to use.

This paper first introduces StatsCasts and describes the collaboration between the three universities leading to the production of these resources. It then reports on the evaluation of the initial phase of this collaborative research project which aims to investigate the strategies required for providing more flexible support to students of introductory statistics with the view to extending this support to other cohorts of students.

## 2.   What are StatsCasts?

StatsCasts are screencasts (narrated recordings of handwritten explanation on a computer screen) that cover a variety of topics from statistics, basic research methodology, and how to use SPSS for statistical analysis. While screencasts may be recorded for a number of purposes (including full lecture content, supplementary support material which may broaden or deepen understanding, or to provide worked examples [15]), StatsCasts are primarily short, focussed video resources developed to explain concepts that may not have been fully understood in class. Initially, based on our many years of teaching introductory statistics, topics chosen for the StatsCasts reflect the concepts that our students have found difficult. In providing explanations of these concepts StatsCasts are segmented into clear steps within a relevant context in order to focus a student's attention, contain appropriate visuals to illustrate a concept to increase engagement with the content, and adopt a conversational style to promote engagement with the presenter [16,17].

It is recognised that difficult and often complex concepts, such as those perceived to be so by novice learners of statistics in introductory courses, may need to be revisited multiple times throughout a course [18]. Given that there is limited time in which to do this within the designated class time of an introductory statistics course, StatsCasts provide an alternative format for content revision. StatsCasts can complement the interactivity, dialogue and availability of immediate feedback that a face-to-face or real-time online "visit" to a mathematics support centre can provide. As such, StatsCasts deliver more flexible learning options to students outside fixed support centre opening hours but do not replace the support given by a tutor or regular class attendance.  In fact, StatsCasts allow students to study statistical concepts from anywhere, on any internet connected device that can play videos, supporting the notion of mobile learning, which "offers educators a means to design learning activities and resources that allow students to individualise their learning" [9, p.220].

Although there is recognition of the benefits of screencasts, some criticism has been levelled at scientific screencasts, including that students are not challenged by bringing up and discussing common misconceptions and that they may be regarded as too passive a medium [20]. While it is acknowledged that screencasts do not allow students to verify their understanding in the traditional sense by obtaining immediate feedback from a tutor, there is a place for screencasts to supplement learning [21].  A study conducted by Loch, Jordan, Lowe and Mestel into the impact on performance in a quiz following the provision of screencasts to review calculus content demonstrated "the very significant effect of screencasts on those questions which are relevant to the screencasts" [21,p.9].  Provision of these additional explanations of concepts regarded as 'troublesome knowledge' may assist students to move forward from 'stuck places' particularly at times when they do not have easy access to immediate feedback from a

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tutor [22]. We therefore argue that StatsCasts have a role to play in supporting statistical learning, and it is this role which is the focus of our research. Furthermore, in response to these reservations, we have specifically designed StatsCasts as short screencasts (the goal is to keep the maximum length near five minutes), maintaining a dynamic and more personal element [21] by adding handwritten annotations to the videos to keep students active in cognitive engagement [23].

## 3. Production of StatsCasts

StatsCasts are produced by tutors and lecturers at the three collaborating universities, with internal funding provided at one of these to pay a tutor to prepare recordings. The StatsCasts specifically discussed in this paper were produced in addition to the lecturer's normal workload to fill the void in flexible, anywhere, anytime support to his students. Topics chosen for inclusion in the StatsCasts reflect those usually covered in most introductory statistics courses – initially those of immediate relevance to his students. While there is common ground on required topics for the StatsCasts across all three universities, content and specific examples vary based on student needs, the focus of the introductory statistics courses being taught and the teaching experiences of those involved.

On the technical side, StatsCasts are produced following agreed specification guidelines to ensure they are compatible with each of the three universities' requirements of open educational resources. These include the format of the video, university branding, copyright/licensing, but also the provision of meta-data. The videos are produced in MP4 format, the current standard for iTunes U videos, which allows them to be played back on most video-enabled devices, including iDevices, as no Flash components are included. Since StatsCasts are developed as open educational resources, a Creative Commons licence (BY, ND, NC) is applied, which means that anyone can download them for free for non-commercial purposes (NC – non-commercial), provided they remain unchanged (ND – no derivatives) and authorship is acknowledged (BY). An agreement was reached between the marketing departments at all three universities to use the combined logo shown in Figure 1, designed by an SUT library staff member.
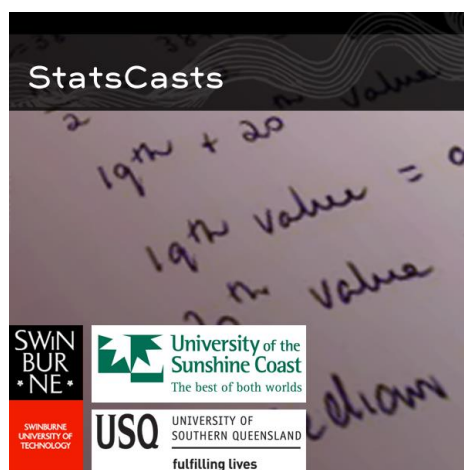


**Figure 1.** The StatsCasts logo

In addition, StatsCasts use similar front and end slides, tailored to the university that has produced the particular video; see Figure 2 for an example of the USC end slide. Since "discoverability" is vital for online resources, each StatsCast is provided with a set of meta-data, which includes not just the name of the narrator and the

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university, but also classification as a StatsCast and the broad area of statistics that the video belongs to, such as inferential or descriptive statistics. The title, short description of content, and keywords related to the specific topic covered are also stored in the meta-data and included in the file when it is downloaded by a viewer.

StatsCasts differ from other online educational resources as they are internally peer-reviewed within the research group for quality control. Each video needs to be approved by at least one other colleague from the other two universities before it may be released outside the producing university. This peer-review process is taken seriously, with some StatsCasts being rejected, and others flagged for editing.
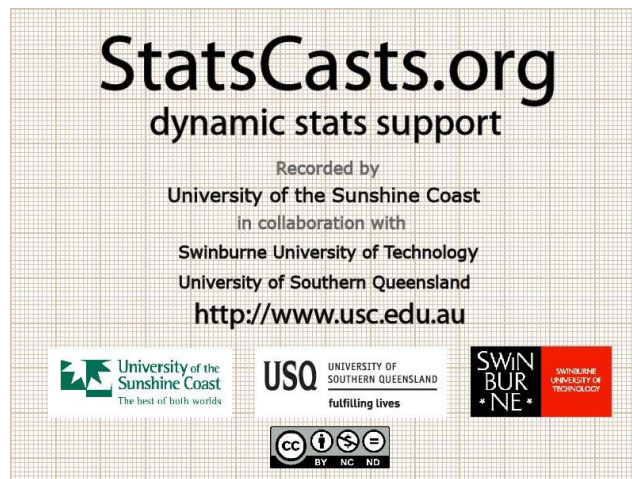


**Figure 2.** The end slide of a University of the Sunshine Coast StatsCast

StatsCasts are hosted by SUT via iTunes U and YouTube, but also on the project website http://statscasts.org. At this stage, only a small number of StatsCasts have been made publicly available, as focus of the project is on production for immediate local use and evaluation. Publication via the web will follow in the next few months.

For the interested reader, we share our workflow from conceptual idea to online StatsCast:

(1) Selection of topic to record – generally one that was of most need to the students we currently teach
(2) Selection of actual content for StatsCast and preparation of script
(3) Recording of StatsCast, new entry in shared Google Drive spread sheet, listing all meta-data
(4) StatsCast is shared with the other two universities via Dropbox for review
(5) StatsCast is reviewed, any suggested edits are made
(6) For StatsCasts that have passed peer-review, credit first and end slides are added, and the meta-data is included in the file
(7) Upload to Dropbox to share with the Swinburne Commons team responsible for iTunes U and YouTube publication. StatsCast is flagged on Google Drive spread sheet as ready to publish
(8) StatsCast is published, and Google Drive spread sheet updated by Commons team to reflect publication.

Following production and local release of the initial USC StatsCasts, the first phase of the research project was to conduct a preliminary evaluation of the introduction of this type of support for first-year students studying statistics.

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## 4. Methodology

In the overarching research project, the following research questions will be addressed:

(1) What are the perceived benefits of accessing StatsCasts, by students and instructors?

(2) How should StatsCasts be designed to maximize effectiveness?

(3) What factors encourage students to use StatsCasts to support their learning?

(4) How are students using StatsCasts to study statistics (e.g., while travelling on mobile device; while studying with other students; to prepare for an assignment or examination)?

In this pilot stage of the research project, only the first question will be discussed in depth. Further examination of this and the remaining three questions will be considered in the continuing investigation beyond this pilot stage.

This study was conducted at the University of the Sunshine Coast (USC), a small Australian regional university (about 8900 enrolments in 2013). The students involved in this study were all enrolled in the course SCI110 *Science Research Methods*, a course that introduces research methodology and statistical concepts. In 2013, 924 students in SCI110 were enrolled, in disciplines such as biomedical science, engineering, environmental health, environmental science, health promotion, nutrition and dietetics, occupational therapy, paramedic science, and sport and exercise science. For all students, SCI110 is a required course in their program, usually in their very first semester at university, so many students are not intrinsically motivated to engage with the course. Thirteen StatsCasts were made available to these students with each screencast being uploaded into the learning management system following presentation of related content in lectures and tutorials (see Table 1 for details on topics, durations of the screencasts, and the number of responses related to viewing each screencast). Students were encouraged to watch the recordings as part of their studies. A short anonymous online survey was conducted asking students to identify which StatsCasts they had watched, how helpful they found the StatsCast (very helpful, a little helpful, a little unhelpful, or very unhelpful), and to give a comment. While it is not possible to distinguish the number of individual students who responded to the survey due to anonymity, the number of views of each screencast by respondents to the survey can be identified in all but seven instances. The responses ($N$=555) across the 13 screencasts were grouped according to how students responded to this question, and a thematic analysis of their comments was performed to gain further understanding of their views.

**Table 1.** Number of responses by StatsCast viewed ($N$= 555)

| StatsCast topic | Duration | Responses | StatsCast topic | Duration | Responses |
|---|---|---|---|---|---|
| Creating Research Questions | 3m 34s | 52 | Confidence Interval for one mean | 4m 48s | 40 |
| Identifying Experimental units / Observational units | 3m 23s | 56 | Sample size for means | 4m 9s | 34 |
| Study Design | 5m 45s | 78 | Sample size for proportions | 3m 26s | 36 |
| Constructing Graphics | 5m 6s | 47 | Paired *t* tests | 5m 0s | 11 |

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| Understanding boxplots | 3m 56s | 56 | Two sample test for means | 10m 24s | 8 |
|---|---|---|---|---|---|
| Tables / Chi-square tests | 5m 19s | 55 | How to select an appropriate test | 4m 58s | 12 |
| Odds / Odds ratio | 4m 47s | 63 | Not identifiable | | 7 |

## 5. Preliminary evaluation

Of the 555 responses, 497 (90%) indicated that the specific StatsCast watched was "very helpful", 48 (9%) indicated it was "a little helpful", and 10 (2%) indicated it was "very unhelpful". There were no responses of "a little unhelpful."

Of the 497 responses that indicated the StatsCasts were "very helpful", 144 did not include a comment. While many comments provided in the remaining responses acknowledged that the StatsCasts were "great" or said "thank you," indications of how the StatsCasts were useful could generally be classified as: (i) useful for revision, (ii) clarifying concepts, (iii) confirming understanding of concepts, (iv) giving the option of watching at their own pace, (v) adding value to visual learners, and (vi) viewing saved time compared with reading (see Table 2 for a sample of typical comments). Students also appreciated that the StatsCasts were clear, concise and to the point. One student who had missed a couple of lectures commented that "the StatsCasts were very helpful in catching up on what I had missed."

**Table 2.** Sample comments on how the StatsCasts contributed for "very helpful" group

| Category | Sample Comments |
|---|---|
| Revision | "The statscasts are so helpful and perfect for refreshing memory, revision and taking note of the important aspects of a lecture." <br> "It was great as revision after covering these topics in the lecture." |
| Clarification | "Very helpful to listen to when you get stuck" <br> "I hadn't understood this until I watched the Statscast" <br> "helped to clarify terminology and give examples" |
| Confirm understanding | "Good to be able to confirm what ive (sic) learnt" <br> "It is good to get little reminders about the content of this course as there is so much to take in." |
| Own pace | "I can pause, print, replay what and when I want." <br> "helps to really understand because you can download them and keep watching as much as you need" |
| Visual learner | "Visually showed what was being done" <br> "Its better for visual learners cause reading about [it] usually doesn't help as much." |
| Time saving | "They were clear and concise. Much more useful than hunting through pages of slides from lecture notes." <br> "saved me hours" |

Forty-eight responses indicated that the StatsCasts were "a little helpful." Of these, 35 (73%) responses referred to issues related to either still being confused, already being proficient with the content, requested more examples, or offered positive remarks (see Table 3 for sample comments). Of the remaining thirteen responses, two referred to technical issues and included that there was a preference for drop-in sessions where questions could be asked; three indicated the screencasts were more than "a little

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helpful" but not as much as "very helpful," but no such category was available, with two of these responses adding that more explanation was needed; eight did not comment.

**Table 3.** Sample comments for "a little helpful" group

| Category | Sample Comments |
|---|---|
| Still confused | "not the statscasts fault.  Still grappling with the concepts. May become very helpful down the track." <br> "still a little confused , more reading and watching may help. penny hasn't dropped yet." |
| Already proficient | "I was already pretty comfortable with this but just wanted to check." <br> "pretty much had this one under control anyway from watching the lecture, but every little bit helps" |
| More examples | "more examples would of (sic) helped more." <br> "I think it would be beneficial to have examples containing relational/comparative RQ's." |
| Satisfied | "utilised as part of my study is helping me understand the subject" <br> "A good straight forward explanation" |

Of the ten responses indicating that the StatsCasts were "very unhelpful" one referred to technical issues with downloading the files and the other nine did not include a comment.  These ten responses came from at most 4 individual students.

The overall perception of the StatsCasts by students was positive, with most students indicating that they were beneficial to their learning and understanding of the relevant concepts. In this initial evaluation it appears that the StatsCasts were mainly used for revising concepts that had already been covered in class. Where the StatsCasts may have not satisfied expectations was in providing more examples or where a student needed to do more work on the topic before full understanding could be achieved.

## 6. Discussion, conclusions and ongoing work

### 6.1. Discussion

While StatsCasts are primarily being produced for students at SUT, USC and USQ, it is believed that they will be of benefit to a wider audience. The focus is on providing supplemental explanations of key concepts covered within introductory statistics courses, and as such cannot be expected to provide a comprehensive view of the content of any introductory statistics course. While some students indicated that they wanted more examples discussed within a StatsCast, others appreciated the succinct explanations given. By their very nature, the StatsCasts were not designed to provide lots of examples on any one topic as this would have been at the expense of keeping them short and concise to reduce cognitive load and keep students engaged. Furthermore, extra examples are generally furnished in lectures and tutorials. StatsCasts are intended to be ancillary to this content.

Early on in the project it was realised that due to the diversity in requirements within the three collaborating universities, there is, at times, a need to produce more than one screencast for each commonly-included topic in introductory statistics. Some courses have a greater emphasis on mathematical aspects than others. For some there is a greater need for procedural explanations with worked examples in addition to the more conceptual discussion, whereas for others the more mathematical aspects are downplayed in favour of a stronger emphasis on conceptual understanding.

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One key aspect to the production of StatsCasts was to keep them short. This may have implications for the selection of topics, including concepts and procedures, which should be included in the StatsCasts collection. Screencasts of topics that do not lend themselves to being broken down into short explanations, while still useful, may need to be identified as being longer and possibly separated from the main collection.

With limited technical resources the development team has concentrated their efforts on applying appropriate pedagogical content knowledge, using correct statistical language, and writing with consistent statistical notation. To this end, careful scripting of each StatsCast prior to recording is implemented to maintain quality. This can be very time consuming, however it is anticipated that the reusability of these resources in providing flexible access to support by students learning introductory statistics will balance this cost.

### 6.2. *Conclusion*

In this paper we discuss the development of StatsCasts including issues addressed by the research team in exploiting this type of support for learning. We present preliminary findings on the perceived benefits of accessing StatsCasts by students. With the diversity in student needs and content focus of introductory statistics courses (particularly in the level of mathematical content), the purpose of each StatsCast needs to be explicitly stated at the outset so that students can obtain maximum benefit from accessing this type of support.

### 6.3. *Ongoing work*

Further investigations will include how to effectively present relevant information on the website about the content of each StatsCast and how the StatsCasts link together to give comprehensive ancillary support for learning introductory statistics. For the next phase of the project students will be invited to participate in a more comprehensive online survey based on a MathsCasts survey held at SUT [24]. In this survey students will be asked how StatsCasts could be improved (e.g., technical quality, content selection, style of explanation) but also how students engage with the StatsCasts. This feedback will then influence the design of the next round of StatsCasts and help find answers to the four research questions we have stated earlier.

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# Introducing Queuing Theory through Simulations

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Queuing theory is usually introduced to students from second year onwards in a university undergraduate programme, as the mathematical principles governing queues can be fairly demanding, making it challenging to introduce any earlier. However, we often see queues and experience queuing in real life. It would therefore be appropriate, relevant and useful to introduce the concept of queuing theory to pre-university students or first-year undergraduates. The approach suggested is through simulation models supported by suitable technology. In doing so, students can understand some basic probability theory and statistical concepts, such as the Poisson process and exponential distribution, and learn how queues may be modelled through simulation, without the need to know all about classical queuing theory. In this paper, we will discuss the role that simulation can play in a classroom to create real world learning experiences for students. To provide a concrete illustration, a set of real data collected in an ATM queue will be used to explain how students can systematically be engaged in a modelling activity involving queues.

**Keywords:** queuing theory; modelling; simulations

## 1. Introduction

Queuing is part of our everyday life. For example, we queue at the checkout counters at supermarkets, for banking services in a bank and to purchase food at fast-food restaurants. This real-life phenomenon, though commonly seen, is not usually studied in pre-university or even first-year undergraduate courses. *Queuing theory* may be incorporated into undergraduate Operations Research or Statistics courses at more senior levels due to its complexity and the demand for mathematical maturity of students. If discussions of queues were done at all at lower levels, it would not usually involve working through the whole process, such as including the collection of real data.

In this paper, we propose that queues be taught to pre-university students or first-year undergraduates, who need not understand all the details of the theory, but can still appreciate a real application of mathematics through simulations. Though some researchers have discussed the use of simulation in teaching mathematics to high school or university students (see for example, Goldsman [1], Reed [2] and Sánchez [3]), they either do not consider modelling or do not present their teaching processes in detail. Our focus is on showcasing how the entire modelling process of the mechanism of queues, from data collection to constructing simulation of queues (not simply using a black box) and to analysis of the queuing model may be introduced to students.

Ang [4] discussed the importance of promoting mathematical modelling in classroom practices and the use of technology as a bridge for the cognitive gap that hinders a student from carrying out a modelling task. The advantages of using simulation as a pedagogical device has been discussed widely (see [1], [2], [3] and [5]). Teachers indicated their beliefs in the usefulness

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of simulation activities in solving problems, giving meaning to and enhancing the understanding of concepts [3]. Students often find active participation in simulation to be more interesting, intrinsically motivating and closer to real-world experiences than other learning modes [5].

Hence in a nutshell, we are proposing the use of simulation and modelling to teach either students who do not have enough background in mathematics and probability theory and need bridging to high level queuing concepts, or students who may not be inclined to proceed to high level mathematics, but can appreciate queues which are a part of life in the modern world. In the process, students are exposed to a whole package of mathematical knowledge; modelling processes, stochastic processes, mechanisms of queues, and real applications.

## 2. Basics of M/M/1 queues

In this section, we will present some basics of queuing theory that instructors may wish to discuss with their pre-university or first-level university students to enable them to partake in the whole modelling process of queues. In particular, we focus on teaching modelling of queues using the M/M/1 model. For simplicity of exposition and convenience of data collection, queues at an automatic teller machine (ATM) are considered. Figure 1 shows a schematic of a typical queue at an ATM. Note that an ATM can also be more reliable in terms of service times since it is not "human" and will not get tired over time.
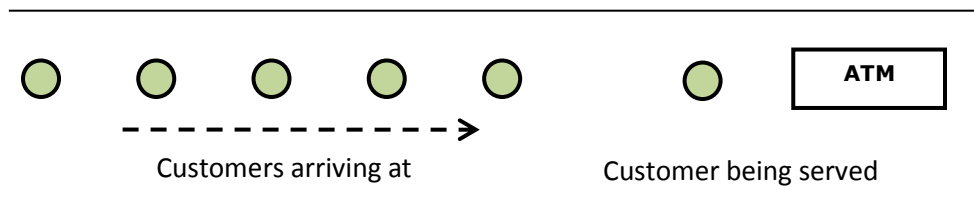


**Figure 1.** A single-server queue at an ATM

In a simple queuing model, the three components involved are the *arrival process, service process* and the *queue structure.* The arrival process typically involves three aspects:
- how customers arrive, for example, singly or in groups (batch or bulk arrivals)
- how arrivals are distributed in time, for example, what is the distribution of **inter-arrival times** (times between successive arrivals)
- whether the number of customers is finite or infinite.

The characteristics of a service process include the following:
- how long the service will take, that is, the **service time distribution**
- number of servers available
- whether each server has a separate queue or there is one queue for all servers.

A queue structure determines:
- how a person is chosen to be served from a set of waiting customers, for example, first-in first-out (or first-come first-served), last-in first-out or randomly
- whether there is balking (customers do not join queue if it is too long), reneging (customers leave queue after waiting for too long), jockeying (customers switching between queues)
- if the queue is of finite or infinite capacity.

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In an ATM queue, customers arrive randomly over time and wait for their turns in a single queue, and the ATM (single-server) serves one customer at a time on a "first in first out" basis. The modelling task is to construct a model that can simulate such a queuing system.

The simplest and most commonly considered queue is the M/M/1 model, where the "1" implies that there is only one server. The first "M" stands for Markov or memoryless and means arrivals occur according to a Poisson process. A Poisson process is a stochastic process (a collection of random variables used to represent the evolution of a system over time) where the inter-arrival times are exponentially distributed. That is, if $a$ represents the average number of customers arriving per unit time, then the probability that the inter-arrival time $T$ exceeds the value $t$ is given by $P(T > t) = e^{-at}$.

The second "M" also stands for Markov and denotes that service times of the server are exponentially distributed. That is, if $b$ represents the average number of customers served per unit time, then the probability that the service time $S$ exceeds the number $s$ is given by $P(S > s) = e^{-bs}$. We assume that both the inter-arrival and service times follow exponential distributions because this distribution is the only continuous distribution that possesses the unique *memoryless* property. That is, the probability of waiting an additional time unit for the next customer arrival does not depend on how long it has been since the previous arrival, and the probability of completing a service within the next given time period is independent of how long the person has been served already.

For the simple queuing system above, there are useful formulae that can be derived under the assumption that the system has reached a **steady state -** that is, the system has been running long enough so as to settle down into some kind of equilibrium position. Or in other words, the operating characteristics of the queue do not vary with time. But note of course that in real-life, systems often do not reach such a state.

Let $\rho = \frac{a}{b}$ be the traffic intensity, that is, a measure of traffic congestion for the server. It is clear that if $\rho < 1$, that is, the average arrival rate is less than the average service rate, then the queue length approaches a constant and the system reaches steady state. Otherwise, the queue grows indefinitely. Then according to the M/M/1 model, the expected steady state waiting time in a queue $W_q$ and the expected total time spent in system $W$ are

$$W_q = \frac{\rho}{b-a} \qquad\qquad (1)$$

$$W = \frac{1}{b-a} \qquad\qquad (2)$$

Though the derivations for the formulae above are complex in classical queuing theory, these equations are actually simple and easy to use. That is, given the arrival and service rates, we can easily calculate the expected times $W_q$ and $W$ in a queuing system. Note that the above and other relevant formulae (e.g., for average queue length) can be derived using *birth-death processes*. Students who are more mathematically mature can be referred to Hillier and Lieberman [6] or Bunday [7] for details.

### 3. Data collection and simulation process

With a basic understanding of queuing theory, students can proceed to collect data at an ATM to obtain estimates of the important parameters $a$ and $b$. For example, students can video-record a

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queue or use a digital watch to record the arrival and finish times of customers on site. The inter-arrival times, service times, wait times and total times (wait time and service time) can then be calculated with the aid of an electronic spreadsheet. As an example, Figure 2 shows a screenshot of an MS Excel worksheet with data that we obtained from observing an ATM queue at our university campus around late lunch time (when there is sufficient traffic flow).



| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | | Ave arrival rate | Ave service rate | | | Ave total time | Ave wait time |
| 3 | | a = 1.087 | b = 1.239 | | | 0:03:25 | 0:02:37 |
| 4 | | | | | | | |
| 5 | cust no. | Inter-arrival time | arrival time | service time | finish time | Total time | Wait time |
| 6 | 1 | 0:00:00 | 13:05:52 | 0:01:31 | 13:07:23 | 0:01:31 | 0:00:00 |
| 7 | 2 | 0:00:10 | 13:06:02 | 0:00:53 | 13:08:16 | 0:02:14 | 0:01:21 |
| 8 | 3 | 0:00:42 | 13:06:44 | 0:01:03 | 13:09:19 | 0:02:35 | 0:01:32 |
| 9 | 4 | 0:00:09 | 13:06:53 | 0:00:43 | 13:10:02 | 0:03:09 | 0:02:26 |
| 10 | 5 | 0:00:05 | 13:06:58 | 0:00:38 | 13:10:40 | 0:03:42 | 0:03:04 |
| 11 | 6 | 0:00:15 | 13:07:13 | 0:00:27 | 13:11:07 | 0:03:54 | 0:03:27 |
| 12 | 7 | 0:00:04 | 13:07:17 | 0:00:35 | 13:11:42 | 0:04:25 | 0:03:50 |
| 13 | 8 | 0:00:49 | 13:08:06 | 0:00:32 | 13:12:14 | 0:04:08 | 0:03:36 |

**Figure 2.** Screenshot of Excel worksheet with data from an ATM queue

In this case, the inter-arrival times in column B are obtained from the arrival times in column C (e.g., cell B7 = C7-C6). The service times in column D are calculated using columns C and E (e.g., cell D7 = E7-MAX(E6,C7)). That is, the service time of a customer depends on whether the customer arrives before/after the previous customer has completed the service. It is clear that total times in column F are the differences between the finish and arrival times, while the wait times in column G are the differences between total times and service times. The values of $a$, $b$, average total and average wait times can then be calculated easily and are reflected in Figure 2.

Students can better appreciate the mechanism of queues and learn about generating randomness through simulation. Instructors can help students to understand the steps of each simulation run using a flowchart as shown in Figure 3. Note that we assume the inter-arrival time and arrival time of the first customer to be 0 (for convenience).

The random inter-arrival times and service times can be generated using "Inverse Transform Method". For example, to generate random numbers from an exponential distribution with parameter $a$, we first generate $r$ uniform random numbers over [0,1]. Then let $r = 1 - e^{-at}$ (cumulative distribution function). It follows that $t = -\frac{1}{a}\log(1 - r)$. Since $r \sim U(0,1)$, we have $1 - r \sim U(0,1)$. Thus, we can simply let $t = -\frac{1}{a}\log(r)$. For more details, refer to [8].

To execute the simulation program, the user will also need to input the number of simulation runs desired, that is, we are actually doing Monte Carlo simulation. The idea is to calculate results many times, each time using a different set of random values from the uniform distribution. The belief is that averaging the results of many simulations should provide a better

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indication of real behaviour.  The output of the whole program will then be the average total time in the system and average wait time of customers, taken over *all* simulation runs. More details are provided as comments within the program, which is provided after Figure 3.
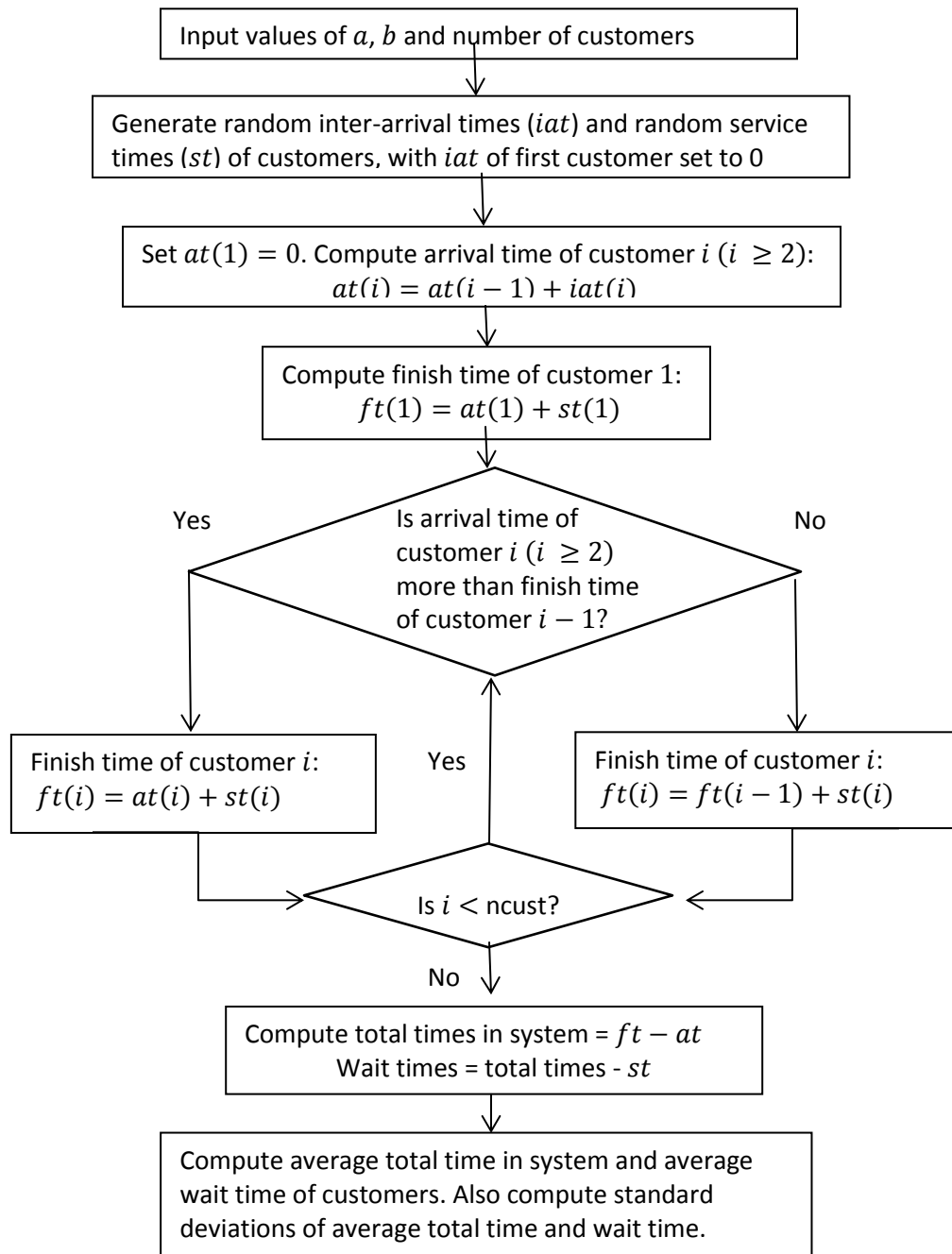


**Figure 3.** Flowchart describing each simulation run

% Simple queuing theory simulation, M/M/1 queue
a = input('Input the mean number of arrivals per minute:');
b = input('Input the mean number of customers served per minute:');

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```
ncust = input('Input the number of customers:');
nsim = input('Input the number of simulations required:');

%Initialise ave_wait_time_matrix, ave_total_time_matrix
ave_wait_time_matrix = [];
% to initialize matrix of average wait times for all simulations
ave_total_time_matrix = [];
% to initialize matrix of average total times for all simulations

for k = 1:nsim   % to run "nsim" number of simulations

        % Notations:
        % at = arrival time of a person joining the queue
        % st = service time (the time spent at the ATM machine)
        % ft = finish time after waiting and being served.
        %
        % initialize arrays:
        at = zeros(ncust,1); % all arrival times are initialized
        ft = zeros(ncust,1); % all finish times are initialized

        % Generate random arrival times assuming Poisson process:
        r = rand(ncust-1,1); % generate "ncust-1" uniform random numbers
        iat = -1/a * log(r); % generate inter-arrival times according to
                             % exponential distribution
        iat = [0; iat]; %to set zero iat of first customer
        at(1) = 0; % arrival time of first customer is assumed 0

        for i=2:ncust
                at(i) = at(i-1) + iat(i); % arrival times of other customers
        end

        % Generate random service times for each customer:
        r = rand(ncust,1); % generate "ncust" uniform random numbers
        st = -1/b * log(r); % generate service times according to
                            % exponential distribution

        % Compute time at which each customer finishes:
        ft(1) = at(1)+st(1); % finish time for first customer

        for i=2:ncust
                ft(i) = max(at(i)+st(i), ft(i-1)+st(i));
                % to obtain finish time for all other customers
        end

        total_time = ft - at; % total time spent (in queue and in service)
        wait_time = total_time - st; % time spent waiting in queue

        for j = 1:ncust
                if wait_time(j) < 0
                        wait_time(j) = 0; % to manually set wait time to be zero
```

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```
                                    % when there are computer errors
                  end
            end

            ave_wait_time = sum(wait_time)/ncust; % compute average wait time
            ave_total_time= sum(total_time)/ncust; %compute average total time
            ave_wait_time_matrix = [ave_wait_time_matrix; ave_wait_time];
            %to add on ave_wait_time of current simulation to matrix
            ave_total_time_matrix = [ave_total_time_matrix; ave_total_time];
            %to add on ave_wait_time of current simulation to matrix

end

ave_wait_time_final = sum(ave_wait_time_matrix)/nsim
% to find average wait time taken over all simulations
ave_total_time_final =  sum(ave_total_time_matrix)/nsim
% to find average total time taken over all simulations

sq_dev_wait_time
= (ave_wait_time_matrix -ave_wait_time_final*ones(nsim,1))
  .*(ave_wait_time_matrix - ave_wait_time_final*ones(nsim,1));
% to find square of deviations of average wait time of each simulation from average
% wait time take over all simulations

std_dev_wait_time = sqrt(sum(sq_dev_wait_time)/(nsim-1))
% to find standard deviation of wait time

sq_dev_total_time
= (ave_total_time_matrix-ave_total_time_final*ones(nsim,1))
  .*(ave_total_time_matrix - ave_total_time_final*ones(nsim,1));
% to find square of deviations of average total time of each simulation from average
% total time take over all simulations

std_dev_total_time = sqrt(sum(sq_dev_total_time)/(nsim-1))
% to find standard deviation of total time
```

## 4. Analysis of results

We will use the values of $a$, $b$ and number of customers observed from one of our data sets to discuss how simulation can be used to verify real data or relate to classical theory. With input values $= 1.087$, $b = 1.239$, ncust $=75$ and nsim $= 1 \times 10^6$, the program output were "ave wait time $= 3.057$ mins" and "ave total time $= 3.865$  mins". The standard deviations of the average wait time and average total time from simulations were 2.246 mins and 2.306 mins respectively. The actual average wait time and total time in our real data are 2.62 mins and 3.42 mins respectively.

Using equations (1) and (2), we have the theoretical times $\boldsymbol{W_q = 5.772}$   and $\boldsymbol{W =}$ $\boldsymbol{6.579}$. These values vary quite a lot from the simulated and real times. They fall just outside one standard deviation from the simulated average times. The discrepancies could be due to the small sample size (75 customers) and probably the short duration (about 70 mins) of the

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experiment. If traffic flow could be monitored over several days, perhaps more accurate results could be obtained. However, we can check that the actual average wait time and total time fall within half a standard deviation from the simulated average values. Thus perhaps we can conclude that M/M/1 model is quite suitable to model ATM queues.

## 5. Conclusion

In this paper, we present an approach to teach the modelling of queues through simulation to students who may not be mathematically mature enough to understand classical queuing theory. Through the entire process, students can learn a great variety of concepts while appreciating the natural phenomenon of queues in real life. Our hope is that pre-university and undergraduate educators will find this work useful in teaching statistics, modelling, and real-life applications of mathematics.

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## Audience Insights: Feed forward in Professional Development

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**Keywords:** Professional Development, variation, perturbation, reflection, enactment

As a result of watching a colleague teach Tanya decided to make changes to her lecturing practice which resulted in very positive changes to students' evaluations of her lecturing. We describe the model of intra-departmental professional development in which she had the opportunity both to observe others teach and examine and discuss her own practice. The variation between a colleagues' lecture practice, her previous experience of lecturing practice, and her own practice perturbed her and provided the initial impetus for her decision. We call this a feed-forward mechanism.

### 1. Introduction.

At a previous DELTA conference Bill Barton asked us to consider how we support student learning as we lecture. He claimed that:

> Lecturing is the interplay between the mathematical essence and the learning culture, and the teaching task is to make them work together to create better learning. The mathematical essence is not just a list of content, but includes the nature and uses of the subject, what it is like to act mathematically, and attitudes towards it. The learning culture refers to the expectations, intentions, and actions of both students and lecturers with respect to university pedagogy [1, p 965].

This paper focuses on two aspects of lecturing: the manner in which we display our attitude toward the mathematics we are teaching, and ways we consider our students. It describes the changes Tanya made to her practice after observing and discussing a colleague's teaching, and a visit to the circus.

We share the story of her teaching history and the change process and look beneath the surface for transferable mechanisms, to ask why she was perturbed and what is generalizable from her idiosyncratic path of change [2]. Her diary and reflections refer to key events in her decision to adopt a new approach toward lecturing. Her aim was to produce a more cheerful, interactive approach to lecturing, to bring more of herself into the teaching situation alongside the mathematics, and to pay more attention to the students' roles and expectations. This paper documents and explores her response to a professional development intervention and the manner in which it impacted her teaching and, according to her student evaluations, her students' engagement in the courses she teaches.

To establish a context for Tanya's reflection and enaction [2], we first describe the Development and Analysis of the Teaching of Undergraduate Mathematics (DATUM) professional development discussion group and its mode of operation. It was in this environment that she identified the type of change she wanted to make. We suggest that perception of variation provided the initial impetus for her perturbation. Finally we ask whether in situations such as this, what is being seen is an example not of the power of feedback but the impact of a feed forward mechanism.

### 2. The DATUM environment

This was the context that afforded Tanya the opportunity to reflect on her practice and to enact the changes in it [2]. It was here that she was able to observe, discuss and reflect on excerpts from her own and colleagues' teaching.

DATUM is a professional development project whose focus is the thoughtful, supportive, shared exploration of decisions made and actions taken during lectures. builds on the theoretical frameworks of teaching-in-context and of the role played by Resources, Orientations and Goals in decision making, developed by Schoenfeld and his Teacher Model Group (TMG) at the University of California at Berkeley [3,4]. We also take note of Speer's admonishment to pay attention to "small, but meaningful, aspects of practice" [5, p.219], "at the very level of detail when development and change appear to occur—the moment-to-moment decisions and practices of teachers" [5, p263].

Small groups of research mathematicians and mathematics educators, usually about 6 people, meet at regular intervals to re-examine their lecturing practice. Before each meeting one of the group chooses a lecture that they are happy to have video recorded. From this recording they select a section of less than five minutes for the whole group to watch and discuss. The discussion is audio-recorded and transcribed. Ahead of time the lecturer has reflected on their intended practice using Schoenfeld's Resources, Orientations and Goals (ROG) as a framework. This ROG is shared with the group.

While the choice of section is entirely up to the lecturer they frequently select a section that illustrates a decision point when they feel they have done something unexpected, or a teaching incident that raised a question for them. We generally found that while discussion initially focussed on the chosen segment we almost always went on to consider a range of issues of learning, practice and mathematics.

This model of professional development has been in development in the mathematics department at Auckland University since 2009 [1,6,7]. In 2013 the founding group divided into three new groups in order to be able to invite new participants. Tanya was one of these participants. Like others she had two recordings made of her lectures during the year. In the first semester she was recorded teaching students in a first course in calculus about continuity. We watched the colleague's lecture that provided the initial impetus for her reflection on practice and enactment of change soon after this. A couple of weeks later we then watched her lecture on continuity. In the second semester her lecture on partial derivatives and tangent planes was recorded and discussed.

### 3. Data

We draw on a variety of data sources in this paper: Tanya's recording of aspects of her teaching history and her diary of events described in this paper, her written Resources, Orientations and Goals (ROGs), video recordings of her lectures, the recorded conversations during DATUM meetings, and end of semester evaluations of the courses she taught in 2012 and 2013. It was after she received the latest evaluations that she shared with me, as the co-ordinator of the DATUM group she joined, the story we tell in this paper.

In the next section Tanya tells the story from her perspective: her history, diary and view of the entire process. Her history gives us some indication of how she may have developed the expertise and willingness to reflect that she brought to the process.

### 4. The story from Tanya's perspective

#### 4.1. My teaching history

I began my teaching career as a secondary teacher in the former Soviet Union. I won a scholarship to Rice University to do a PhD in pure mathematics that I completed in 2003. Marriage brought me to New Zealand where I was employed on temporary part-time contracts as a teaching post-doc at Massey University. For the next 8 years I tried to pursue a research career at the same time as being a part-time working mum to three kids. Trying to prioritise my three roles in life: research, teaching and being a mother was very difficult. In

2012 I was happy to accept a job offer from the University of Auckland to become a Professional Teaching Fellow – permanent full-time employment for the first time in my life! Finally I could concentrate on one path and put my undivided energy into it.

### 4.2. From my diary

**Table 1.** Timeline of chain of events in 2012

| Feb | I am pleased to be invited to join a DATUM group. |
|---|---|
| Mar 14 | I wrote a ROG (Schoenfeld, 2011) for my lecture on Continuity (MATHS 108 is a Stage 1 Calculus/Linear algebra course taught to an audience of 300 students per stream). |
| Mar 15 | Lecture on Continuity. |
| Mar 21 | Discussion of colleague's (Simon) lecture on Equivalence Relations to second year students. |
| Mar 24 | Watched my lecture to choose a 'slice'– I was beginning to think that my teaching style is a bit different from Simon's. |
| April 4 | Discussion of my lecture. |
| After April | At DATUM meetings I raised the topic of different teaching styles – my reflection process was brewing. |
| June | Lecturing Evaluations of MATHS 108 completed – feeling a bit upset about them – my active reflection begins and simmers during marking. |
| July | Mid-semester break visit to the circus and a breakthrough in reflection. |
| End of July | Semester Two begins. I am teaching MATHS 208, Stage II Calculus/Linear Algebra course – new teaching style from day one. |
| Oct 15 | Lecture on Partial Derivatives and Tangent Planes for MATHS 108, Stage 1 Calculus/Linear Algebra course. |
| Oct 25 | Discussion of my lecture. |
| Late Oct | My Lecturing Evaluations of MATHS 108 and 208 completed – a significant change. |

### 4.3 My story of change

In 2012 as part of the professional development project DATUM I observed a number of different ways lecturers behave in class. In particular I was very impressed with Simon's lecturing style  mesmerizingly enthusiastic and cheerful. When I was watching the video of his lecture I was thinking that my lecturing style is quite similar to his - energetically charged and interactive. When I watched my own lecture it was a complete shock to me.  I saw that I wasn't as cheerful as I thought and certainly I was not coming across as an enthusiastic person who smiles easily. I only smiled a few times during the entire lecture. This was a revelation to me  the image that I had in my head was not matching the reality.

All these thoughts were going through my mind but I had very little time on reflection  it was week three of the semester, my first semester as a Professional Teaching Fellow at the University of Auckland. I was in survival mode  just trying to get through the semester without making poor judgments. In the last week of the semester I received student feedback from my lecturing evaluation done on-line.

In response to the question 'What improvement would you like to see?' students said things like: "A bit more energetic when doing lectures" "Have more sense of humour and try not to talk in one tone all the time." "More class involvement/activities." "Not much. She is quite good, but her voice is slightly monotonous." "Smile :-)" The last comment felt like a knife was stabbed through my heart  that was the last drop to get me into a state of dissonance. I was seriously frustrated.

My first impulse was to just go to the next lecture and start smiling all the time  easy, I thought. During the next lecture it became obvious to me that if I started smiling after every

sentence I would look like a lunatic  it was simply impossible to do within my existing teaching style. I was coming to the conclusion that I needed to change my teaching style altogether in order to be able to smile more often, to show the students my energetic engaging self.

The semester ended. I had a few weeks of peace and a few weeks of marking and time to ponder. It was time to complete the reflection process and get ready for the next semester. In the holidays I took my kids to a circus. I found myself watching circus performers very closely while thinking about a comparison between a circus audience and a student audience. In both cases you pay money to come to a circus or to a lecture. At a circus you get pure entertainment and an audience is in a happy state clapping every minute or so. At my lecture students are hopefully learning but are not close to a state of being entertained so that they can clap every minute demonstrating their gratitude. Here it was  the moment of breakthrough - at the circus. I knew exactly what I was going to do next semester and it was not me doing cartwheels during lectures  it's better: I am going to take myself into the lecture, to introduce myself into the lectures alongside the mathematics.

During my first lecture for MATHS 208 to about 300 students I began with an introduction  I spent ten minutes telling them the story of my life with some career advice here and there to keep it relevant. Most of them were smiling at me gratefully. After opening up to 300 people it felt very natural for me to introduce a Ukrainian chocolate prize for answering questions correctly. Ukrainian chocolate is a wrapped candy that I am able to throw quite far  far enough to reach any student in a lecture room. I am not a natural thrower hence the entertainment! The students loved it. I never have to wait for an answer  the students compete for speed as well as for correctness. It's like a game for them – an entertainment. Everyone is having fun at my lectures now, including me.

In the following sections we will examine aspects of the change process from a research perspective. First we will consider the importance of reflection and the role variation [8] played in provoking the perturbation or dissonance to which she refers. Second it is useful to consider aspects of lectures as performances. Finally we consider the role of a feed-forward mechanism in the change process.

## 5.    The change process

### 5.1    Reflection

When Tanya returned to her room after the discussion of Simon's lecture to watch the video of her teaching she paid very careful attention to her demeanour.  She was reflective paying "active, persistent and careful consideration" [9, p. 6], to how she presented herself. And what she noticed perturbed her. She subsequently received positive feedback during the discussion of her own lecture on continuity. No-one else commented on the aspect that concerned her.

We know how valuable feedback is. What do we know in the forward direction? She knew she needed to think about the future in the light of what she had observed. Towards the end of all subsequent DATUM discussions she would raise a question relating to style of delivery – asking people what they thought. It was clearly playing on her mind.
Like Clarke and Hollingworth we view lecturer change as 'growth and learning' [2, p. 943] as they describe in their interconnected model of professional growth. While their model was developed and has been more widely used in school settings we find it useful in this tertiary situation. The particular value of this model is that it recognizes the idiosyncratic and individual nature of teacher professional growth" [2, p. 947] and enables the identification of

particular ''change sequences'' and ''growth networks.'' It incorporates the idea that ''learning is an integral part of generative social practice in the lived-in world'' [10, p. 35]. This allows what they refer to as the social ''situatedness'' of learning to enter the equation. It is clear that the discussions, the viewing of lectures and the student evaluations perturbed Tanya provoking dissonance, and that reflection on these and on her role as a lecturer or performer impacted on the outcome.

### 5.2    The role of variation.

Tanya perceived a variation between her previous experience of watching lectures and the one she saw Simon give. This was the initial impetus for her re-examination. In analysing Tanya's response to watching Simon's lecture we draw on ideas from variation theory.

Runesson [11] makes a compelling argument that variation is a necessary condition for learning. Her research is based in the classroom with children but we contend the arguments she presents work equally well in lecturer learning situations. She states that "From a variation theory perspective, to learn implies to experience, understand, perceive or see something in a different way,"[11, p. 397] that variation is the primary factor needed for discernment, which will lead to learning [8]. In order to discern or notice the aspect we have to experience a variation in that aspect. Further in variation theory learning is always about learning something, "learning has an object. In variation theory the object of learning is the capability to do something with something" [8, p.400]. Our question is thus not only why Tanya experienced a variation but also how she learned and what she did with what she had learned.

In the following discussion after Simon's lecture we had been talking about how she encourages participation by getting students to nod when the following interchange occurred:

> Tanya: But your point of difference that you differ from many other math lecturers, the ones I have seen in my life, who have taught me, he's (sic) always in a good mood, he's loving  this, look at his face. Every pause you make you turn around and smile to the class. It's amazing!
> Simon:  But I am really enjoying this because of the course and because of the students and that's why I chose this one to be videoed because I am teaching another class I am not enjoying as much, even though I like the mathematics more, it's because of the class.

We argue that this perturbed her to begin to consider lecturing as a performance.

A number of researchers who work in teacher development draw analogies between learning and journeys [12]. The impetus to undertake the journey is referred to by Cobb, Wood and Yaeckel [13], who write of a trajectory of learning, an analogy that carries with it a sense of direction and initial thrust.

### 5.3    The role of perturbation and dissonance.

Tanya wrote "*that was the last drop to get me into a state of dissonance. I was seriously frustrated.*" Clarke writes that for a teacher who is for some reason perturbed a "healthy disequilibrium" has arisen leading them to re-examine their classroom practice [14].  A Best Evidence Synthesis of professional learning opportunities that impacted on student learning found that the effective professional learning situations usually began by creating dissonance of some type for the teachers [15]. The dissonance was designed to disturb in some way their existing views of teaching or learning.

### 5.4    The role of performance in lectures.

Later in the discussion of Simon's lecture we considered the use of notation and symbols in mathematics and the need to translate them for students into ordinary language. At previous

meetings we had talked about the different ways in which lecturers verbalise symbolic statements.

> Tanya: Maths is like a foreign language – you take long sentences that …. A concept and then you shorten it into very precise notation and if you don't know what the notation means it's like you don't speak the foreign language - you have no chance to understand it or what's behind it.

Immediately after this came the following interchange that focussed on the role of the 'theatrical or staged pause' and the performance that is a signal to think.

> Simon: I wish I could speak more slowly
> Judy: But you stop every so often … you are good at pausing
> Barry: There are a number of quiet moments …There's also 'this' (gesticulates head scratching). I like the head scratching – as signal that you have stopped to think
> Simon: It's hilarious – like a performance. Yes it's a conscious way to say you are supposed to stop and think
> Barry: Yes that is the point
> Simon: I sort of stare at the board – and I should say – I want you to stare at the board too but I can't bring myself to say that. So I stare at the board for them … It's mean to be a signal for them
> Judy: Contemplative. Is this staged contemplation?!

What perturbed Tanya was the mismatch between her lecturing experiences and what she saw Simon doing, the possibilities she perceived for herself and the mismatch between how she saw herself out of the classroom and the person she took into the lecture theatre "*I chose to shift my own behaviour in the direction of the one that best matched my personal way of interacting when not in lectures.*"

From the above discussion about performance in lectures it is apparent Tanya took many messages to help her achieve the change she envisaged – about speed of delivery, translating for students, pausing and signalling and the importance of performance. She does not mention these in her own reflection but it is clear from her student evaluations and from the second video that she did more than decide to be cheerful. The role of the students in the 'meaning making' in mathematics became prioritised in her lectures. It is interesting that she talks about the entertainment in terms of fun more than in terms of access. Her evaluations make it clear that she is both talking more slowly and pausing, providing more scaffolding for understanding the language and smiling.

As luck would have it Simon and the first author recorded and observed her lecture on Partial Derivatives and Tangent Planes. All the concepts were demonstrated on the surface of a large black umbrella with the aid of string, rulers and other props that Tanya took out of a capacious black bag – the resemblance to Mary Poppins was inescapable!! It was not just fun - the metaphors were considered and powerful and the communication of ideas paramount. The change was obvious.

## 5.5    *The role of feed-forward in the change process*

We ask whether, as we watch and consider others' teaching, a feed-forward mechanism may be activated? The role and importance of feedback in the learning cycle of both students and teachers is the subject of many studies and is summarised by Hattie in his much cited 2009 meta-analysis.

> Feedback is most powerful when it is from the student to the teacher. When teachers seek, or at least are open to, feedback from students as to what students know, what they understand, where they make errors, when they have misconceptions, when they are not engaged—then teaching and learning can be synchronized and powerful. Feedback to teachers helps make learning visible [16, p. 173].

What might feed-forward be? We envision it as somewhat different from the Feedforward mechanism in computer networks or Marshall Goldsmith's model [17]. The notion arose during an exchange with an interested outsider about the processes in the DATUM project. She asked whether the lecturers got feedback on their performances in lectures. We replied, "Not really – it's more like feed-forward." By this we mean exactly what happened to Tanya as she watched Simon's lecture, his actions opened up future possibilities for her teaching. This idea is highlighted in the following interaction between two participants in the project during the discussion of how one had chosen to approach the teaching of continued fractions.

Simon: It would have been nice **to have said** …
Greg reminds him that the future is ours to change, and of the aim of the project.
Greg: **You can always** – that's what this project is about.
As with all professional development initiatives the big question is – what effect did the change have on student learning. In the next section we present data to evidence the positive impact the changes Tanya made had on her students.

## 6.    Impact of the changes on students

At Auckland University student responses in evaluations are collected in two ways. The first is a Likert scale fixed format prompts section.  Table 1 shows the mean response, where 5 is strongly agree and 1 is disagree, to three crucial prompts for the courses Tanya taught in 2012. The very substantial positive changes, with class sizes of 300 in Maths 108 and a total of 550 in two classes of MATHS 208, speak for themselves.

| Item | MATHS 108 Semester 1 | MATHS 108 Semester 2 | MATHS 208 Semester 2 |
|---|---|---|---|
| **The lecturer responded to students' questions in a constructive way** | Mean 4.01 Std.Dev. 0.88 | Mean 4.57 Std.Dev. 0.68 | Mean 4.43 Std.Dev. 0.71 |
| **The lecturer stimulated my engagement in the learning process** | Mean 3.69 Std.Dev. 1.00 | Mean 4.49 Std.Dev. 0.77 | Mean 4.25 Std.Dev. 0.80 |
| **Overall, the lecturer was an effective teacher** | Mean 3.94 Std.Dev. 0.99 | Mean 4.58 Std.Dev. 0.73 | Mean 4.53 Std.Dev. 0.66 |

**Table 2.** Mean response from Evaluations.

The second section in the evaluations allows students to make open responses. The size of the classes means a wide range of responses are given. However themes are relatively easy to identify. In the first semester for MATHS 108, while many students were satisfied, the following are typical of the negative responses: "Tanya is very fast when she explains any concept or goes through examples. I suggest she slows down and gives us a moment to take it in and copy down any notes/examples." "Focus more on ensuring that students understand than just rushing to get through all the slides." "Increase student participation in class." "I notice that you focus on just one part of the lecture, or just on the sheet in front of you. A little more eye contact with different sets of students in different parts of the hall should fix that"

The responses from the students in the both courses in the next semester are markedly different. "Tanya provided very good examples and worked through everything at a good speed in a way that everyone could understand." "She is the BEST lecturer this semester. She is very cheerful, very interactive with students. Stimulates our learning. The atmosphere in her lectures is also very great!! I always look forward to going to her lectures." "The fact that she takes her time … explaining each step and assumptions she is making helps me to understand. She has good communication skills which always helps." "I am not a big fan of

Ukrainian chocolates but I was really impressed with her ability to get the class to interact." "Tanya's use of props e.g. the umbrella was very helpful to visualise concepts."

A particularly interesting subset of students was in her class in both semesters. Some of these were in MATHS 208 and others were the students that failed 108 in semester one and were repeating it. This was fortuitious in terms of their responses making valuable comparisons.

The following are open responses from some of these students: "I was very impressed with the change in teaching method/style from MATHS 108 to 208. Very impressed with the way Tanya expressed MATHS 208 stuff in lectures. Thank you." "I had her as a lecturer for 108 and feel she has improved remarkably as she has slowed down and explained things in a much succinct and clear manner." And finally: "The lecturer is much better to teach this year than last year. She always smiles and looks enjoyed to teach MATHS 208. Do you remember me? Haha! I suggested you to smile often in the lectures last year. I hope I pass this course and take stage 3 of maths courses with you teaching!"

## 7.    Concluding remarks

As Clarke and Hollingsworth highlight productive professional development paths are idiosyncratic [2]. And while that is true there are also transferable lessons to be taken from Tanya's professional development journey: watching excerpts from colleagues' lectures and discussing decisions made can stimulate reflection, and re-viewing our own practice may be uncomfortable at times but can be the beginning of generating positive change. We will continue to explore the idea of a feed-forward mechanism in provoking change. We find it interesting that while writing this paper Tanya realised that she is now able to remember and draw on practices she remembers from 'the best lecturer I ever had' at Rice University, something she could not do in her old teaching style.

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## Undergraduate mathematics outcomes: The mantis shrimp spectrum

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**Keywords:** undergraduate mathematics; learning outcomes; mathematical behaviours

Undergraduate mathematics worldwide has had more or less standard mathematical content and skills outcomes that have been stable over several decades. But are these outcomes all we really want our students to learn? What other outcomes do we assume will occur, or hope will result, from our programmes? Which of these outcomes do mathematics lecturers regard as most important? Finally, do we have a common view about undergraduate outcomes? Interviewing lecturers in New Zealand and overseas is the first phase of a project to identify all learning outcomes and develop ways to observe whether they occur. We report on the development of a multifaceted "outcome spectrum", and describe initial interview results, including differences from expectations and differences from what is written in the literature.

## 1. Introduction

The mantis shrimp is the world champion of colour vision. It has such good eyes it can perceive both polarised light and multispectral images. If you combine the number of visual pigments with the other filters it has in its eye, it can distinguish sixteen different types of pigments. We only have three, leaving us wondering what the mantis shrimp perceives that we are prevented from seeing. Relying exclusively on traditional means of assessment in undergraduate mathematics, we see only the content and skills a student learns. What other outcomes are desired? How can we develop our awareness of the full spectrum of outcomes?

We are a group of researchers, mostly based in a university mathematics department, who aim to develop a framework of learning outcomes by interviewing lecturers, employers, students and alumni of undergraduate mathematics. In what colours and shapes does the community want student learning to occur? The project LUMOS (Learning in Undergraduate Mathematics: Outcomes Spectra) assumes that there are more desired outcomes than are stipulated in course documentation. Mathematics education literature includes outcomes that are highly valued but not assessed in traditional tests and assignments, for example, students' mathematical thinking, their mathematical processes, their understanding of the field in broad terms, their ability to work together and communicate mathematically, and their attitudes towards the subject [1, 2, 3, 4]. Aspects of these have been researched for specific contexts in New Zealand, for example, "soft skills" required in vocational activity [5]; affective factors [6]; and self-efficacy in chemistry [7]. However, no comprehensive catalogue of outcomes has been researched, nor observed, for undergraduate mathematics.

Why is a complete understanding of undergraduate mathematical learning outcomes of strategic importance? We are in a time of increasing pressure to change how we offer undergraduate education. This is particularly true in mathematics because of increasing demand for quantitatively competent graduates in all fields, and because there has been only little change in university courses in response to new student conditions, new technologies, or new knowledge of effective pedagogy. In order to properly evaluate developments in undergraduate delivery, we need to understand more clearly what we are aiming to achieve, and have ways to observe whether we are achieving them. The aim of LUMOS is to link

student learning to pedagogical interventions. It is, of course, an impossible quest because "the goal is never clearly defined, and the myriad of variables and external influences mean causal links are obscured." [1]. Nonetheless improving our ability to discern learning outcomes remains an urgent task.

Additional motivation for the project comes from the evidence that existing, traditional course delivery approaches need to be modified to achieve some of the learning outcomes desired by employers and graduate courses [5, 8]. This is particularly the case for engineering and mathematical sciences education. Innovations need to be proposed and trialled, and some NZ work has been done, for example, by Klymchuk, Zverkova, Gruenwald, and Sauerbier [9] and Sneddon [10]. Other NZ research relevant to LUMOS includes work on secondary/university transition [11] and a major NZIMA-funded project on secondary/undergraduate mathematics learning and teaching [12]. But we also need a basis on which to judge the result, and on which to adopt, modify or abandon these approaches.

Internationally, there are projects with similar objectives. Epstein [13] recently reported results of the Calculus Concept Inventory, a tool designed to distinguish the effect on deeper understanding of calculus in 1st year courses of different styles of instruction. We hope to reproduce some of this precision with these, and with other, objectives.

In this paper we describe the first two iterations of an undergraduate mathematics outcomes framework: the first posited on the basis of the literature and our experience, and the second developed using input from interviews with experienced lecturers who teach undergraduate mathematical sciences.

The framework will form the basis for next phase of the LUMOS study, the development of instruments by which we can extend our limited perspectives to something closer to that of a mantis shrimp for the observation of learning outcomes, and thereby explore the consequences of different pedagogical approaches to course delivery. Ultimately we aim to develop a "practical scheme by which we can observe, analyse, and effectively report student learning for a particular course." [1]. Later phases of LUMOS involve collecting data on three teaching innovations and reporting on the full spectrum of student learning outcomes in each. These aspects are described by Barton and Paterson [1], but are not the subject of this paper.

To the extent that the learning outcomes are predetermined, the study is positivist: that is, we assume that we can observe and record in a group profile student behaviours that are evidence of particular learning outcomes. For example, we accept the value of tests and examinations to give us information about the understanding of taught content and the ability to perform particular mathematical tasks under defined conditions. We also assume that attitude scales measure a relatively stable state of mind that has relevance for students' current and future mathematical behaviour. Part of the project will involve statistical confirmation of the reliability and validity of these measures.

On the other hand, we also wish to attempt the description and recording (as a group profile) of important characteristics that we know are neither well-defined nor previously captured in a robust way. In this respect the study, particularly in its early stages, will need to be more ethnographic. That is, as researchers we try only to describe what we observe and do not infer the existence of ideas such as deep understanding, or feelings such as anxiety. . As we gather learning outcome data in specific contexts, we will feed it back into the development of the framework and subsequently into future observations.

## 2. The initial description of the spectra

This paper describes the generation of the first two iterations of the framework. The initial framework was included in the funding proposal. It was built on existing learning

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frameworks created in the secondary school context, in particular the Watson-Mason Question Typology [14], Artigue's [15] triple level framework developed to evaluate technology use, and Sfard's [16] Participation/Acquisition analysis of learning theories. The Pirie-Kieren [17] and Schoenfeld [18] models of problem-solving also informed some detail of the framework. The framework adopted two models of tertiary mathematical understanding: Dubinsky's APOS [19] and Tall's Three Worlds models [20].

At that stage we stated that the categories of learning outcomes were likely to include those in the table below.

**Table 1.** Mathematics framework in the original proposal

| **Undergraduate Mathematics Learning Framework** |
|---|
| Categories of learning outcomes are likely to include:<br>• Specific content knowledge.<br>• Specific mathematical technical skills.<br>• Conceptual understanding<br>• Advanced mathematical thinking<br>• Mathematical problem solving<br>• Mathematical processes/modes/behaviours, such as conjecturing, exploring, abstracting, persisting, pattern identification, proving, generalising, and metacognitive behaviours.<br>• Mathematical representational versatility<br>• Mathematical horizon knowledge<br>• Mathematical communication and collaboration<br>• Affective aspects associated with mathematics (attitudes, beliefs, orientations, confidence, interest, willingness to engage). |

Are these the outcomes the lecturers who deliver the courses want, expect and value? Are there others? Which are top of their list of priorities? What aspects of a particular outcome do they most want to see? Are some comparatively unimportant? What do lecturers expect as outcomes from pre-requisite courses? To find answers to these questions we needed to interview lecturers in the mathematical sciences, that is, mathematics, statistics, engineering science, economics, finance, physics, biological sciences. At a later stage we will interview employers and alumni.

Together the mathematics educators in the research team drafted and refined the question protocol in Table 2 below. The aim of the interviews was to encourage lecturers to share a much wider range of outcomes than they usually list for a course. It was interesting to note that they seemed to enjoy the opportunity to examine and elaborate their underlying aims.

The interviews were conducted by the authors, and Christchurch collaborators. The interviewees had responded to a general request and were thus self selected. The sample size and variation in fields of research interest and years of experience leads us to believe the sample is more or less representative of lecturers in the mathematical sciences in New Zealand. Data was collected from 25 lecturers at the Universities of Auckland, Victoria, and Canterbury who taught a range of undergraduate mathematical science courses: pure and applied mathematics, statistics, and mathematical modelling courses in engineering science.

The interviewees were asked to respond to the questions with two different courses in mind. First they were asked to consider the lowest course they had taught during the preceding two years. They were then asked about the highest recently taught undergraduate

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course. Final questions focussed on global learning outcomes they espoused for science or engineering graduates.

**Table 2. .**Protocol of questions and prompts for interviews with lecturers.

| |
|---|
| What **key content and skills** would you want students to fully have by the end of the course? |
| What else would you want the students to gain from this particular course? |
| Let me ask you again about desired outcomes in some different ways: |
| • How would you like a student who had passed this course to perform when, for example, they are confronted with a mathematical problem ? |
| • Is what you have told me the same as what you expect from students from this course when they enter your subsequent classes ? |
| ***Prompts***: *(If not mentioned above) what about:* |

| | |
|---|---|
| • *the ability to solve problems* | • *use technology* |
| • *communicate and collaborate* | • *knowing history and applications,* |
| • *use logic* | • *argue, justify, conjecture, prove* |
| • *internal and external connections* | • *confidence, liking for, respect for mathematics* |
| • *notation & symbolisation* | • *beliefs about mathematics, e.g. its beauty, applicability* |

Which of all the above is a top priority in this course?

Data was also collected online, and at international conferences as opportunities arose. Conversations after presentations at these conferences, frequently heated, have contributed to the framework. For example, at a UK university, a discussion after a seminar went on for more than another hour over the question of whether or not there was some particular item of mathematical knowledge that was essential for every mathematics graduate to know so well it is always accessible. No resolution was found.

In developing the data-based framework we paid attention to the relationship between learning models and models of teaching and lecturing (for example, Ball's Mathematical Knowledge for Teaching and MQI evaluation instrument [21], and Barton's LATUM model [22]) and studies of the transition between school and university [23]. The framework was also informed by studies of characteristic behaviours of productive mathematicians [2, 24, 25, 26, 27]. Underpinning the project is the theory of academic development at tertiary level [28].

## 3.  The (Current) Mathematical Framework

The framework is now at the stage where six main categories are more or less established, but the first and second levels of sub-categories are known to be incomplete. Data from the interviews has enabled us to identify some first level sub-categories as in Table 3 below.

**Table 3.** Current mathematical framework

| Content | Skills | Processes | Mathematical behaviours, modes, or habits | Affect | General graduate outcomes |
|---|---|---|---|---|---|
| Curriculum topics | Routine techniques | Logical thinking | Perseverance | Feelings about mathematics: e.g. respect, lack of fear, interest | Critical thinking |
| Deeper under-standing | Using definitions | Argumen-tation Proving and disproving | Playing with problems | Beliefs about mathematics: e.g. its usefulness, its importance | Desire to learn |
| Examples | Develop a "toolbox" | Abstracting Generalising | Using one's complete tool box | Attitudes to mathematics: e.g. should it be compulsory | Collaborating |
| Horizon Knowledge: e.g. overview of the field; historical knowledge | Visualising Graphing Creating diagrams | Conjecturing/ Hypothesising /Testing | Speaking and reading the language of mathematics | | Communicating |
| Notation & symbolic conventions | Checking answers | Problem-solving | Making errors | | Being responsible |
| | | Pattern identification | Mathematical maturity | | |
| | | Advanced mathematical thinking | Academic integrity | | |
| | | Represen-tational Versatility | Metacognitve mathematical thinking | | |
| | | Modelling | | | |

In the expanded framework we have included some examples, and illustrative quotations from the interviews with lecturers. In Table 4 below we include a section of the expanded framework in mathematical processes as illustration. (The codes distinguish between Researchers and Tutors because we believe we discern broad differences in approach. Does the reader?)

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**Table 4.** Section from the expanded framework from the Processes category

| First level of subcategories | Second level of subcategories | Examples | Quotes from the interviews: (R) indicates a research mathematician, (T) a tutor |
|---|---|---|---|
| **Logical thinking** | Using definitions | A function is continuous iff… | sophistication in the understanding of how a definition works (R) |
| | Constructing a step-by-step process or argument | Understand mathematical induction | Be logical and rigorous (T) <br><br> Ability to think clearly and logically (R) <br><br> The idea of modelling: what are facts, what are assumptions (R) |
| | Analysing an argument (from more than one perspective) | Double counting in a combinatorics problem | Be analytical (T) <br><br> Flexibility of approach (T) |
| | Grasp formal structure | Understand the role of functions in mathematics | Graphs: To know what they are and their link with groups, and hence linear algebra (R) |
| **Argumentation** | Explanations | "Explain your working here" | That mathematics is built on a process of careful formulation and proving of consequences, and this applies to everything they have learned (R) |
| | Challenging | Contradict others | Ability to argue and question (R) |
| | Rethinking | Be open to criticism and adapting ideas | Mathematics is the scientific method. Mathematics is not theorem/proof, but common sense (R) |
| **Proving** | Proving <br><br> Counter-example production | Prove Stokes Thm <br> "Show xxx cannot be true" | Content needed for life is high school level; university outcome is … logical thinking (hence proving is important). (R) |
| **Generalising** | Extending meanings <br><br> Finding other contexts | Seeing group structures in different and new contexts | Deeper grasp of more fundamental mathematical principles (because they are important for solving problems and understanding why eigenvalues and eigen-functions are so important (R) |
| **Abstracting** | Recognising fundamental relationships <br><br> Symbolising | Recognising the identity element as having particular roles | Formulate PDEs from a physical description, distilling the important features (R) <br><br> Be able to think from the real world to the math formulation creatively (R) <br><br> See the abstract in the specific situation (T) |
| **Pattern identification** | | Deriving formulae for sequences | Flexibility of mind to make connections and think in a wider sphere (R) |
| **Conjecturing** | Hypothesising | Mathematical modelling | to think systematically, invoking prior knowledge (R) |
| | Testing | Computing results from a model and test against reality | |
| **Representational Versatility** | | Swapping from algebraic to graphical representations | Graphs: To know what they are and their link with groups, and hence linear algebra (R) |
| **Advanced mathematical thinking** | [Nothing corresponding to this category yet found in the data] | | |
| **Problem-solving** | Able to take a problem apart | Recognise the role of different variables and be able to replace them with others | Step back from the problem (T) <br><br> Breaking up complex objects into simple ones that can be put together (R) <br><br> Set up good notation – it's not cosmetic – an integral part of problem solving (R) |
| | Identify what you need to know and find it out | Recognise when an integration is possible using known methods | Ability to go and learn what you don't know (R) <br><br> Able to do unseen problems, exhibit a mode of thinking not just memorise routine skills (R) |

### 4. How does the categorisation of the spectra compare with our expectations?

It is not surprising that the current framework resembles the original one in many important respects. What has become interesting is the prioritising of outcomes by lecturers and how they articulate different types of outcome. As we have listened to the lecturers a number of our original expected outcomes have been validated: lecturers are more concerned with process than specific content; developing problem solving skills is an important part of mathematics; the ability to communicate ideas is valued; statistics course outcomes include empowering students to deal with media bombardment. The most significant of these, the first, confirms previous research in New Zealand [12].

On the other hand, two types of responses have piqued our interest. Some are things we expected to hear, but which have not been mentioned; others are things we did not anticipate, but which have been brought up by one or more lecturers.

There is considerable literature on advanced mathematical thinking [2, 20]. To date none of our interviewees have alluded to this, neither directly nor in another form that we recognise. We did not prompt this particular outcome, so we do not know what they would say if asked, but we assume that most lecturers would have some form of advanced thinking in mind that they want for their post-graduates at least. Did we not recognise their allusions to this? Perhaps lecturers do not see this as an aim at the undergraduate level? Or perhaps the idea of advanced mathematical thinking is not one they accept.

A similar lack of evidence of concern was found with respect to representational versatility. Again it may be that lecturers call this by a different name, or just assume that students need to be able to work in different notational systems and with different representations of the same idea. But they did not mention it, despite the prompt about both notation and symbolisation, nor did any reference occur when discussing technology—the usual platform for changing representations.

Most surprising, given its frequency in the literature on affect at school level, was that no-one mentioned that they wanted their students to like mathematics more. When prompted they continued to deny this as a desired outcome. As one lecturer said "It is too much to ask them all to like mathematics, but they must appreciate its strengths and limitations—that is, they must develop a respect for mathematics." (R) This notion of a respect for maths and its capabilities was echoed by a number of people. "They need to need to respect it as a tool and see the possible uses for it" (T). Others referred to "the ubiquity of mathematics in science."

An unexpected response was to hear mathematicians speak about analogies in the context of learning outcomes. The word was used in at least two different ways. First lecturers emphasised the value of comparing analogous systems. "Students need recognise the limitations of what they are doing—to use analogies in order to solve or categorise problems" (R) and "to be able to model by analogy, for example, in a spring or truss we have displacement and force, in a capacitor we have voltage and current, and in a hydraulic system we have pressure and volumetric flow rate. The same models and equations can be used in the three different situations to relate the variables in equations."(T)  Secondly they spoke about analogies in the sense of metaphors for helping students remember or visualise situations. "I use the 'frog in spa pool' analogy when discussing grad, divergence and curl." Despite not understanding this analogy ourselves, the implication was that students needed to develop such tools.

Lecturers also spoke of the importance of knowing what you do not know, recognising this and knowing where to go to find it out: "They need to have academic integrity—not hiding from what they do not know"(R) "This includes recognising when you are stuck and being able to find the information from texts."(R)

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## 5. Concluding remarks

We are at the very early stages of recognising the multiple colours and forms of the desired learning outcomes, having interviewed only one group of stakeholders with respect to undergraduate mathematics. Nevertheless we are starting to see better, to develop mantis shrimp eyes, and have already been surprised at the wider spectrum of possible learning outcomes that is emerging.

Unlike much of our experience of Mathematics Department tea-room arguments, the considered responses of lecturers during their interviews exhibit strong understanding of the needs and aspirations of all undergraduate students, mathematics majors or not. We have gained a sense of shared purpose that reaches far beyond the specific matters of content that shape much day-to-day debate.

We look forward to the next phases of the project in which the wider research team (which includes research mathematicians, professional teaching staff, mathematics educators and higher education researchers) tries to observe each colour in this full spectrum of outcomes, and to see how it develops through our various undergraduate courses.

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# Mathematics Bridging Courses and Success in First Year Calculus

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**Keywords:** mathematics bridging courses; calculus; student preparedness

Students entering university with insufficient mathematics preparation for the courses they intend to study is an increasing problem. We show evidence of how withdrawal rates, failure rates, and final marks in a first year calculus unit are strongly associated with the level of mathematics studied at school, the assumed knowledge published for the degree and enrolment in a bridging course. Bridging course students were, on the whole, able to pass their first semester university calculus-based subject; however, they did not achieve at the level of their mathematically well-prepared peers.

## 1.    Introduction

In 2012, Australia's Chief Scientist in his report to the Prime Minister argued that 'Mathematics, Engineering and Science (MES) are fundamental to shaping the future of Australia, and the future of the world' [1].  Yet, despite the importance of mathematics to society, the number of students studying the higher levels of mathematics in the senior year(s) of secondary school in Australia is falling [2] and similar declines have been reported in many other Western developed countries [3, 4].

In New South Wales (NSW) students can choose to study mathematics in senior secondary school at one of four levels for their Higher School Certificate (HSC) – the qualification students receive at the end of year 12.  These levels are an elementary level – with no calculus content and considered not suitable for tertiary study – called General Mathematics, an intermediate level called HSC Mathematics and two advanced levels called HSC Mathematics Extension 1 and HSC Mathematics Extension 2. Of the students eligible for an Australian Tertiary Admission Rank (ATAR), a rank used to select school leavers for admission to university, the percentage of students studying mathematics at an intermediate or advanced level for their HSC has fallen from 61% in 1992 to 35% in 2012 – an alarming trend [5].

In Australia, university entry criteria may also inadvertently exacerbate the problem of low uptake of the higher levels of mathematics in senior secondary school [6]. Students are admitted to university primarily on the basis of their ATAR which allows the comparison of students who have studied different combinations of subjects for their end of Year 12 qualification. At many universities, including the Authors' institution, there are no subject prerequisites for entry into degree programs. Rather students are advised of the level of senior secondary mathematics that is 'assumed knowledge' for each degree, but nevertheless may be offered a place in a degree solely based on their ATAR score. This creates a 'tension' between a student gaining access to a certain degree and being adequately prepared mathematically for that degree [7]. Students, who have elected not to study intermediate or advanced mathematics at senior secondary school, have a difficult choice; either they accept

that they have closed their access to quantitative disciplines for the immediate future or they enrol in a degree program for which they are mathematically under-prepared.

The study of mathematics at university features not only in science and engineering degree programs but also in many other tertiary disciplines including pharmacy, economics, agriculture and information technology. Failure to study mathematics at an appropriate level may have serious consequences for a student's success at university. In a study conducted at a Canadian university, Kajander and Lovric [8] concluded that students' performance in a first year calculus course was strongly correlated to the amount of time they spent learning mathematics in the final years of secondary school. In an Australian study, Rylands and Coady [9] analysed data on the performance of first year university students in four mathematics and mathematics-related subjects. They concluded that a student's secondary school mathematics background, and not their ATAR, has a dramatic effect on pass rates: 77% of students with only elementary mathematics failed a basic mathematics subject.

Large universities may address the diversity in the mathematics preparation of commencing students by offering first year mathematics subjects at a number of levels [6]. Many universities, including the Authors' university, attempt to ameliorate the difficulties encountered by mathematically under-prepared students by offering preparatory programs (Bridging Courses) that enable students to obtain prerequisite or assumed knowledge in mathematics *before* commencing their degree program [10].

In Australasia, mathematics bridging courses have been part of the tertiary preparation scene for many years but there has been little research on their effectiveness [9-11]. In 2006, Galligan and Taylor [11] posed two (of four) unanswered questions within bridging mathematics as:

How is success defined in bridging mathematics activities?

Are successful bridging mathematics students successful university students?

For the first question, there are inherent difficulties in defining and measuring success in bridging courses. Godden & Pegg [12] suggest that formal evaluation of bridging mathematics programs may be contrary to the aims of the programs, and undermine their major strengths of flexibility and student-centred approach. They argue that traditional evaluative techniques are 'just not possible' and 'risk losing the essence of the support and assistance so necessary for these students'.

For the second question, internationally, bridging mathematics programs have been shown to be highly effective at resolving skill deficiencies for some students [8, 13]. In a large US study, Bahr [13, p.442] found that 'remediation has the capacity to fully resolve the academic disadvantage of math skill deficiency' for the quarter of students who 'remediated successfully', but the likelihood of successful remediation declined sharply as the 'depth of remedial need' increased. The latter finding echoes Wood's [14] remark that bridging programs do not work for very mathematically weak students.

In this paper, we do not attempt to define or measure success within the bridging courses themselves. Rather, we first examine the results of students enrolled in a first semester university mathematics subject known to have a sizeable number of students who are demonstrably mathematically under-prepared. We then ask and answer the questions:

How do the results of mathematics bridging course students compare to those of similarly under-prepared students who do not take the mathematics bridging course?

How do the results of mathematics bridging course students compare to those of students who studied the appropriate level of mathematics in senior secondary school?

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## 2.    Methodology

### 2.1    *Context of the study*

The University of Sydney is a major research-intensive university in Australia with over 50,000 students. Mathematics is taught in first year at four levels ranging from an introductory calculus subject to advanced mathematics subjects. Students are advised of the level of mathematics appropriate for them, but some degrees have requirements that lead to students enrolling in subjects for which they are demonstrably mathematically under-prepared [15].

For this reason our study concentrates on the performance of students in the first semester subject called Differential Calculus. This subject has HSC Mathematics Extension 1 (advanced mathematics) as assumed knowledge and relies on that content knowledge. Its first semester companion subject is Linear Algebra and students usually go on to complete second semester subjects called Integral Calculus and Modelling and Statistics. The four subjects together make up one-quarter of a full-time load for a year.

Our university offers two different mathematics bridging courses; 2 Unit and Extension 1 in February each year. The bridging courses include 24 hours of class teaching held over 12 days. In 2013, they had a total enrolment of 365 students; 42% (n=155) in the Extension 1 course. The courses are fee-paying and open to all, including students enrolling at other universities. The Extension 1 bridging course is for students who have previously studied HSC Mathematics (intermediate) and wish to enrol in subjects that have an assumed knowledge of HSC Mathematics Extension 1(advanced) including Differential Calculus. The 2 Unit bridging course is for students who have previously studied General Mathematics (elementary) or no mathematics in senior secondary school and are commencing degrees or enrolling in subjects with an assumed knowledge of HSC Mathematics (intermediate). As many of the 2 Unit bridging course students subsequently enrol in university mathematics or statistics subjects in other faculties or are eligible to enrol in our introductory calculus subject, we will not consider these students further.

### 2.2    *Data collection*

Data were collected from the University's database on all students who were enrolled in Differential Calculus for the years 2012 and 2013.  Further information was obtained from internal School of Mathematics and Statistics databases and the Faculty of Science bridging course database.

Students were only included in the study, if:
  they had applied to the Universities Admission Centre (UAC) for the first time in the two years previous to their enrolment at the University, and
  the subjects they studied for the HSC were recorded together with their marks.
  Any additional criteria will be given at the beginning of a section.

### 2.3    *Limitations*

Our bridging course students were self-selecting and were prepared to commit the time and money to learning mathematics prior to entering university. Many other variables including motivation levels and the amount of mathematics support sought and provided are unknown and cannot be controlled. Our bridging course students may not be representative of bridging course students at other universities.

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### 3. HSC Background in Mathematics vs. Success

In this section, we will report on students who studied some level of mathematics for their HSC and completed Differential Calculus. Figure 1 shows the distribution of final marks in Differential calculus grouped by the highest level of mathematics studied for the HSC. The results for students who studied General Mathematics (elementary) or no mathematics are not shown due to the very small number of students (<10 students). We found very strong evidence ($p < 0.0001$) that students who had completed higher levels of mathematics at school attained appreciably higher final marks in first semester calculus. In 2012, the means for students who had completed HSC Mathematics, Extension 1 and Extension 2 were 49.9, 59.5 and 69.5 respectively. The corresponding means for 2013 were 50.4, 58.4 and 67.1 respectively. The estimated differences in means of between 8 and 10 marks between these groups were all statistically significant. In the subsequent sections, we divide the students who had studied HSC Mathematics for their HSC into two groups; those students who enrolled in the Extension 1 mathematics bridging course and those who did not enrol in the bridging course. We then explore to what extent enrolling in a bridging course can ameliorate this difference in marks.
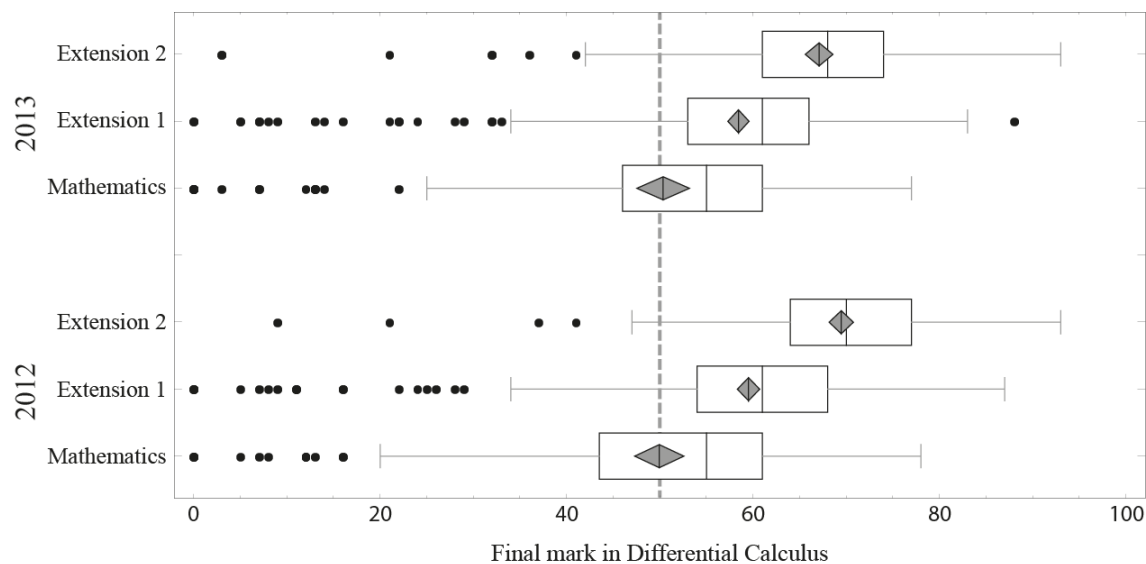


**Figure 1.** Boxplots of the distribution of final marks in Differential Calculus by the highest level of mathematics studied for the HSC. The shaded diamonds indicate the mean and its 95% confidence limits. The pass-fail boundary at 50% is indicated by a vertical dashed line.

### 4. Mathematics Bridging Course Students

In this section, we will report on all students who studied HSC Mathematics (intermediate) as the highest level of mathematics for their HSC and enrolled in (but not necessarily completed) Differential Calculus. All percentages will be rounded to the nearest whole number. Table 1 shows summary demographics for the students in 2012 and 2013 that satisfied these criteria. The demographics are very consistent from one year to the next. Each year's sample has about 200 students; almost all are 20 years old or younger, about three-quarters are male, over 80% are enrolled in science, engineering or computer science degrees and about 40% chose to attend a bridging course. Throughout this paper we will refer to the students who enrolled in the Extension 1 Mathematics Bridging Course as the BC group, and those who did not as the non-BC group. Note that there are no 2 Unit bridging course students in the non-BC group.

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**Table 1.** Background information on all students who had studied HSC Mathematics (intermediate) as the highest level of mathematics for their HSC and enrolled in (but not necessarily completed) Differential Calculus.

|  | 2012 | 2013 |
|---|---|---|
| Total number of students satisfying criteria for this study | 200 | 196 |
| Male students | 146 | 141 |
| Students 20 years old or younger | 198 | 196 |
| Students enrolled in degrees or combined degrees with an engineering, science or computer science component | 176 | 164 |
| Students who enrolled in the bridging course | 81 | 78 |

We compared some pre-existing attributes of students in the BC group with those in the non-BC group and found no evidence ($p > 0.05$) of a difference in gender, ATAR or mark achieved in HSC Mathematics (intermediate) at school. This suggests that these factors are not associated with whether students chose to enrol in a bridging course. However, the published assumed knowledge for their degrees was an important factor. The students were enrolled in over 20 degree programs, which we classified into two groups according to whether or not the programs had HSC Mathematics Extension 1 (advanced) published as the assumed knowledge in UAC Guide 2013. The degrees with Extension 1 as assumed knowledge included all engineering degrees and combined degrees such as the Bachelor of Engineering/Bachelor of Commerce, the Bachelor of Resource Economics and the Bachelor of Project Management. The degrees that did not have Mathematics Extension 1 as assumed knowledge included the Bachelor of Science, Bachelor of Computer Science and Technology and the Bachelor of Liberal Arts and Science. Most, but not all, the degrees in this latter group have HSC Mathematics (intermediate) as assumed knowledge. The number of students in each category is given in Table 2. The data show very strong evidence of an association between bridging course enrolment and the assumed knowledge for the student's degree (Fisher's Exact Test, $p \leq 0.0005$). For the degrees with assumed knowledge of HSC Mathematics Extension 1 (advanced), slightly over half the students with only an HSC Mathematics (intermediate) background chose to do the bridging course.

**Table 2.** Number of BC and non-BC students grouped according to the level of assumed knowledge published for their degree. The *p*-values are given for Fisher's Exact test.

|  | 2012 | | 2013 | |
|---|---|---|---|---|
|  | Non-BC | BC | Non-BC | BC |
| HSC Extension 1 not assumed | 74 | 30 | 64 | 22 |
| HSC Extension1 assumed | 45 | 51 | 54 | 56 |
|  | $p \leq 0.0005$ | | $p \leq 0.0005$ | |

## 5. Impact of Bridging Course

We analysed three indicators that might be related to the impact of enrolling in bridging courses: withdrawal rates, failure rates and final marks. We show below that participation in a bridging course was correlated with favourable changes in each of these indicators. We also then compared the final mark of bridging course students to those who had studied HSC Mathematics Extension 1 at school. We will refer to the latter students as the HSC Extension 1 group.

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## 5.1 Withdrawal rates

Students who were awarded a final mark in Differential Calculus will be considered to have completed, those who were not will be classified as withdrawals. Table 3 shows strong evidence (Fisher's Exact test: p<0.0001) of an association between the BC group and low withdrawal rates. In 2012, only 5% of the BC group compared to 34% of the non-BC group withdrew before completing Differential Calculus. The corresponding withdrawal rates for 2013 were 9% and 37% respectively.

**Table 3**. Student withdrawals and completions for BC and non-BC students. Withdrawals include students who discontinued not fail and discontinued fail.

|  | 2012 |  | 2013 |  |
|---|---|---|---|---|
|  | BC | Non-BC | BC | Non-BC |
| Withdrawals | 4 | 40 | 7 | 44 |
| Completions | 77 | 79 | 71 | 74 |
|  | *p* ≤ 0.0001 |  | *p* ≤ 0.0001 |  |

Tables 4 and 5 subdivide these groups further depending on whether or not HSC Mathematics Extension 1 was the assumed knowledge for their degrees. While substantial fewer students withdrew from the subject Differential Calculus if Mathematics Extension 1 was the assumed for their degree, nevertheless for both groups there was an association between the BC group and low withdrawal rates. The evidence suggests that enrolling in a bridging course has a positive impact on a students' retention in university mathematics subjects irrespective of the assumed knowledge for their degrees.

**Table 4.** As in Table 3 but only for students in degrees with HSC Mathematics Extension 1 as assumed knowledge

|  | 2012 |  | 2013 |  |
|---|---|---|---|---|
|  | BC | Non-BC | BC | Non-BC |
| Withdrawals | 2 | 12 | 3 | 11 |
| Completions | 49 | 33 | 53 | 43 |
|  | *p*=0.023 |  | *p*=0.003 |  |

**Table 5.** As in Table 3 but only for students in degrees where HSC Mathematics Extension 1 was not assumed knowledge.

|  | 2012 |  | 2013 |  |
|---|---|---|---|---|
|  | BC | Non-BC | BC | Non-BC |
| Withdrawals | 2 | 28 | 4 | 33 |
| Completions | 28 | 46 | 18 | 31 |
|  | *p*=0.002 |  | *p*=0.007 |  |

## 5.2 Failure rates

Table 6 summarises the pass and failure rates for those students who completed Differential Calculus. For the 2012 data, the failure rate for students who had enrolled in the bridging course was less than half that for those who had not enrolled; 17% and 38% respectively. The 2013 data also shows a smaller failure rate for bridging course students; 23% compared to 36%, but the evidence for an association is somewhat weaker. Table 7 compares the pass and failure rates of the students in the BC group to those students in the HSC Extension 1 group. Although the failure rates for the HSC Extension 1 groups are about 5% lower, there

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is no statistical evidence of an association between these groups and passing or failing Differential Calculus.

**Table 6.** Number of students failing and passing for the BC and non-BC groups. The category Fail includes Absent fail.

|  | 2012 |  | 2013 |  |
|---|---|---|---|---|
|  | BC | Non-BC | BC | Non-BC |
| Fail | 13 | 30 | 16 | 27 |
| Pass or above | 64 | 49 | 55 | 47 |
|  | *p*=0.004 |  | *p*=0.072 |  |

Table 7 compares the pass and failure rates of the students in the BC group to those students in the HSC Extension 1 group. Although the failure rates for the HSC Extension 1 groups are about 5% lower, there is no statistical evidence of an association between these groups and passing or failing Differential Calculus.

**Table 7.** Number of students failing and passing for the BC group and students who studied HSC Mathematics Extension 1 at high school.

|  | 2012 |  | 2013 |  |
|---|---|---|---|---|
|  | BC | Extension 1 | BC | Extension 1 |
| Fail | 13 | 49 | 16 | 79 |
| Pass or above | 64 | 415 | 55 | 414 |
|  | *p*=0.12 |  | *p*=0.18 |  |

### 5.3 Final marks

The results for students in the BC group and non-BC group, and the BC group and HSC Extension 1 group were compared. We found evidence that the students' background and enrolment in a bridging course are each associated with appreciably higher final marks. Figure 2 shows the distribution of final marks for the non-BC, the BC and HSC Extension 1 groups. In 2012 the mean marks for these three groups were 46.7, 53.2 and 59.6 respectively. The estimated mean difference of 6.5 marks between the non-BC and BC groups was statistically significant (t=2.50, df=154, p=0.015, with a 95% confidence interval from 1.3 to 11.8 marks). The estimated mean difference of 6.3 marks between the BC and Extension 1 groups was also statistically significant (t=3.87, df=539, p=0.0001, with a 95% confidence interval 3.1 to 9.5 marks).

Similarly, in 2013 the mean marks for these three groups were 47.3, 53.5 and 58.4 respectively. The estimated mean difference of 6.2 marks between the non-BC and BC groups was statistically significant ( t=2.17, df=143, p=0.031 with 95% confidence interval 0.6 to 11.8 marks). The estimated mean difference of 4.9 marks between the BC and Extension 1 groups was also statistically significant (t=3.00, df=562, p=0.003, with a 95% confidence interval 1.7 to 8.2 marks).

In summary, the mean marks of the BC group were approximately halfway between those of the non-BC group and the Extension 1 group.

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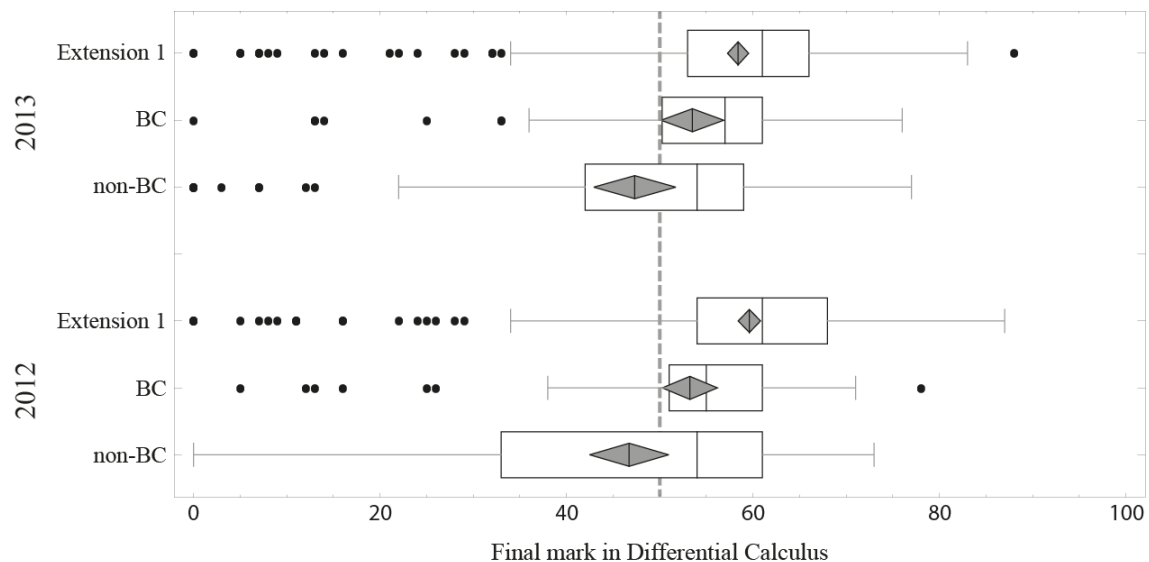
**Figure 2.** Boxplots of the distribution of marks for BC, non-BC and HSC Extension 1 students. The data is represented as in Figure 1.

## 6.    Discussion

In this study, we do not examine students' success during the mathematics bridging course but look beyond the bridging course to investigate the outcomes for a group of bridging course students in their university mathematics subjects relative to their cohort.

We found that, in general, students who commenced university mathematically under-prepared for their university mathematics subjects were significantly more likely to withdraw from or fail those subjects. This will come as no surprise to university mathematics teachers and is consistent with the research of Rylands and Coady [9]. It is of concern, however, that students may make decisions not to study the higher levels of mathematics in senior secondary school based on the short term goal of gaining a university place, not realising that by doing so they may severely compromise their future success at university.

Consistent with previous research [8], we found that the students, who had previously studied intermediate mathematics at senior secondary school and were mathematically under-prepared for their studies but had invested their time and money prior to university by enrolling in a mathematics bridging course, were significantly less likely to withdraw from and more likely to succeed at their university mathematics subjects compared to students who did not enrol in a bridging course. While improving student success is pleasing, reducing the rate of withdrawal for some students may be more problematic. For many students, not withdrawing from their mathematics subjects may be the appropriate action even at the risk of failing, but for others it may be more prudent to withdraw and enrol in a subject for which they are better suited mathematically. This highlights the need for good and timely advice for students as they select their university mathematics subjects.

While the mathematics bridging course students in this study were generally successful in their university mathematics subjects, our bridging course students did not achieve at the level of those who studied advanced mathematics at senior secondary school. Furthermore, our study does not consider whether attending a mathematics bridging course improves the outcomes in university mathematics subjects for students who studied elementary mathematics (or no mathematics) at senior secondary school. For these students, the research suggests that the gap between elementary mathematics and intermediate

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mathematics may be a bridging too far for attendance at short bridging courses prior to university to realistically address [13, 14]. Therefore, secondary school students should be discouraged from thinking that attending a mathematics bridging course is a panacea for failing to study an appropriate level of mathematics in secondary school – it is not an adequate alternative.

We do not claim a causal link between mathematics bridging course attendance and a student's success in his or her university mathematics subjects and hence a measure of its efficacy. Previous research suggests that students who enrol in mathematics bridging courses value the bridging courses not only as a means of ameliorating prior difficulties with mathematics and improving their mathematics learning but as a means to facilitate their transition to university [16]. The success of our bridging course students in their university mathematics studies may point to the vital contribution that mathematics bridging courses play as a vehicle to nurture the student's 'will to learn' which Barnett [17] suggests is of crucial importance to students' academic persistence 'ahead of both knowledge and skill' in the face of uncertainty and difficulty [17, 18].

## 7.    Conclusion

Our study highlights the importance of universities accurately informing prospective students of the appropriate level of mathematics to study at senior secondary school for their degree programs and the possible consequences for students of not doing so. Although 75% of underprepared students who enrolled in a bridging course were able to pass their first semester university calculus-based subject, they still did not achieve at the level of their well-prepared colleagues. Whether or not this disadvantage persists in non-calculus based university mathematics subjects or to later semesters is a subject for our future research.

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# Interactive teaching using tablet PCs: Designing effective questions

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**Keywords:** tablets, tablet PCs, questions, interactive, mathematics, learning

Clickers have been shown to help students learn concepts when used with carefully written multi-choice questions. Tablet PCs can be used in a similar way with the advantage that students can also submit their working and strategies. But what types of questions are best suited to this interactive use of tablet PCs? In this preliminary study, students' perceptions of a range of types of questions were analysed. Students in a course in discrete mathematics and statistics reported that most questions were useful for learning. This result is important as it is much easier for teachers to write questions for tablet PCs by formatting existing questions than to write meaningful multi-choice questions for clickers. Questions benefitted from a structured layout such as a table or gaps to be filled. Students also reported that questions that encouraged in-depth thinking and discussion, such as open questions, common misconceptions, or choosing a best strategy were particularly useful for their learning.

## Introduction

It is important that students actively engage with their learning as this leads to improved learning, particularly when the engagement involves hands on activities, feedback [1], and peer discussion [2]. Such pedagogical principles can be incorporated into learning activities with the innovative use of technology.[3]

To improve student engagement, Mazur [4] introduced clickers so that students could answer multi-choice questions during a lecture. He used clickers in a way that caused students to help each other understand concepts in physics. Mazur's evidence-based interactive clicker activities are now widely used in US universities.[5] The activities have been found to help students understand concepts in many different subject areas and improve student engagement and participation, for example [6, 7].

Tablet PCs can be used in a similar way to clickers. However students and teachers can also write directly on tablet PC screens with a stylus, which means that many more types of questions and answers are possible in addition to the multi-choice questions used with clickers. In this preliminary study with a class of 22 tertiary students, student perceptions of a range of different types of questions on tablet PCs are investigated. The results will be used to inform the design and focus of a more in-depth study.

## Background

Active involvement of students in Mazur's learning activities that use clickers has been shown to improve student performance in tests.[8]  Mazur's learning system, known as Peer Instruction, begins with students' preparation before class and involves the following activity sequence during lectures:

- Lecturer introduces a concept and then displays a multi-choice question to class.
- Students submit individual answers on clickers, discuss answers with their neighbours, and then resubmit their answers.
- Lecturer shows tally of answers, asks a few students to explain their answers to the class, and then summarises.

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Anderson et al. [9] used tablet PCs to enhance established pedagogies such as engagement, feedback, and peer instruction. In their case study, using tablet PCs in this way was found to enable and support active learning. Other studies support this.[10, 11] Anderson et al. used the following activity sequence:

- Teacher introduces a concept, then distributes a question to students' tablet PCs.
- Students submit their solutions and working.
- Teacher selects, displays to the class, and incorporates student solutions into a class discussion.

An observation in Anderson et al.'s [9] study was that peer instruction occurred spontaneously when a question was at just the right level for students to create similar but different solutions. In a similar study, Razmov and Anderson [12] often asked students to work in groups of two or three while sharing a tablet and observed that this promoted collaboration and in-depth discussion, particularly for open questions.

A key difference between Mazur's clicker activities and Anderson's tablet PC activities is the timing and frequency of peer instruction. With clickers, students submit an individual answer before discussing the question with peers. With tablet PCs, peer instruction can occur at any time after students receive a question but only when the nature of a question prompts students to discuss it.

Another difference between Mazur's clicker activities and Anderson's tablet PC activities is the nature of the questions. Mazur's multi-choice questions, called concept tests, require students to have a thorough understanding of a concept in order to choose a correct answer. Writing this type of question is both difficult and time consuming so Mazur has set up the Peer Instruction Network [13] to facilitate the sharing of questions amongst teachers. Anderson's questions on the other hand are the type of questions that teachers normally use in their classes. They are either closed questions that test a skill or a concept or they are open questions where more than one correct answer is possible. Thus, it is much easier for teachers to write questions for tablet PCs, than for clickers.

The special feature of using questions on tablet PCs is that the teacher can display students' full answers including working. It is therefore possible to discuss not just solutions, but more importantly the strategies used in finding solutions. But the disadvantage of using tablet PCs is that peer instruction may not occur. In order to promote peer instruction in this study, students worked in pairs and shared a tablet PC. This study could then focus on investigating what types of questions are best suited to students working in pairs on tablet PCs.

**Setting**

Tablet PCs were used to teach a first year discrete mathematics and statistics course on a business computing degree. Prior to the use of the tablet PCs each class session consisted of a one-hour lecture followed by a one-hour tutorial. Since using tablet PCs, each class session is now a two hour interactive teaching session. The teacher introduces each concept and then students work in pairs on questions on their tablet PCs in a learning activity that follows Anderson et al.'s [9] activity sequence (see Background). Students usually work in pairs but in some class sessions there is a group of three and sometimes students work on their own.

The one-semester course is taught in alternate semesters by the authors of this paper and the study was conducted in the first semester that tablet PCs were used when there were 22 students in the class. The first author was the teacher and the second author attended all classes as research observer.

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**The questions**

The questions were created on Powerpoint slides and were delivered to students using Classroom Presenter. [14] The design of questions was constrained by the physical characteristics of the tablet PCs as writing directly on the screen of a tablet PC with an electronic pen is not quite as easy as writing with pen and paper, and the writing area is limited. However, tablet PCs allow the use of colour, writing, highlighting, drawing, and typing, so questions can be designed to take advantage of these features.

Many of the questions were designed to simply provide practice with the concept that had just been taught, but a range of different question types were created, for example, open questions with multiple answers for encouraging discussion.[5] Questions were created with the expectation that most students would be able to attempt them. The aims were that all students would participate, that there would be peer instruction, and that the learning activities would encourage rather than discourage. All questions were designed for a pedagogical reason as recommended by Anderson et al. [9]

Another use of tablet PCs is by teachers when they are presenting a lecture. Many of the guidelines for lecturers using digital ink [15] appear to be applicable to questions written for students, in particular:

- Plan enough room to write.
- Split a slide into two if there is not enough room.
- Make diagrams large enough to write on.
- Practice with the technology.

We tested our questions by answering them on a tablet ourselves. This testing led to changes, for example, more room provided for an answer, complex questions divided into two questions, or part of the answer filled in, so that they wouldn't take so long to do.

We wrote eleven different types of questions. For each question, we chose a question type that we judged to be suited to the content and the pedagogy we intended using for teaching that content, as recommended by Anderson et al.[9] Many questions were based on existing questions for the course.

**Method**

With ethics approval, three types of data were collected about tablet PC questions. First, one or two questions were selected for evaluation in each class session. Students completed question evaluations using tablet PCs immediately after doing selected questions. They indicated reasons for whether or not a question helped their learning and gave each question an overall rating on a five point Likert scale from 1 (low) to 5 (high). Students responded individually and provision was made on the evaluation screen for up to three students in a group to respond. There was also room for students to write further comments. See Appendix.

The second type of data collected were the answers to the questions that students submitted during the lessons and will be referred to as "student submissions". The third type of data collected were observer's comments and teacher's reflections written during the course and notes taken during our many discussions of these.

**Analysis of data**

Several different analyses were carried out as follows.

First, to obtain an overall picture of the types of questions used in this course all questions were put into categories according to type. The number of questions of each type that was evaluated was also recorded.

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Second, for those questions that were evaluated by students, their reasons for why a question was useful or not to their learning were totalled and percentages calculated.

Third, to assist making comparisons of overall student perceptions for different questions, a "question rating" was calculated for each question, This was done by averaging the ratings given by students in their question evaluations. Students' ratings were discarded if they were ambiguous, but if the intent was clear although outside the given range, they were recorded as 1 (low) or 5 (high).

Fourth, student submissions were marked by the authors and the number of correct submissions recorded. When a question had several parts, the answer was only recorded as correct if all parts were correct.

Finally, when analysing the data for each question, observer and teacher reflections were also considered.

Students completed the question evaluations after they had submitted their answers and the class discussion of submissions had occurred. It should be noted therefore that question evaluations and question ratings apply not just to the question but to the learning activity as a whole, including answering questions, viewing student submissions, and teacher-led discussion.

## Results and Discussion

In this section all questions used in the course are first categorised into type, to give an overall view of the types of questions in this study. Data for the questions that were evaluated by students are then examined for question features that students reported helped or hindered their learning. Results are discussed then summarised as a set of suggested guidelines for writing questions for tablet PCs.

### *Types of questions*

There were 154 questions written for this course, and eleven different types of questions were identified. For each type of question, the number of questions in the course and the number of questions evaluated are shown in Table 1.

**Table 1:** Types of questions

| Type of question | Description | Number of questions in course | Number of questions evaluated |
|---|---|---|---|
| Text | Text and mathematical symbols only. None of the distinguishing features of the other categories. | 86 | 6 |
| Complete a table or fill in the gaps | Tables or gaps to indicate where students should write different parts of their answer. | 35 | 14 |
| Diagram | Drawing or labelling a diagram or chart. | 6 | 1 |
| Colour-in | Shading areas on Venn diagrams. | 5 | 1 |
| Compare solutions | A question and two strategies were given with all algebraic steps shown in full. Students were asked to compare the strategies and comment. | 3 | 0 |
| Find mistakes | Find and correct mistakes in a full worked solution. | 5 | 0 |
| Best strategy | Decide which strategy will lead to the solution most efficiently. | 2 | 1 |

| Common misconceptions | Designed to alert students to common misconceptions. Questions consisted of two parts that looked similar, but required different strategies. | 2 | 1 |
|---|---|---|---|
| Matching | Match or categorise items. | 3 | 2 |
| Open | More than one appropriate or correct answer. However, it was also possible for students to give incorrect answers. | 5 | 3 |
| Open and diagram | Open questions that also required students to draw a diagram or chart. | 2 | 1 |
| **TOTAL** | | **154** | **30** |

Of the 154 questions, 86 (or 56%) involved only text and mathematical symbols. These were closed questions with a specific answer about a process or concept. The other 44% of questions covered a variety of different types as shown in Table 1.

Students rated table questions higher than text questions. The average question rating was 4.0 for table questions and 3.7 for text questions. The authors observed several advantages of providing a structure for students' answers such as completing a table, adding to a diagram, or filling in gaps. A structure gave students an indication of how to get started on an answer and helped them lay out their answer in a way that would fit on the screen. Because all submissions followed a structure that students were already familiar with it was easier for students and teacher to read the submissions, and this facilitated discussion.

### Student perceptions of usefulness of questions for learning

Question evaluations were administered regularly throughout the course. During the 22 class sessions in which tablet PCs were used, students filled in a question evaluation on 30 occasions. Students soon became familiar with the format of the questionnaire and filled them in very quickly. Students appeared to complete them willingly for most of the course but there were signs of "questionnaire fatigue" towards the end of the course, particularly for a few students. The number of question evaluations received for questions ranged from 10 to 22 with a median of 18. The wide variation was due to the number of students who attended each class session as well as the occasions when students didn't submit question evaluations.

Students rated most questions positively. The average question rating for all questions evaluated was 4.0 and three quarters of the questions had ratings of 3.8 or more. Students recorded that most questions helped them learn with 96% of all responses for all questions reporting that the question helped learning, and this ranged from 85% to 100% for different questions.

Question ratings ranged from 2.4 to 4.7, and students also indicated when they found questions too hard or too easy. To investigate students' different perceptions of questions, the percentage of students who found a question too hard or too easy is plotted against the question rating in Figure 1.

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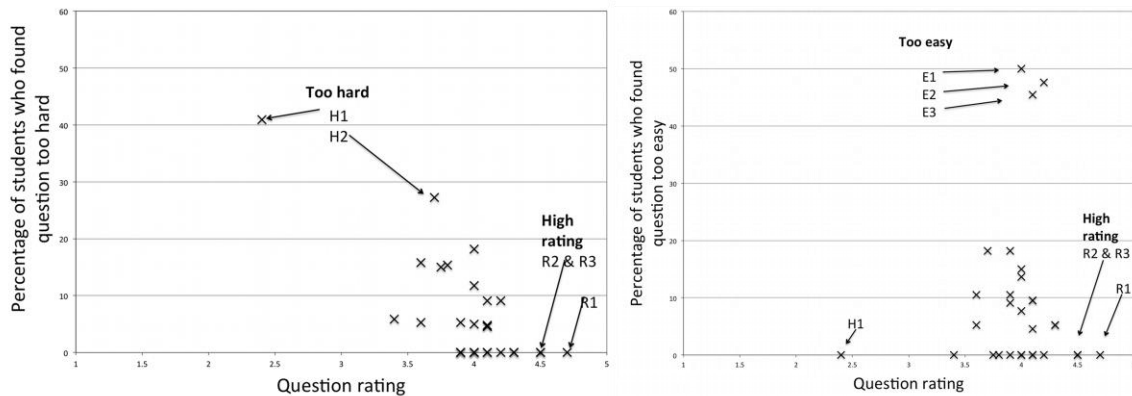
**Figure 1:** Percentage of students who found a question too hard or too easy versus question rating.

Questions that stand out from the others are labelled in Figure 1. Two questions stand out for being too hard (H1 and H2), three for being too easy (E1, E2 and E3), and three for having the highest question ratings (R1, R2 and R3). These eight questions will be examined in more detail.

### Hard questions

Two questions that stood out as being too hard will be considered in this section.



**Figure 2:** Question H1

The question labelled H1 (see Figure **2**) had the highest percentage of students (36%) finding it too hard and nobody found it too easy. It also had the lowest question rating of 2.4, the only rating that was below 3, the midpoint on the Likert scale. Only one submission was correct although three pairs worked through most of the problem correctly but made a minor error in the final step. One pair stated on their submission that they couldn't do it and five students reported in the questionnaire that they didn't know what to do. Students' comments on the question evaluations included "Frustrating" and "Too much thinking required". The observer also noted that this question was too hard and that many students needed help from the teacher to get started.

For this question, students needed to understand and apply several concepts about the order of carrying out operations. It may be better to separate it into two smaller questions, for example, $- - 1 - (3 - - 2)$ and $- 4 * - 3 \wedge - 2 + 3$. There would be fewer concepts in each question, and it would be easier for students to identify all their errors because this is difficult when errors affect subsequent steps.

Although 36% (8 out of 22) of students found this question too hard, 55% said this question was useful for their learning and all students reported that it made them think. It appears that students found it useful to view and discuss a variety of errors after finding a question hard.

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Functions and truth tables

1. Complete the truth table
2. Simplify the expression for the function

$f(x, y, z) = xy\bar{z} + \bar{x}\,\bar{y}z + xyz + \bar{x}\,\bar{y}\,\bar{z}$

=

=

| $x$ | $y$ | $z$ | $f(x, y, z)$ |
|---|---|---|---|
| 1 | 1 | 1 | |
| 1 | 1 | 0 | |
| 1 | 0 | 1 | |
| 1 | 0 | 0 | |
| 0 | 1 | 1 | |
| 0 | 1 | 0 | |
| 0 | 0 | 1 | |
| 0 | 0 | 0 | |

**Figure 3:** Question H2

For the question labelled H2 (see Figure 3), only 11 out of 22 students submitted a question evaluation. Of these 27% reported that it was too hard. However, 18% found it too easy. There were 12 answers submitted so either one pair submitted two different answers or there were two students working on their own. Nine of the submissions were correct. In one of the incorrect submissions, the table was not filled in and the teacher noted at the time that the layout of this question needed improving. The observer commented that several students needed the layout explained before being able to get started.

Students may have had trouble starting this question because of the way it was structured. The Boolean expression that was needed to complete the truth table was given after the truth table. It may have been better to guide students by providing information in the order that it would be needed.

Although the low number of question evaluations submitted and high number of correct submissions means this question may not have been as hard as it appears on Figure 1, this question provides an example of a key observation made earlier in this study. It is important to provide a structure that guides the students as to the format of the answer required and encourages them to get started correctly.

### Easy questions

Three questions that stood out as being too easy will be considered in this section.

Strategy

$3x + 2y - 4z = -7$    (1)
$2x + 8y - 3z = -9$    (2)
$5x - 4y + 2z = 18$    (3)

Which strategy is best?

A. Eliminate x

B. Eliminate y

C. Eliminate z

**Figure 4:** Question E1

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Students found easy questions helpful. For Question E1 (see Figure 4), half of the students found it too easy, yet all of them said it was useful for their learning. So it appears that this question, which was designed to encourage students to think about and choose a good strategy before working out a problem, helped most students learn even though half of them thought it was too easy.

The teacher observed that the multi-choice nature of the answers provided no indication of students' reasoning. Nine out of ten submissions identified the shortest strategy with the easiest calculations, but it was not until a class discussion of the other submission that students' misunderstandings were revealed.

This question is really a concept test as defined by Mazur [4] as it is a multi-choice question that tests understanding of a particular concept. Mazur's activity sequence for clickers incorporates students' justification of answers. With multi-choice questions on tablet PCs, more in-depth peer and class discussion may occur if students are also asked to justify their answer.
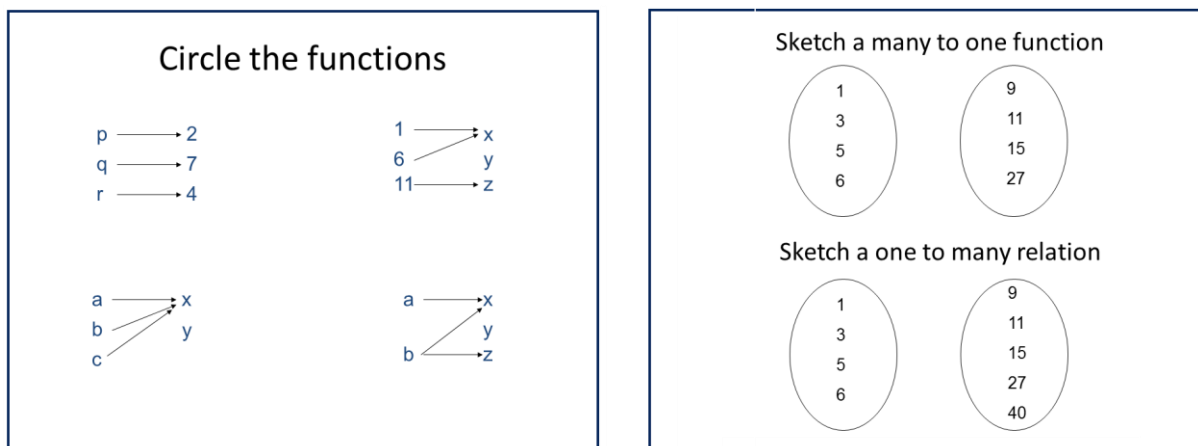


**Figure 5:** Questions E2 and E3

The data for Questions E2 and E3 (see Figure 5), also illustrate that students can find easy questions useful. Although 50% of students found Question E2 too easy, all student submissions were correct, and all students found it useful for their learning. In Question E3, 28% of students found it too easy, although one student found it too hard. Like Question E1 and E2, all students found it useful for their learning.

The observer noted that students were particularly interested in other students' submissions for Question E3. This is an example of an open question as many different correct answers are possible. Although some students felt it was too easy, it may have been at the right level for all students to learn from either creating their own answer or viewing other students' answers. It may also be that completing easy questions contributed to students' confidence. There is potential to further investigate the reasons for students' high rating of easy questions.

### Highly rated questions

The three questions that stood out because they had the highest question ratings will be considered in this section.

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For each problem, state whether it is about permutations or combinations and why.

1    15 students apply for 6 different jobs. In how many ways can these jobs be filled?

2    There are 10 different cars but only 6 car parks.

    (a)    How many different ways can the cars be parked?

    (b)    How many ways can 6 people get a park?

**Figure 6:** Question R1

Question R1 (see Figure 6), which was designed to alert students to common misconceptions, stood out as it received the highest question rating of all questions at 4.7. In addition, all students said it was useful for their learning because it focussed on a specific point and challenged them at just the right level.

It appears that being asked to decide whether a question involves permutations or combinations helped students to focus on this point. Yet, fewer than half (4 out of 9) of the pairs of students submitted an answer and no submission answered the question fully by including a reason. Thus, the number and nature of student submissions for this question provided useful and immediate feedback to the teacher about the students' stage of learning.

1    You are looking at cars on Trademe. What sort of relationship would you expect between age and price?

2    You are looking at cars on Trademe. What sort of relationship would you expect between age and mileage (kilometerage!)?

**Convert binary <—> hex**
Complete the table

| Binary | Hexa-decimal |
| --- | --- |
| 1011 1101 | |
| | 2F |
| 1101011110.01011 | |
| | BC |

| A | B | C | D | E | F |
| --- | --- | --- | --- | --- | --- |
| | | | | | |

8 4 2 1

**Figure 7:** Questions R2 and R3

Both Questions R2 and R3 (see

Figure **7**) had the second highest question rating of 4.5. All students found Question R2 useful for their learning because it made them think, and 93% reported that it challenged them at just the right level. All pairs submitted correct answers and students were exposed to several different ideas when the submissions were displayed and discussed. The teacher and observer noted that the variety of answers led to more discussion and prompted students to further thinking. The open nature of this question may have contributed to its usefulness for students, by encouraging in-depth discussions.

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Question R3, on the other hand, was a closed question that included a table for writing the answer and hints that may prompt peer discussion. All students recorded that it made them think and challenged them at the right level. This is an example of a high rating for a standard type of question for this topic. Like all questions in this study, it was chosen as being suited to the type of content and for a pedagogical reason, in this case to help students become familiar with a new concept by practising a standard skill. It would be easy for teachers to prepare this type of question as they can start with existing questions and only need to consider adding a structure that encourages students to get started.

In summary, the highest rated question was designed to address common misconceptions and the next two highest were an open question and a standard type of question with a table. Thus, one of the three highest rated questions was a traditional question and the other two were designed to refine or deepen students' understanding of a concept. The latter will be termed "thinking questions" and include the last five types of question in Table 1, i.e. best strategy, common misconceptions, matching, and open questions with or without a diagram.

Thinking questions are over-represented in the highly rated questions as only eight were evaluated and two of these were in the top three. This compares with twenty standard questions, with just text or with tables added, that were evaluated but only one was in the top three. Therefore, it is worth considering all the thinking questions that were evaluated in this study. The mid-point on the Likert rating scale was 3.0 and the average question rating for thinking questions was 4.2. This compared with 4.0 for questions with a table, and 3.7 for questions with text only. It appears that students found thinking questions particularly useful to their learning. In addition, the observer and teacher reported that these questions prompted deeper discussion.

## Guidelines for writing questions

A few questions stood out from the others as being too hard, too easy or having a particularly high question rating. It appears that several question characteristics may improve students' perceptions of the usefulness of a question for learning. These characteristics are summarised as a set of guidelines for writing questions. They are not a complete set of guidelines but rather those that arose in this study and should be read in conjunction with the guidelines in the Background section of this paper on which the questions in this study were based.

- Separate a question into two if it requires students to apply too many concepts.
- Provide a structure for students' answers that encourages correct solutions.
- Include easy questions as students can find these useful.
- Ask students to justify their answers to multi-choice questions so that their strategies can be seen.
- Write questions that challenge students at the right level.
- Write questions that focus on a specific point.
- Write questions that encourage students to think, by incorporating different strategies and common misconceptions.
- Write open questions to stimulate in-depth discussion.
- Choose a type of question that suits the content.

## Conclusions

The purpose of this preliminary study was to find out about types of questions that students find useful for learning. However, it was clear from the question evaluations from students that most students found most questions helped them learn. This result is important as it is

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much easier for teachers to start with questions that they normally use in class and format them for tablet PCs than to write the multi-choice concept tests that suit the use of clickers. The result also provides general support for following the principles used in this study when writing questions for tablet PCs, especially the principle that questions should be written with a pedagogical goal in mind as recommended by Anderson et al. [9], and also the principle of applying relevant guidelines from those designed for teachers using digital ink when lecturing [15] to questions designed for students using digital ink on Tablet PCs.

A number of questions stood out from the others and analysis of these led to suggestions that were summarised as a set of guidelines for writing questions. This study focussed on student perceptions and was also limited by the small sample size, and lack of information from students who chose not to submit all equation evaluations. Furthermore, analysis of question type was done after the data was collected so the proportion of each question type evaluated did not reflect that of all questions used in the course, and two question types not evaluated.

The guidelines can now be explored in more depth, for example by investigating rewritten versions of the questions that were too hard or too easy, and the nature of questions that generate in-depth discussion. Limitations and results of this study can also be taken in to account in the research design of further study.

When combined with the recommendations from other researchers in the Background section, the additional guidelines in this study will help teachers write questions for use with tablet PCs that students may find useful for their learning. This has important applications to the increasing use of technology, such as tablets and "bring your own device", in interactive learning activities.

## Acknowledgements

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**Appendix**



**Figure 8:** Evaluations completed by students on tablet PCs after answering selected questions

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# Mathematics and Ocean Swimming

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**Keywords:** poor mathematics preparation; failure; blame.

Mathematics is often taught in first year as a service subject. It is important that mathematics academics provide a good service to those whose students they teach. The income of many mathematics groups in universities in Australia largely depends on this teaching. At times mathematics academics are seen as not succeeding in this teaching and are blamed for the lack of skills of the students taught, or blamed for not being able to pass more students.

It is claimed here that mathematicians are often given a very difficult task, that learning mathematics has some aspects of what has been called "complex learning" and that some mathematics students are involuntary learners. It is up to mathematicians to educate those whom we serve about the challenges faced and about what is realistic for their students. An analogy which might assist in this is presented.

## 1        Introduction and background

Mathematics[1] is considered important for many students. There is agreement that mathematical knowledge is necessary in engineering [1] and science [2, p. 6],[3], including the life sciences [4, 5, 6, 7, 8, 9]. The importance of mathematics in ensuring the continued prosperity of Australia is clearly stated in [10, p. 4]. Mathematics is usually a compulsory part of any bachelors degree focussing on business, economics, science, management, IT and marketing to name a few. Yet more students in Australia are dropping mathematics in the final years of secondary school, and of those who study mathematics, the proportion choosing the lower level options is increasing [11, p. 6,19,20], [2, p. 3,5]. This has even been noted in the Australian national media [12].

Australian industry groups have shown concern over the lack of mathematical and other skills, as can be seen in The Industry Skills Councils' report on language, literacy and numeracy skills [13, p. 6]: "Australia is also unique among all participating countries in that students with an immigrant background in Australia now outperform students without an immigrant background." Similarly, in the UK "37.6% of employers reported numeracy problems in the current workforce." [14, Exhibit 13].

Because of the importance of mathematics in many areas, at university many students study mathematics as part of a degree that is not a mathematics degree. Consequently, much service teaching is done by mathematics staff and considerable income is gained from this for the mathematics school or department.  As 60% of Australian Government research and development funding goes to Australian universities and most of that is influenced by what undergraduate students study [15], mathematics undergraduate teaching, which is largely service teaching at some universities, supports a significant amount of the mathematics research in Australia.

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[1] "Mathematics" here means "mathematics and statistics".

## 1.1    *Mathematics as a complex learning outcome*

It is sometimes said that "mathematics is hard". In one sense it is; for someone who knows no mathematics at all it takes some time to get to university level. Children start learning about numbers when they begin school; 12 years later some come to university to continue their study of mathematics. Mathematics takes time to learn. Taylor writes [16, p. 298]: "This aspect of mathematics is reflected in its hierarchical nature and its long history. This in turn contributes to the fact that the study of mathematics, like the study of foreign languages, is inherently difficult."

Mathematical knowledge has some of the characteristics of what Yorke and Knight [17] call a "complex outcome", though their focus was very different (employability). They use "complex learning" to refer both to the outcomes and the processes that promote them. Complex learning has four characteristics (p. 569–570). Of these, two clearly apply to mathematics:

(1)    It is advanced, in the sense that it involves mastery of large amounts of abstract … material, and the discarding of primitive understandings in favour of more sophisticated understandings ….

(2)    It takes a long time and sustained practice.

Yorke and Knight continue (p. 571) "More broadly, the promotion of complex learning involves students being exposed to learning experiences of progressively increasing complexity, with cognitive 'scaffolding' … being removed as the students' expertise and their metacognitive capacity develop." This certainly applies to mathematics.

## 1.2    *Two challenges*

The complex and long term nature of learning mathematics presents challenges. Cockroft [18, p. 100] writes:

> It therefore seems that there is a 'seven year difference' in achieving an understanding of place value which is sufficient to write down the number which is 1 more than 6399. By this we mean that, whereas an 'average' child can perform this task at age 11 but not at age 10, there are some 14 year olds who cannot do it and some 7 year olds who can.

A seven year difference at age 11 suggests a bigger difference at age 18 when many students come to university. Many universities in Australia have few or no mathematics prerequisites for most degrees, and many take in mature age students who might not have studied mathematics for a decade or more. Unless a university measures the level of mathematical ability of incoming students and streams them in first year mathematics, one has excellent students who have done Advanced[2] mathematics in their final year of secondary school in the same class as students who have little grasp of any secondary school mathematics.

There is evidence that taking lower levels of mathematics in secondary school has a huge effect on performance in a first university mathematics subject. Rylands and Coady [20] give an example of a first year basic tertiary mathematics subject in which all students who had completed Advanced mathematics at secondary school passed the subject, whereas most who had completed Elementary mathematics the year before failed. The difference in achievement and understanding is clearly significant among those who studied mathematics

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[2] Advanced, Intermediate and Elementary for Australian secondary school mathematics subjects are defined in Barrington [18, p. 1].

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at secondary school the previous year. However, in a first year university subject there will also be those who have not done mathematics for over a decade. This presents a huge challenge for mathematics teachers; coping with near innumerate students in the same class as with mathematically mature students who have excellent skills; it's more than a "seven year difference", it is the "deadly divide".

The fact that many students studying mathematics in first year do not intend to graduate with a mathematics degree and whose priorities are elsewhere can present another challenge. Some students find themselves having to learn some mathematics without wanting to do so; they are involuntary learners. O'Grady and Atkin [21] studied the effects of incentives (financial rewards) and sanctions (possible withdrawal of allowances) on voluntary and involuntary learners enrolled in training programs. A pilot was successfully completed, with involuntary learners responding to the sanctions and incentives, and the government continued with the incentive/sanction scheme. However, the authors note that "making an adult attend a training programme through the use of directions and sanctions does not, for a significant number of attendees, mean full engagement and participation with the programme."

Universities do not offer financial rewards for success in mathematics subjects, nor do they withdraw allowances for failure; universities are more constrained than the government body offering the program studied by O'Grady and Atkin [21]. We thus have the involuntary learner problem on top of the deadly divide.

We could fail many students or we could drop standards. Dropping standards does not serve the good students, those for whom we teach, and society as a whole. If we aim low so that the weak students can succeed, are we being fair to the good students and do we help them to reach their potential? When we fail many students we can be blamed for poor teaching.

### 1.3    Blame

The challenges of teaching diverse groups of students, poorly prepared students and involuntary learners often lead to high failure rates. Mathematics groups and academics can be blamed for high failure rates; they can be pressured to lower standards and told that they are not good teachers. Examples of those who teach mathematics being blamed when students fail can be found, even though one would expect that most statements blaming teachers are not published.

Middleton and Spanias [22, p. 77,81] cite some studies from the 1980s in which secondary school teachers are blamed: "Students tended to derive satisfaction from a task when they were involved in successful work, and they tended to blame their dissatisfaction on the teachers." From Rooney [23, p. 13] in 1998, again in school: "The blame for negative perceptions focused on the teacher who was described as the main problem". Even recently in Nigeria an article appeared in the Vanguard blaming teachers for failure in mathematics: "Expert blames teachers for failure in Mathematics" [24].

In the US, on the large number of tertiary students having to do remedial mathematics, Kirst [25] writes: "Many blame high schools for a lack of academic rigor . . . ". However, some blame universities, as is seen as Kirst continues ". . . but high schools contend that they are unaware of the content or stakes concerning university placement. Some policymakers contend that high remediation rates are caused by lax admission standards that do not encourage a rigorous college-prep curriculum." A similar claim has been made about the UK [26]: "Some universities do not advertise the level of maths needed to comfortably study particular subjects for fear of hindering applications." That could be the case in

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Australia, take for example, Belward et al. [27] on the lack of mathematics prerequisites for entry to a Bachelor of Science in many Australian universities.

It is easy to blame, but what about the challenges, the involuntary learning and the deadly divide, described in the previous section? Surely too much is expected of those teaching mathematics in many circumstances. However, do academics who are not mathematicians, and whose students mathematics academics teach, understand this or are they willing to listen? We claim here that sometimes the answer is no. Academics are good at learning and at succeeding academically. However, not all academics are physically fit or good at sport. Perhaps those non-mathematics academics will see some of the challenges of teaching swimming to unfit non-swimmers who hate water; this can serve as analogy for the problems of teaching mathematics to poorly prepared involuntary learners. Imagine that you are unfit, can't swim and hate water, and that on a cold morning you are standing at the edge of a pool wearing only your swimmers knowing that you have to jump in and swim one kilometre of butterfly.

Let's develop this analogy; maybe non-mathematics academics will listen if we talk about something other than mathematics.

## 2       The challenges of teaching ocean swimming

The above is rather dry reading, so this section presents the far more interesting fictional challenges of the School of Sun, Sand and Surf, Faculty of Water, Waves & Wishful Thinking, Underwater University, Australia. The purpose is to share my experiences with the subject *Ocean Swimming 101*, and in particular to discuss some of the challenges encountered. The subject is seen by a few students to be "a doddle", but the majority perceive it to be very difficult and so avoid it if they can. The subjects *Basket Weaving for Elephants* and *Tap Dancing for Reindeers* attract many students who would really be better prepared for life in Australia if they chose *Ocean Swimming 101*.

### 2.1     The purpose of the subject: why various student cohorts take it

*Ocean Swimming 101* is core in the programs Bachelor of Fishing (Rock Fishing), the Bachelor of Surfing, the Bachelor of Indigenous studies (Tropical Islanders), the Bachelor of Diving and the Bachelor of Shark Husbandry. It is also a prerequisite for the subjects on Indigenous Fishing, Shark Care and Ocean Rescue, among others. Some students take it because they love swimming.

### 2.2     Assumed knowledge

We prefer students to have succeeded with at least Intermediate swimming in secondary school. Unfortunately many have only done (and often done poorly) the Elementary subject Swimming with Floaties. We do, however, always get a few students who have done Advanced swimming. These should be a joy to teach, but our attention and curriculum is directed to the majority of very poor swimmers, so the good swimmers are often frustrated by the level of most of the subject.

We find that fewer and fewer students are doing Advanced swimming at school. We have anecdotal evidence that students are being advised that if they are not aiming for the Olympics then they will be better off doing Intermediate swimming or Swimming with Floaties, as it will improve their university entrance score. That the students are at a higher risk of drowning later in life seems not to be a consideration.

We would, of course, prefer to have swimming prerequisites for incoming students. Staff cannot cater for the weaker students; those who did poorly at secondary school can

sometimes not swim at all, or can only swim a few metres and will drown if left in deep water. However, like most universities, prerequisites were removed some time ago and there is no prospect of them being reintroduced soon.

## 2.3    Assessment

The final ocean swim is a mandatory assessment component. Students must pass this in order to pass the subject. Students are dropped one kilometre offshore and are required to swim to shore in under one hour. As readers will be aware, a top swimmer can swim one kilometre in about 10 minutes, so this assessment item should not be difficult.

However, it is this final swim that deters many students. Some of us have overheard conversations much like "Ocean Swimming, that's too hard. I'm doing Basket Weaving for Elephants. It's all essays and I can waffle my way through the assignments, *and* there is no final swim."

## 2.4    The big problem

*Ocean Swimming 101* is not a popular subject, and the failure rate is over 40%. Why is this so? Staff in the School of Sun, Sand and Surf (SSSS) are finding more and more that students are very poor swimmers, and that over time they are getting worse. We all know that early high school students are capable of learning to swim one kilometre in a pool, yet many of our intake have not learnt to do this. They have avoided swimming—"it's too hard". They are not encouraged to put in the time required to become competent swimmers. They are told that it is OK to be a bad swimmer, to be unfit, to be physically weak, and to be afraid of water. Some even proudly boast that they are hopeless swimmers. I recently heard an announcer on the radio say unashamedly that she was a good cricketer, but couldn't swim a length of the pool.

Without basic swimming skills it is not possible to achieve the outcomes of *Ocean Swimming 101*, as they build on the swimming skills that students are assumed to have. It is completely pointless to try to teach someone bilateral breathing or to cope with navigation in a large swell when they can barely swim 20m. In fact, it is worse than pointless as it makes many students feel inadequate and destroys their self-confidence.

Ocean swimming is not just a collection of skills that anyone can master in a week, these skills can only be gained by those who already have a solid base. Bridging courses have been tried as a bandaid measure, however, we all know that unfit students with no muscle tone or aerobic fitness and no technique can at best improve by an infinitesimal amount in one or two weeks.

## 2.5    Failure rate, the "Drown Rate"

We usually fail around 40% of students. What can be done?

(1) Lower standards. It has been suggested that the final swim be held in a local pool, or replaced by a 50m ocean swim. Both these options lower the level to that of a secondary school subject, and do not prepare students for the degrees containing this subject, nor do they prepare the students for the subjects for which this is a prerequisite. Without something equivalent to the final ocean swim we can't ensure that the subject outcomes have been achieved. The same goes for the suggestion that passing the final swim not be required for a pass in the subject.

(2) Improve our teaching, use innovative teaching methods and flexible learning. This ignores the reality. Many of our students do not get into the water. For some, if they

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do they sink because they have very few skills. When it comes to the crunch they put off the necessary work, or they start, and as soon as they find the water cold, or are a little breathless, they give up or put off the work for next week. Those that are weak must develop the stamina, technique and physical ability necessary to swim one kilometre. Many of our students do not have these attributes and will not develop them without much hard work and time over the 13 weeks of semester. It is just impossible to develop these skills in a few days of cramming.

(3) Replace the final swim with a sequence of shorter ocean swims. This has been suggested, but then it was changed to "strongly encouraged" to do this. Staff of SSSS unanimously vetoed this proposal as we all know that the only students who will act on this are the keen ones. The students who need most to swim are the ones most likely to ignore any encouragement. The argument that killed this proposal was "if you are out beyond the breakers and get into trouble, do you want your life to be in the hands of someone who has only been encouraged to swim, or someone who has actually proved that they can swim one kilometre to shore?"

(4) Have several mandatory short and easy swims early in the semester. A pass in these is necessary for a pass in the subject. These were suggested so that students would know if their skills were not enough for the subject. It was hoped that the threat of forced withdrawal would scare some of them into the water for some practice, and so give them a better chance of achieving the subject outcomes. It was then suggested that students be "strongly encouraged" to do these swims. SSSS staff then dropped the whole idea as those who needed it most they said would be the least likely to do anything if only strongly encouraged.

One might at this point think that *Ocean Swimming 101* is a very difficult subject. However, some of our students find it so mind numbingly boring that they do not engage, fail to pick up the few (easy for them) skills necessary to pass the subject, and fail. They know that they can swim one kilometre, but without some work they will not learn bilateral breathing or the other necessary skills, and often lose tone and fitness during the semester.

For a reasonable Advanced swimming student we could, if we could target our teaching at these students, have them achieve all outcomes by midsemester. We do such students a disservice by treating them as though they know nothing of swimming, and giving them work much below their abilities. We are disappointed, but not surprised, when these students disengage and fail the final swim, or are not even aware that it is mandatory.

## 3    Conclusion

Those who teach mathematics are sometimes blamed for the lack of learning of some of their students. This has been happening for many, many years, in many contexts and at various levels. As Australia increases the proportion of the population with a bachelor's degree this situation will only become worse as the deadly divide becomes wider. Mathematics can be difficult to learn for some, requiring much time and effort, so students who are many years behind will most likely still be many years behind after a semester of mathematics, no matter how engaging the subject and how many innovations used. This challenge is compounded if weak students are in the same class as excellent students.

It is up to those who teach mathematics at various levels to explain to those to whom we provide a service what the challenges are and why, for some students, we can't do what they want in the time given. No one else will defend us. The ocean swimming analogy may be of use in explaining some of our difficulties. Until those outside mathematics understand

the current challenges in teaching undergraduate mathematics, discussions on realistic solutions for dealing with poorly prepared students cannot begin. Realistic solutions could include not allowing poorly prepared students entry to bachelor's degrees containing mathematics, or requiring poorly prepared students to enrol in and pass mathematics at a foundation college or similar (perhaps studying for several semesters) before beginning bachelor degree studies.

Two other useful analogies are learning to speak a language and learning to play a musical instrument—how many of us could learn to speak fluent Chinese or learn to play the piano in a semester?

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# Letters & Numbers: A Vehicle to Illustrate Mathematical & Computing Fundamentals

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The television quiz program *Letters and Numbers*, broadcast on the SBS network, has recently become quite popular in Australia. This paper explores the potential of this game to illustrate and engage student interest in a range of fundamental concepts of computer science and mathematics. The Numbers Game in particular has a rich mathematical structure whose analysis and solution involves concepts of counting and problem size, discrete (tree) structures, language theory, recurrences, computational complexity, and even advanced memory management. This paper presents an analysis of these games and their teaching applications, and presents some initial results of use in student assignments.

## 1. Introduction

The television quiz program *Letters & Numbers*, broadcast on the SBS network, has only recently become quite popular in Australia. In various European countries the same essential format (with relatively minor variations) has been popular in Europe for decades; in the UK it is known as *Countdown*, while in France it is *Des chiffres et des lettres*.

This paper explores the game's potential as a vehicle to illustrate and engage student interest in a range of fundamental concepts of computer science and mathematics. The Numbers Game in particular has a very rich mathematical structure whose analysis and solution involves concepts of counting and problem size, discrete (tree) structures, expression trees, expression tree evaluation, recurrences, formal language theory, parsing and generation, computational complexity, and even advanced memory management.

The fact that so many significant computer science concepts can be presented in the context of this game makes it stand out as an example problem to use for discussion and analysis. It can even serve as a running example to demonstrate different topics arising in the same class. Finally, since it is a game, popular on television, it has engagement appeal as a class example. Students tend to like games, even maths games, as part of learning new theory [1]. An analysis of the *Letters & Numbers* game and its teaching applications in Discrete Mathematics, Mathematics of Computing, and Computer Science is presented. Also discussed is use of the game as a basis for assignment work in a Discrete Mathematics class at Bond University.

This paper is organised as follows. Section 2 discusses the Letters Game. Section 3 discusses the Numbers Game, the underlying language of it, its size, and the computational cost of simple solution procedures. Section 4 describes three programming implementations of Numbers Game solvers. Section 5 discusses the teaching applications of the Numbers Game, and Section 6 discusses the experiences of using the Numbers Game as an assignment in an introductory Discrete Mathematics class. Finally, Section 7 offers our conclusions.

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## 2. Letters game

Nine letters of the alphabet are chosen, with one contestant choosing a sequence of *vowels* or *consonants*. Duplicates are possible. Then both contestants are given 30 seconds in which to form the longest possible English word from a subset of the chosen letters. Such words must either exist in the standard dictionary used on the program or be well-defined extensions of dictionary words. To solve the problem on a computer is straightforward. Given electronic dictionary files (just words as simple text), it is a relatively trivial matter to write code to scan the dictionary for acceptable words. An obvious approach is to first compute the frequency of each letter in the original collection of nine. This information may be stored conveniently as an array, $A$ of 26 non-negative integers. As the dictionary scan proceeds, a similar array $B$ is computed for each word encountered. If, for any dictionary word, each element of $B$ does not exceed the corresponding element of $A$, then we have a "hit" and that word is output as a solution.

There is nothing very profound here, but it can be a useful programming exercise for elementary programming subjects. The first author has used essentially this problem as a programming exercise for undergraduates over several years in the 1990s at Bond University, and details may be found in [2]. Algorithmically, this approach is linear in the size of the dictionary. While potentially more efficient approaches using only permutations of subsets of the original nine letters and a dictionary encoded in an efficient lookup structure like a retrieval tree [3] can be imagined, they can only offer a constant factor speed-up. The linear cost on the size of the dictionary is not prohibitive enough to warrant the extra effort and complexity given the speed of modern processing.

The letters part of the game is not discussed any further, except in the discussion of its use in a student assignment at Bond University in Section 6. It is the numbers part of the game which has a much richer mathematical structure, and that forms the subject of the remainder of the paper.

## 3. Numbers Game

In this part of the game, one contestant is invited to choose a "combination" of six numbers. In normal mathematical parlance, the term "combination" refers to a *subset*; however, in this game, it does not refer to a set at all, but rather to two set cardinalities. Even this is not quite true as the collection of numbers ultimately chosen may contain duplicates. For example, in response to the invitation to choose a combination, a contestant may typically say "two large and four small numbers". These are references to two underlying sets of possible small and large numbers, respectively S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and L = {25, 50, 75, 100}.

In fact it is not the contestant but "Numbers Queen", Lily Serna, who chooses the numbers by selecting the specified quantities of large and small numbers from two separate collections of face-down cards. Duplicates are possible, so selection may be regarded as being with replacement. For the choice "two large and four small numbers", an example of the final "set" of chosen numbers might be {2, 3, 2, 7, 50, 75}. Strictly, this is not a set but a *multi-set* (some authors use the term *bag*), since duplicates may be present.

Extreme choices such as all large or all small are permitted but rarely chosen, as there seems to be a perception that such extremes may make the next part (generating the target number) more difficult. Frequent "combination" choices are three each of large and small, the so-called "family mix" of two large and four small, and less commonly, the "rat-pack" of six small numbers.

Once the six numbers (hereafter called *operands*), with possible duplicates, are selected, Lily then presses a button, which presumably directs a computer to generate a pseudo-random three-decimal-digit number (hereafter called the *target*) from 101 to 999

inclusive. Contestants then are given 30 seconds to find a way of combining any or all of the six operands, with the usual arithmetic operations of addition, subtraction, multiplication and division, and parentheses where necessary, with the aim of generating the target. There is no special benefit to players in using more or fewer operands; rather the aim is to generate the target exactly, or within 10 either side. It is not explicitly stated, but nevertheless clear that no intermediate result may be non-integer, or negative.

## 3.1.  *Problem size and Catalan numbers*

Suppose we wish to generate arithmetic expressions with $n$ operands. The corresponding binary expression tree has $n$ operand nodes (leaf nodes) plus $n-1$ operators (internal nodes), giving $2n-1$ nodes in total. Consider now the *interior tree* generated by the $n-1$ interior operator nodes. The number of distinct such trees is given by the Catalan number of index $n-1$, which we denote $C_{n-1}$.

$$C_{n-1} = \frac{(2n-2)!}{(n-1)!n!} \qquad (1)$$

There are four possible operators and these may be chosen $n-1$ times with replacement. Thus, the number of choices here is $4^{n-1}$. Now graft $n$ operand nodes onto this tree. Since order matters and operands can only be used once, the number of ways of doing this is the number of permutations of $n$ objects chosen from $6$. This is

$$\frac{6!}{(6-n)!} \qquad (2)$$

The multiplication rule now gives us the total number of trees with $n$ operands:

$$T_n = \frac{(2n-2)!}{(n-1)!n!} \times 4^{n-1} \times \frac{6!}{(6-n)!} \qquad (3)$$

This expression ignores the possibilities of semantically equivalent trees. It also ignores unsuitable, semantically invalid trees such as those which generate division by zero or improper final or intermediate results. Examples are $5-7$ or $25/4$ or $10/(3-3)$. In our various implementations, the code checks for these and avoids constructions of such (sub)trees.

The expression on the right side of Equation (3) cannot be simplified in any significant way, however if we consider the quotient $T_n/T_{n-1}$ massive cancellation occurs and, with $T_1 = 6$, the useful recursive form of Equation (4) results.

$$T_n = \frac{8(2n-3)(7-n)}{n} T_{n-1} \qquad \text{if} \quad 2 \le n \le 6 \qquad (4)$$

The number of trees as given by either Equation (3) or Equation (4) is shown in **Error! Reference source not found.**.

**Table 1:** Number of expression trees with $n$ operand nodes

| $n$ | $T_n$ |
|---|---|
| 1 | 6 |
| 2 | 120 |
| 3 | 3,840 |
| 4 | 115,200 |
| 5 | 2,580,480 |

| 6 | 30,965,760 |
|---|---|

### *3.2.    Efficiency of solution generation*

A brute force approach to solving the Numbers Game is simply to generate every possible expression, or equivalently, every tree described in Section 3.1, and then evaluate them. Clearly, this is computationally expensive, with cost proportional to the number of possible expression trees. Furthermore, this cost is not just in run time, but also in memory to store the viable structures. A naive Haskell implementation of this search (which uses copying to create new trees, and garbage collection instead of explicit deallocation) running on an Intel i5 processor with 4 Gb of RAM runs for several hours without showing any signs of completion. Clearly, such a naive approach is impractical.

However, there is much scope for optimisation of the solution generation process. First, division by zero is not defined, which introduces the notion of an invalid or illegal expression. If such an expression appears anywhere in the final expression tree, the whole expression is invalid. Hence, when an illegal intermediate expression is discovered, the construction of the remaining possible super expression trees can be halted as none will be valid. Also, while not invalid as expressions, intermediate values that are negative or proper fractions are never used in game play, which leads to the notion of an implicit rule that such expressions are illegal as discussed in the beginning of this section. In every case of our empirical solution generation, expression trees containing instances of these three kinds of invalid expressions account for well more than half of all the possible expressions given the operands, but for the most part represent an order of magnitude more possible trees. Not generating these trees yields significant savings in time and space cost and brings solution generation back into tractable realms of time and space cost, though the space usage is still very high: on the architecture described above, it takes approximately 40 seconds to generate all viable solutions and allocates over 79 Gb of heap space with maximum heap usage at any one time of nearly 0.5Gb.

Beyond invalid expressions, different intermediate expression trees can be constructed that evaluate to the same value. For example, given operands 1, 2, 3, 4, 5, 6, we need only generate one of $2 \times 3$ and $3 \times 2$. Trees representing these two simple expressions are not the same, but the trees both belong to the same equivalence class, with an appropriate definition of equivalence or equivalence relation. It is not difficult to exclude "duplicates" in the sense of this relation, however there are some more subtle forms of equivalence. Commutative equivalence can be checked syntactically, but general value equivalence requires the expression (sub) trees to be evaluated.

Even if equivalently evaluating expressions are not discarded in construction, significant savings in memory can be achieved by using *memorisation* [4]. Here, each unique (sub) expression tree is constructed only once and stored. When a sub-expression tree would be duplicated in another tree, a pointer into the expression tree storage is used instead. Clearly, this makes the solution generation much more complex, but results in massive memory usage savings.

### 4.  Implementations

The authors have created three implementations of solvers for the Numbers Game: one in Haskell, mentioned in Section 3.2, one in Excel/VBA, and one in Delphi 7. Each uses a modified exhaustive search algorithm.

The Haskell implementation is the most straight-forward and is a naive implementation of brute force generation of all possible solutions, without generating intermediate solutions representing invalid expressions as described previously. It takes about 40 seconds on a modern architecture to solve the Numbers Game for 6 operands, and uses a

great deal of memory. This is because new structures in Haskell are created from old ones by copying, and the forests of expressions trees are stored in ordinary lists. No effort was made to use Haskell structures more efficiently, which is certainly possible. The Excel/VBA model is again a brute-force implementation, but expressions are stored as string values in arrays, as opposed to tree structures in lists. This implementation also considers commutative expression equivalence and does not generate commutatively equivalent expressions. It may takes up to about 2 minutes to solve the Numbers Game on a modern architecture, although many examples from the *Letters & Numbers* program are solved in less than 20 seconds.

The Delphi implementation is a direct translation of the Excel/VBA model into Delphi, again using string values to store expressions and arrays to store collections of expressions. Interestingly, it ran about 100 times as fast as the VBA/Excel model, generating and checking for feasibility all possible 2,398,368 candidate trees/expressions in about 2 seconds on a modern architecture running Windows 7. Its significant out-performance of the Haskell implementation is due to its more compact and efficient string storage of expressions and use of arrays for collections rather than lists. Otherwise, all implementations are essentially the same algorithm.

## 5. Teaching Applications

The rich mathematical and computational structure of the Numbers Game make it amenable to use as an example in a wide variety of teaching applications for Discrete Mathematics, Programming, and Complexity Analysis. Such application areas are summarised in Table2 and discussed here.

### 5.1. *Arithmetic expression trees*

A topic we believe is accessible to first year students with only modest background in algebra is that of arithmetic expression trees (AETs). Such trees show the recursive structure of arithmetic expressions and allow abstract representation as a recursive structure (binary tree). This canonical form may then be flattened into a linear form (prefix, infix or postfix string) using various recursive traversal algorithms. A simple, stack-based algorithm can be used for evaluation of postfix expressions, where parentheses are not necessary to preserve meaning. A conventional infix expression becomes ambiguous when more than one binary operator is used. Only by convention does multiplication have precedence over addition, and parentheses are used to force order of evaluation, if necessary. The important distinctions between form and content, syntax and semantics can be pointed out.

### 5.2. *Counting and Recurrences*

Determining the size of the Numbers Game problem for a given number of operands is a non-trivial exercise in counting, as discussed in Section 3.1. It provides a practical example of using factorial in counting, and also introduces Catalan numbers. Analysing the number of different tree structures also serves as a vehicle to see the recursive nature of (expression) trees and provides a good example for studying recurrence relations. Even in a weaker class where such topics are not formally included, the Numbers Game offers an opportunity to showcase some of the principles of recursion and recurrence relations, and various means of computing them, and even a brief glimpse at their asymptotic properties [5].

### 5.3. *Data structures and algorithms*

Expression tree creation and manipulation is standard fare for a data structures class. The Numbers Game is a simple application requiring binary expression trees. Evaluating an expression tree is an example of post-order traversal. Creating the forest of possible

expression trees for a given set of numbers is an interesting application of permutation generation and recursion.

Also common in data structures classes is using a stack for postfix expression evaluation. Again, the Numbers Game is a good example on which to apply this algorithm. If the possible expressions are considered as strings, then the problem can be solved by generating the possible permutations of operators and operands, pushing them on stacks, and applying the stack evaluation algorithm. The (recursive) approach to generating all permutations may require the management of a collection of stacks-in-the-making, if added complexity is desired.

Finally, the collection of operands can require some linear structure such as an array or list, so solving the game can require multiple data structures and interactions amongst them.

### 5.4.    *Languages and language theory*

The language of possible expressions in the Numbers Game has very interesting aspects, which makes it a very useful example for teaching concepts of language theory. A simplified version of the game with a small number of operands (say, 2 or 3) can be used for examples of simple finite-state automata, but as examples that are not artificial sequences of a's and b's, as so many are. It may also be a useful example for instructors to discuss relative merits of regular expression notation and finite-state automata. The fact that the language is finite gives the theoretical result that it is a regular language, but it is a regular language without a satisfying representation using either regular expressions or finite-state automata, and can thus serve as a segue into more powerful means of defining languages.

Binary expressions are a classic example used when teaching context-free grammars (CFGs), as they require the interesting notions of precedence and associativity. Initially, it seems the Numbers Game language is just such an expression grammar, but the requirement that the operands be used only once introduces a multiple counting requirement that CFGs cannot represent. Again, this is a useful example for instructors to demonstrate the limitations of CFGs, and the need for yet more expressive language definition tools. So, again, there is a nice segue into the next level, attribute grammars, representing context sensitivity.

Finally, the Numbers Game poses the very interesting language question of generating strings in the language, rather than merely recognising and parsing them as discussed above. Such generation, of course, forms the basis of the brute force method of solving the problem computationally. This is a very challenging programming exercise in typical third generation/object-oriented languages like C or Java, requiring sophisticated use of several data structures and an algorithm for creating permutations of the operands. Such an exercise would be suitable for an advanced programming class. Alternatively, this example can also be used to highlight advantages of programming languages more suitable to the task with richer type mechanisms (such as Haskell), or with more language support for generating structures based on constraints (such as Prolog). This would be an excellent exercise for a comparative programming languages course.

### 5.5.    *Computational complexity and advanced programming*

The Numbers Game is an interesting teaching example in computational complexity analysis for two reasons. First, the time cost analysis[1] requires calculation of the number of possible expression trees, and therefore interesting application of Catalan numbers and recurrence relations. Secondly, it is an example with very significant space cost, which is not so common in standard data structures examples. The Numbers Game is an example for students where,

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[1] Linear in the number of possible expression trees.

even though the time cost may seem reasonable, the space cost definitely prompts exploration of more efficient solutions.

**Table 2:** Summary of Teaching Applications

| Topic | Course |
|---|---|
| Counting and problem size | Discrete Mathematics |
| Discrete structures and trees | Discrete Mathematics<br>Data Structures and Algorithms |
| Expression trees | Discrete Mathematics<br>Data Structures and Algorithms |
| Catalan numbers | Discrete Mathematics |
| Expression evaluation (Stacks and Trees) | Data Structures and Algorithms |
| Tree traversal | Data Structures and Algorithms |
| Finite State Automata | Discrete Mathematics<br>Programming Languages and Compilers |
| Regular Expressions | Discrete Mathematics<br>Programming Languages and Compilers |
| Computational complexity | Theoretical Computer Science<br>Data Structures and Algorithms |
| Formal grammars | Programming Languages and Compilers |
| Parsing and generation | Programming Languages and Compilers |
| Memory allocation and de-allocation | Programming Languages and Compilers |
| Garbage collection | Programming Languages and Compilers |
| Memoisation | Programming Languages and Compilers |

Algorithmic aspects of more efficient solutions have been discussed in Section 3.23.2, but additional memory manipulation aspects can be applied to the problem: explicit memory allocation/deallocation, garbage collection, and *memoisation*. These are important topics in advanced programming or compiler design classes, and again the Numbers Game serves as a useful running example to explore them. Apart from providing a programming application for these concepts, these aspects of improving efficiency provide a useful comparison between programming languages and a forum for discussing the current trend in programming languages away from explicit memory management and away from explicit pointers into memory. For example, in Java one cannot control memory allocation and deallocation, and in, say, Haskell or Python, one cannot implement *memoisation*. This is not to suggest that these languages are deficient, but rather to provide an interesting demonstration of the price paid for the expressive superiority of these languages.

## 6.  Experience using Letters and Numbers as a student assignment

In Bond University's Bachelor of IT, there is just one mathematics unit, known as Analytical Toolkit. It consists of a typical set of introductory discrete mathematics topics, ending with a few weeks of very basic introduction to probability and statistics. In the January semester of 2012, it was decided to set an assignment for the students based on the *Letters & Numbers* game. Since most of the class were first semester students, and, from past experience, their mathematical backgrounds are typically very poor, this exercise had to be carefully planned. Further, most students in this class do not have any programming experience. How could we present sufficient background material for the students firstly to understand the problem, and

secondly to implement some kind of a solution? It was decided that the idea could only become feasible if we:

1. reduced the scope of the problem by relaxing the Numbers Game to 3 operands instead of 6,
2. cast the problem into an environment where at least some of the solution logic may be expressed with having to write code, and
3. supplied the class with some skeleton code which outlines an overall solution strategy.

These were achieved by putting a reduced version of the problem into Microsoft Excel 2010 with VBA code and Excel formulae and tables. For the Letters part of the problem the class was supplied with public domain word lists, again in Excel. A session on elementary VBA was also taught.

To give an idea of what was supplied to the students, and what they in turn were required to produce, Section 6.1 presents an abridged version of the suggested steps for the Numbers part for the game. Figure 1 shows a sample snapshot of the Excel spreadsheet for the Numbers Game.

| | A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | x | y | z | lparen | rparen | operator1 | operator2 | target | Expressions | ExprValues | solutionIndex | solution |
| 2 | 1 | 5 | 3 | ( | ) | + | + | 20 | (1+5)+3 | 9 | 37 | (1+3)*5 |
| 3 | | | | | | + | - | ◄ ☐ ► | 1+(5+3) | 9 | | |
| 4 | xyzPerms | | | | | + | * | | (1+5)-3 | 3 | | |
| 5 | 1 | 5 | 3 | | | + | / | | 1-(5+3) | -7 | | |
| 6 | 1 | 3 | 5 | | | - | + | | (1+5)*3 | 18 | | |
| 7 | 5 | 1 | 3 | | | - | - | | 1*(5+3) | 8 | | |
| 8 | 5 | 3 | 1 | | | - | * | | (1+5)/3 | 2 | | |
| 9 | 3 | 1 | 5 | | | - | / | | 1/(5+3) | 0.125 | | |
| 10 | 3 | 5 | 1 | | | * | + | | (1-5)+3 | -1 | | |
| 11 | | | | | | * | - | | 1+(5-3) | 3 | | |
| 12 | Generate Expressions | | | | | * | * | | (1-5)-3 | -7 | | |
| 13 | | | | | | * | / | | 1-(5-3) | -1 | | |
| 14 | Clear Expressions | | | | | / | + | | (1-5)*3 | -12 | | |
| 15 | | | | | | / | - | | 1*(5-3) | 2 | | |

**Figure 1:** Excel model of Numbers Game for 3 operands

Based on student feedback, this exercise was a success. Further, Table 3 presents the results of a survey of students about the assignment.

### 6.1.  *Numbers Game recommended steps*

1. You are given a skeleton spreadsheet with essential areas already named. Please use these names, although you are free to define any extra names that you may need.
2. Note that at least some names are defined so that the VBA code may use them as a reference point for the Offset function. You may quickly check the cells determined by any name simply by clicking on it in the Name Box; alternatively use the Name Manager for complete details of any Name definition.
3. The user enters three numeric operands (numbers from 1 to 100) in A2:C2.
4. The target number must be from 1 to 100 and controlled by a slider (scrollbar).
5. Write formulae to put all permutations, without replacement, of the three input operands in the range A5:C10.
6. Put all permutations, with replacement, of the four fundamental arithmetic operations in the range E2:F17.

7. When the button Generate Expressions is clicked, your code must generate all 192 arithmetic expressions (corresponding to all binary expression trees) which may be formed from the three operands. Put these in the range H2:I193. Column H has expression; column I has the formulae which will compute the values of the expressions.
8. Column J uses an Excel lookup function to find the location of the first solution.
9. Column K uses this location to find the first solution, and displays it.
10. The user is expected to slide the scrollbar to search for solutions. When a solution appears, it must be automatically highlighted by use of conditional formatting.
11. Test your model with the input operands 3, 4, 5 and determine which targets may be generated.

**Table 3:** Student survey responses

| Question | SA | A | N | D | SD | # Responses |
|---|---|---|---|---|---|---|
| Before the assignment, I did not think of the numbers game as a structural problem | 0 | 5 | 1 | 0 | 1 | 7 |
| It was interesting to see the numbers game as a structural problem | 5 | 2 | 0 | 0 | 0 | 7 |
| Before the assignment, I did not see how a search for the solution could be constructed algorithmically | 1 | 1 | 3 | 1 | 0 | 6 |
| I could understand the algorithm, but I found coding it difficult | 1 | 2 | 1 | 2 | 1 | 7 |
| This assignment has increased my skills in applying patterns and structures to solve problems | 4 | 3 | 0 | 0 | 0 | 7 |
| I am competent at arithmetic | 0 | 7 | 0 | 0 | 0 | 7 |
| I enjoy arithmetic | 1 | 6 | 0 | 0 | 0 | 7 |
| I understand English grammar well and use it correctly | 3 | 3 | 0 | 0 | 0 | 6 |

### 6.2. *Student feedback on the assignment*

Although the class was small and the number of responses even smaller, feedback was uniformly positive, as shown in Table 3, and with the following comments.

1. *"Very interesting assignment for applying algorithms to solve problems."*
2. *"Found it to be quite fun and enjoyable however due to my programming background it was easy. Other new programmers might think otherwise as many people had difficulty with the syntax."*
3. *"Although there was a steep learning curve I found this assignment very interesting."*

### 7. Conclusion

The games from the show *Letters & Numbers* are demonstrated to be useful vehicles for computation and mathematical concepts, especially the Numbers Game which is very rich. The treatment of the Numbers Game as a simple syntactical structure problem, rather than a complex algebraic problem, is a tremendous metaphor for how computation is and can be applied to many seemingly unsolvable or difficult to solve problems. Concepts from a wide variety of typical Computer Science and Discrete Mathematics subjects in a degree

programme apply to the numbers game, making it a useful teaching vehicle with wide application.

This paper has presented an analysis of the show's games, along with a discussion of how it can used for teaching concepts ranging from simple counting and tree structures up to complexity analysis and advanced memory management. At every stage, the requirements of solving the Numbers Game strongly affirm why Mathematics is important in a Computer Science or Information Technology degree.

One final comment: since the time of writing of the initial draft of this paper, one of the authors has moved to a different university. Consequently, there is expected to be a much greater opportunity to test the ideas described herein with much larger numbers of students, and importantly, with many having significantly stronger mathematical backgrounds. The current plan is therefore to adapt the present material with a view to setting a more challenging assignment, and to make some attempt to measure learning.

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# A constructivist approach to mathematics laboratory classes

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This article discusses a framework for mathematics education that includes higher-order levels of learning. Theories of learning are also examined, with constructivism discussed in more detail. The constructivist philosophy is applied to structuring a computer-laboratory program in mathematics that differs from the traditional structure where the development of skills in a computer algebra system is a major objective. A structure is proposed of guided mathematical investigations, which makes use of dynamic visualisation, with the aim of helping students construct their own knowledge and develop conceptual understanding.

**Keywords:** constructivism; conceptual understanding; dynamic visualisation; animation; mathematics computer laboratory; flipped classroom

## 1    Introduction

Many tertiary mathematics courses include a computer laboratory component in their curricula. Maple, Mathematica and Matlab are popular choices for such programs, which usually run in parallel to the lecture series. Often the aim of the laboratory program is to consolidate lecture material as well as to teach students the basic skills of navigating particular software. Despite this usage of computers, the literature reports that the uptake of technology in the mathematics curricula of secondary and tertiary institutions has been slow and that there has been continued emphasis on pen and paper techniques, especially in many assessment tasks [1].

The graphics capabilities of desktop computers make them a valuable visualisation tool that can be exploited in curriculum designed to develop conceptual understanding in mathematics. Conceptual development using computers has been the focus of many studies [e.g., 2, 3, 4] and the potential for using the computer as a visualisation tool has been widely recognised. This article focusses on structuring a computer laboratory program to exploit this resource with the aim of enhancing conceptual understanding amongst students.

A number of authors have reported problems with the use of computers in mathematics courses. Predominantly amongst these is student frustration with software syntax [5], often resulting in the mathematics being lost whilst the student tries to negotiate the syntax [6, 7]. Galbraith et al. [8,p.250] identified the need for the transferability of skills after students were exposed to a Maple-based tutorial program; to succeed in pen and paper assessment tasks "students must transfer their learning and expertise substantially from a software supported environment to written format." The issue of overhead in both time and effort for students to exploit the capabilities of a Computer Algebra System (CAS) were identified by Vlachos and Kehagias [9]. A further objection voiced by students is that laboratory programs are irrelevant to the rest of the course and unhelpful in preparing them for examinations [5, 7].

To help alleviate the burden of learning a CAS, some authors reported that they armed students with aids, such as templates, to make the familiarisation and use of software easier. For example, Franco et al. [10] provided instructions on the use of a CAS and an interactive

file in order to focus students attention on the mathematics. Tonkes et al. [7] designed a structured workbook that provided graded proficiency in Matlab. The workbook "contains mathematical ideas, concepts in numerical analysis and Matlab syntax interspersed with a sequence of examples and followed by student exercises."(p.753)

In view of the frustrations and problems reported in using a CAS, this article discusses an approach to developing a mathematics computer laboratory program that does not emphasise the learning of a CAS. Using constructivist principles, we view the computer laboratory environment as an opportunity for students to construct their own knowledge of a topic. Computer graphics is invaluable in providing dynamic visualisations by generating animations to support concept development. We believe that this facility is too beneficial to be ignored. Although we emphasise this approach to designing a computer laboratory program, we do not wish to diminish the value of CAS-based programs in the mathematics curricula and see them as valuable for students, particularly those specialising in mathematics. We do, however, see the laboratory setting as an opportunity to apply constructivist principles to assist students in developing concepts. An alternate opportunity may arise with the upsurge of the flipped classroom approach to education in tertiary institutions.

We start off by discussing two perspectives of learning and teaching mathematics. We follow that with a discussion of learning theories that can be applied to mathematics education. The principles discussed in these sections are utilised in developing a mathematics laboratory program described in the subsequent section and a sample activity is provided. A discussion of current evidence of success ensues and direction for further research is given.

## 2    Education frameworks for mathematics learning

A number of useful education frameworks have been developed, many of which are applicable to the learning and teaching of mathematics. By discussing two of these, we focus on different levels of learning that occur in mathematics classrooms.

### 2.1    Anderson and Krathwohl's taxonomy

A useful framework for mathematics learning and teaching is provided by Bloom [11], who identified six levels of cognition. The framework was later revised by Anderson and Krathwohl [12], who made some adjustments to the naming and ordering of the levels, but more importantly, added levels of knowledge to interact with the levels of cognition. The taxonomy is presented in Figure 1 below.

| Knowledge Dimensions | Cognitive Processes | | | | | |
|---|---|---|---|---|---|---|
| | Remember | Understand | Apply | Analyse | Evaluate | Create |
| Factual | | | | | | |
| Conceptual | | | | | | |
| Procedural | | | | | | |
| Metacognitive | | | | | | |

**Figure 1.** Anderson and Krathwohl's revised taxonomy of learning and teaching.

The cognitive processes levels describe the type of activity that a student undertakes and are described as follows:
 (1)    Remember: Retrieving facts, definitions or formulae from memory.
 (2)    Understand: Obtaining meaning from instructions.
 (3)    Apply: Carrying out a procedure by executing a given sequence.

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(4)     Analyse: Breaking material into its constituent parts and relating the parts to one another.
(5)     Evaluate: Making judgements based on some criteria.
(6)     Create: Putting elements together in a new way.

The knowledge dimensions refer to the subject matter being learnt. A brief description of the four levels follows:

(1)     Factual knowledge: Refers to discipline-specific knowledge, facts and terminology required to be functional in a discipline.
(2)     Conceptual knowledge: Includes classifications and generalisations of structures pertinent to a discipline.
(3)     Procedural knowledge: The skill to perform processes or algorithms.
(4)     Metacognitive knowledge: Knowledge about how to go about solving problems and awareness of one's own cognition.

Anderson and Krathwohl's [12] taxonomy provides a useful framework when planning learning objectives, learning activities and assessment tasks. The taxonomy may reveal a concentration of effort in current practice at the lower levels of cognitive processes (remember, understand and apply) at the expense of higher processes (analyse, evaluate and create). Knowledge dimensions may also be concentrated at lower levels.

Using the common understanding of the term conceptual knowledge in mathematics education, we would argue that conceptual understanding represents a higher knowledge dimension than procedural knowledge. This is evident when students can perform an algorithm in a given situation without much understanding of what the algorithm does and when it is appropriate to apply it. Much research has been focused on achieving a balance between conceptual and procedural knowledge in mathematics curricula. The common concern expressed in the mathematics education literature is that currently there is an overemphasis on procedural knowledge and that student outcomes can be enhanced by placing more emphasis on conceptual understanding [2-4].

### 2.2     *Skemp's instrumental versus relational understanding of mathematics*

Skemp [13] identified two levels of learning: instrumental and relational. Instrumental learning refers to to the ability to apply a given sequence of steps without knowing why they are appropriate in a given situation and what the process means. It is the application of rules without understanding. Relational understanding on the other hand includes the knowledge of the appropriateness of the application of a process in a given situation. It is an understanding of how the parts relate to one another. This framework can be viewed as a simplification of the model presented by Anderson and Krathwohl [12].

Using this framework, we often see students working at an instrumental level, learning algorithms without understanding what they do and when it is appropriate to apply them. A common aim amongst students is to obtain temporary knowledge in order to achieve good marks in tests and examinations. Often this approach is reinforced by faculty through its assessment procedures and pedagogy. It takes determined effort to revise current practices and include activities in the curriculum that encourage relational understanding.

## 3     Education theories

In the last section we discussed learning frameworks, which can assist curriculum developers to design activities and assessments that encourage higher levels of understanding. In this

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section, we turn our attention to education theories that can assist us in understanding how students learn and the type of learning that takes place.

### *3.1    Behaviourism and Piagetian stages of development*

The most popular learning theory in education in the USA during the first half of the twentieth century was behaviourism [14]. Behaviourism is based on the premise that learning can be induced through a system of rewards and punishments. Learning is modelled as the transmission of knowledge from the teacher to the learner, where the learner's mind is like a blank slate that can be written upon. Learning under this model is perceived to be memorisation of facts.

By the 1960s, Piaget's theory of general cognitive development had come into prominence [15]. Piaget [16] identified four stages of children's cognitive development. Sensory-motor stage is the first (from birth to approximately 18 months), followed by the pre-operational stage (to about seven years). The third stage is the concrete operational stage (to about eleven to fifteen years), which leads to the last stage, formal operations, which comprises abstract thought and deductive reasoning.

Piaget's theory of learning was seen as problematic by some researchers when empirical data suggested that logical thinking operations are domain specific and not independent of the contexts of the operations. Thus an individual can exhibit operational thinking in one domain but not another [17].

Almy et al. [18] found that individual children wavered over a period of time on their ability to successfully perform Piagetian conservation tasks. This refuted Piaget's theory that children reach a cognitive developmental level that persisted. The implication for the classroom is that it would be fallacious to think that students of a certain age or grade have reached a prescribed level of development that can be assumed when teaching a new concept. Almy et al.'s evidence stresses not only the importance of ascertaining students' commencing level, but that levels previously achieved by an individual may not be their current cognitive level.

### *3.2    Constructivism*

Duit and Treagust [15] declared that changes in a student's conceptual thinking are key aspect of the constructivist view to learning, which gained prominence in the 1980s and 1990s. The context of the concept was an important aspect of the theory, unlike Piaget's notion of developmental stages that are independent of context. Initially it was thought that students' misconceptions of a phenomenon had to be extinguished and replaced with a correct one. Later research showed that previous conceptions can and do exist with the new [19]. Therefore, knowing what conceptions students currently hold is valuable information in planning a topic.

Constructivism is based on the notion that knowledge is obtained through the active building of concepts from existing ideas. For example, it would be pointless trying to teach the notion of ordering of fractions if a student is unable to order whole numbers. The concept of a fraction is also necessary. A synthesis of these two ideas is required before a student is able to rank fractions in size order. Indeed, knowledge of the order of whole numbers alone may be a hindrance to ordering fractions, since fractions are inversely proportional to their denominators when the numerators are one.

Many researchers have expressed the notion that understanding is achieved when new ideas are woven into existing ideas and knowledge. Understanding requires more than just a statement of the fact; it requires forming a relationship with existing knowledge. New

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knowledge does not exist in isolation. Piaget [16] referred to this connection as assimilation (the integration of reality into a structure) and equilibration (internal reinforcements that enable the elimination of contradictions). Thus to present a new idea in isolation is less beneficial than to make connections with a student's existing knowledge. In tertiary education, many instructors make the assumption that students have the required prerequisite knowledge before commencing a unit of study, but often this knowledge is sketchy at best. Ausubel [20,p.vi] pointed out that "The most important single factor influencing learning is what the learner already knows".

### 3.3     *Social constructivism*

Social constructivists hold the view that conceptual changes in an individual are influenced by social surroundings. Whereas mainstream constructivism views conceptual changes as taking place within a person's mind, social constructivism views it more from the perspective of changing one's relationship with the world [15]. Learning is seen as a part of social interaction, where active discussion takes place to support learning. Under this paradigm, discussion amongst students should be encouraged.

## 4     A constructivist approach to mathematics laboratories

In view of the above learning and teaching frameworks and theories, how can we best utilise the mathematics computer laboratory environment to enhance student outcomes? How can students be encouraged to achieve a deeper understanding in preference to a superficial one for the sole purpose of passing exams and tests? Can the laboratory program be designed to assist students develop a deeper understanding of mathematical concepts? By adopting a constructivist view, we take an approach to developing a computer laboratory program that differs from the traditional one of teaching students CAS skills (albeit a valuable goal in itself). The program we advocate is one based on mathematical investigations aimed at enhancing conceptualisation.

As an illustrative example of a constructivist approach, consider the topic of three-dimensional geometry, which is typically covered in advanced calculus courses. It is often assumed that students have a good understanding of the integration of a function of a single variable, conic sections, equations of planes and even quadric surfaces and traces of surfaces from previous studies. From a constructivist perspective, it is necessary to test this assumption so that students can link the new concepts to their existing knowledge. In an experiment with a class, we found that the prerequisite knowledge limited when we tested students by asking them to write down an example of an equation of a parabola, an ellipse and a hyperbola, and to write down the equations of traces of a given surface. The equation of a parabola posed little problem for students and some were able to present the equation of an ellipse. Few were able to provide the equation of a hyperbola. Students struggled with the concept of deriving the equation of a trace from the equation of a surface. The assumption that students have the prerequisite knowledge was largely problematic in this instance.

Consider a typical three-dimensional geometry problem: "Find the traces of the surface $x^2 + y^2 - z^2 = 1$ in planes $x = n$, $y = n$ and $z = n$. Identify the surface and sketch it." Using the traditional approach, students use an algebraic approach to find the equations of the traces, which students should recognise as the equations of conic sections. Some students would be able to complete this task, generally without visualising the surface or the traces. Many students would either get lost in the algebra or not be able to identify the equations of the traces as conic sections. Few would have the conceptual understanding of what these algebraic manipulations actually represent. However, a computer program can provide a

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visual image of the surface and the traces, and even without prior knowledge of conic sections students can achieve success. A web-based source for this task is freely-available and provided by Keynes et al. [21]. This source is a companion web site for a textbook. A screen capture of the program is shown in Figure 2. (To access this particular page, from the home page click on Calculus 6E, then Browse Visuals and Modules, 13 Vectors and the Geometry of Space and then choose M13.6A Traces of a Surface.) With the aid of this program, students can explore the traces of the surface in orthogonal planes. They can work with the algebraic equations of the traces and see their graphs. Working in groups, students can enhance their learning through discussions as espoused by social constructivism philosophy. Students can explore critical values of $n$ where the traces change, something that is not readily seen in the equations alone. Further explorations from this source ask students to use the traces of an unknown surface to derive its equation, as shown in the screen capture in Figure 3. (Access this by clicking on the arrow next to the heading and choosing Traces of surface C.)
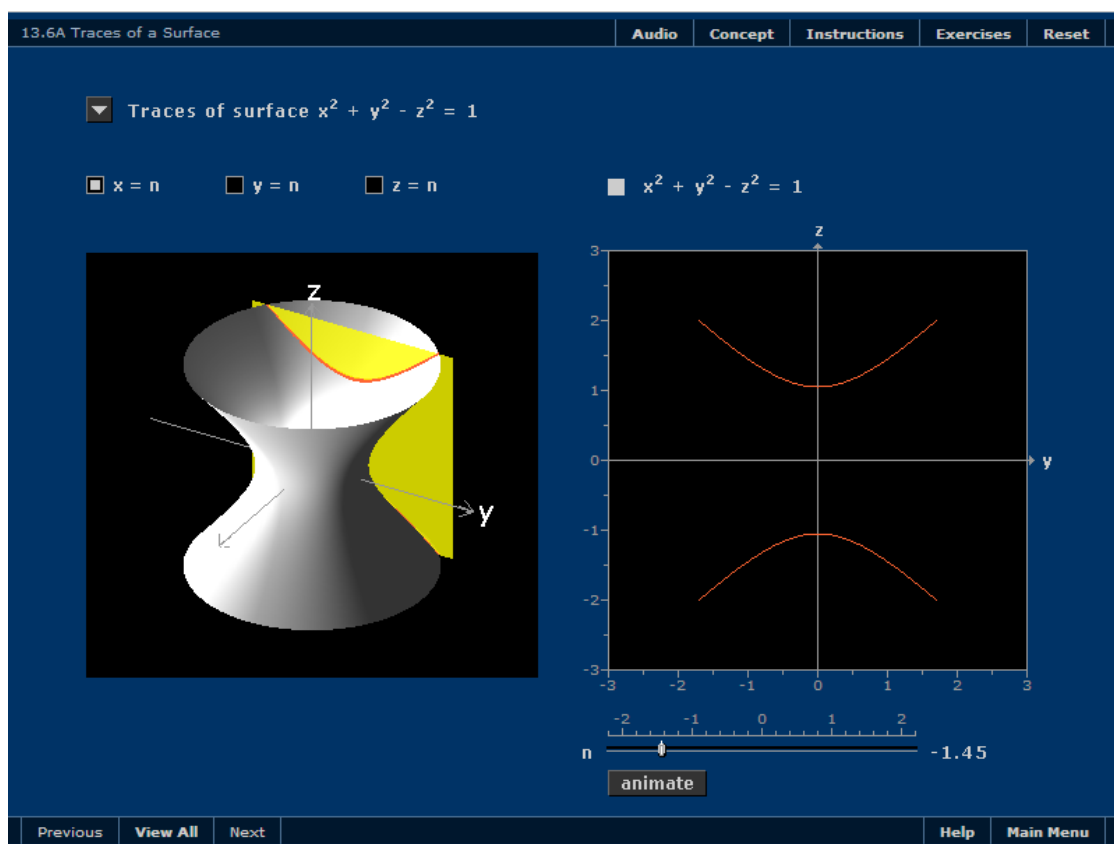


**Figure 2.** Web program of the traces of a given surface.

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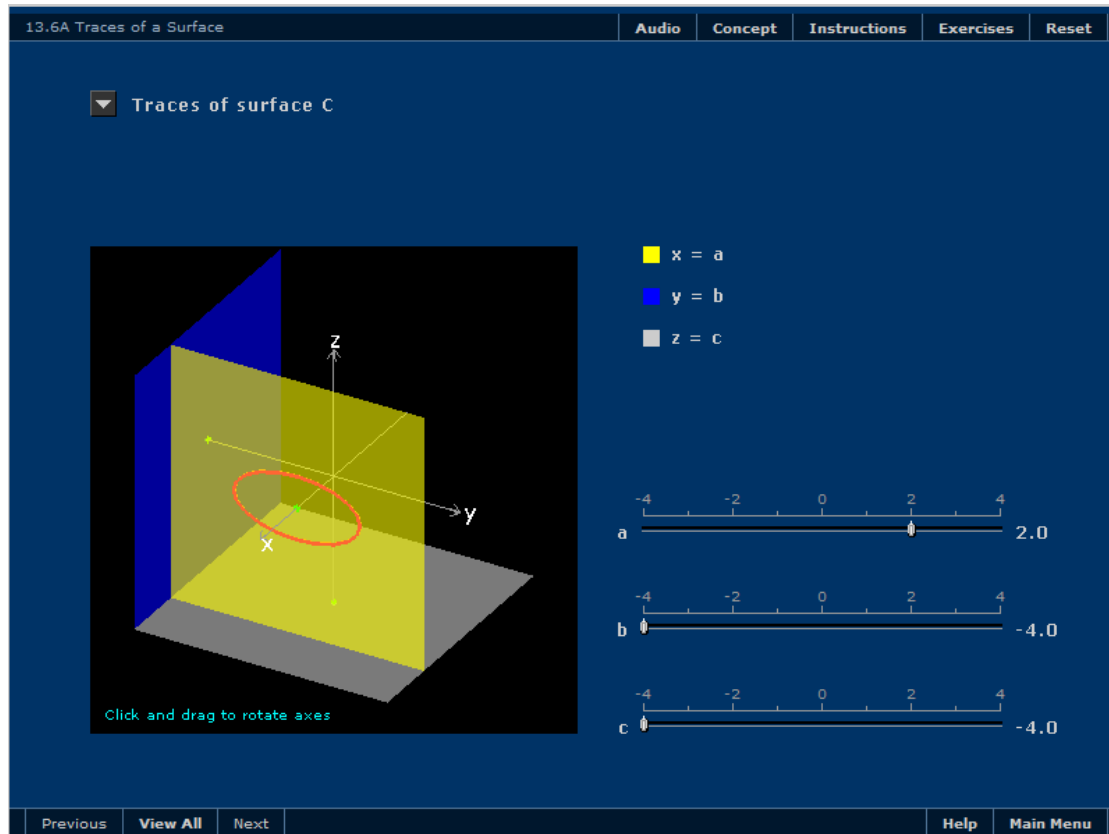
**Figure 3**. Web program to derive the equation of a surface from its traces.

Other examples of programs from that source can be used in a constructivist approach to learning. Many modern CASs have developer's tools that enable the set-up of exploration programs that work in much the same way as the web program given above.

Although we have used this approach in a computer laboratory program, it can be used in other settings, including with assignments and homework exercises, and in the flipped classroom scenario that is being implemented by many institutions today.

## 5    Conceptualisation and technology integration

Some studies have reported success in enhancing conceptualisation by placing more attention on this aspect of learning. For example, Heid [4] established an experimental group of 39 students where concepts were emphasised in both delivery and assessment over the first 12 weeks of an applied calculus course. Most of the algorithmic calculations were relegated to calculators and computers during this time. Only during the last three weeks of the course was skill-building emphasised. She compared learning outcomes of this group against a control group of 100 students who followed a traditional course that emphasised procedures. She found that despite the reduced focus on skills, students in the experimental group performed almost as well in the final exam that assessed skills. She also found that the experimental group displayed a better understanding of concepts, as measured by tests focused on assessing that aspect.

The use of technology in the mathematics curriculum has also met with some success. Using a 68-item instrument on a sample of 172 first-year tertiary mathematics students, Cretchley et al. [6] reported that students responded positively to the visualisation offered in a Matlab laboratory. However, this is tempered with reports from researchers of student frustration and the overhead required in using a CAS that was discussed in a previous section.

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This prompts us to ask if it is possible to use the technology without the overhead of learning a CAS. The answer lies in the use of multimedia, which is not CAS dependent, or to develop an exploratory program within a CAS that can be operated in a point and click mode. A survey of students in a mining engineering course exposed to a mathematics laboratory program of investigations provided encouraging results [22]. However, because of the small sample of nine students completing the survey from a class of 17, the results were inconclusive from a quantitative viewpoint. Some evidence of the success of the program can be garnered from the qualitative data gathered using open-ended questions in the survey instrument. Indicators of success included comments such as "[the program was] very helpful as practical sessions [make it] much easier to remember concepts" [22,p.C292], "The Maths Lab is good and is in line with modern Technology" and "All the content learnt in the lectures was reinforced in the labs and tutorials which helped me understand better". Responses to the structured questions of the instrument also indicated that students felt that the program provided an opportunity to explore concepts developed in lectures. Comments from an semester end student evaluation survey in a unit with an investigative laboratory program further indicated the success of the program with comments such as "For me, the labs were really helpful to understand/visualize the course material … [using] different 3D structures".

Further research is planned to evaluate the investigative computer laboratory approach, such as described above. The aim of the research is to compare learning outcomes and attitudes of students in an experimental group who undergo the intervention against a control group who follow a traditional CAS-based program. Researchers or practitioners who are interested in being involved in this project should contact the leading author.

## 6    Conclusion

In this article, we have discussed how higher levels of learning are often not pursued by students when their main goal is to obtain good marks. This practice is often unintentionally encouraged by pedagogy and assessment policies when procedural skills development is the focus. In contrast to the instruction paradigm's model of a classroom as being a place where students' knowledge is topped up by the expert, constructivism views learning as being created in the mind of the learner by linking new ideas to their existing concepts. We have used the constructivist approach to suggest a structure for a laboratory program as one where resources are made available to the learner to explore concepts in a visually dynamic environment. This approach allocates computer facilities for students to spend time developing their conceptual understanding through a series of guided mathematical explorations. This approach emphasises the importance of conceptual understanding and could go some way towards addressing the concerns expressed by researchers [e.g.,3] of high dropout rates and limited conceptual understanding and an inability to apply knowledge in non-routine situations, even by students with high grades.

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# Addressing dualism in mathematical abstraction: An argument for the role of Construal Level Theory in mathematics education

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Learners of mathematics often struggle to balance the apparently conflicting demands for abstract thinking as well as (often simultaneous) concrete cognitive engagement. Conflicting demands of successful mathematical engagement have been addressed in the literature pertaining to procedural versus conceptual approaches to mathematical learning as well as in the literature on cognitive and meta-cognitive mathematical demands. Construal Level Theory offers an opportunity to understand both these dualities as aspects of the same psychological response to contextual priming. In addition, Construal Level Theory can be understood to illuminate student difficulties with heuristic strategies in mathematical problem-solving. The focus of Construal Level Theory on abstract and concrete cognitive construals as a consequence of psychological distance provides a useful lens for teaching and learning opportunities. We argue that Construal Level Theory offers an opportunity to draw together several strands of mathematics education theory and to help educators address learning challenges in the classroom.

## 1. Introduction

Mathematics is often regarded by students as a difficult subject to study. One among many reasons for this difficulty is the necessity for practitioners of mathematics to simultaneously engage with the work abstractly, bringing conceptual knowledge to bear, and to engage with the work at a more concrete level, using algorithms or algebraic procedures. Novice mathematicians, a role students of mathematics are necessarily filling, often struggle to bring an appropriate level of abstraction to bear on mathematical problems. In order to help students to adopt the appropriate level of abstraction it is important for teachers to first be conscious of what the appropriate level is and then communicate effectively with students to help them adopt it.

There exists an innate duality in mathematical engagement which has been addressed in multiple forms in the mathematical education literature. One example of an attempt to give expression to this duality is apparent in the discussion on procedural and conceptual knowledge; another distinction is the cognition and metacognition contrast. In the case of both of these imposed structures, it is not possible to fully separate the different modes of thinking and both are required in varying degrees for effective problem solving. As a general matter, increased procedural proficiency benefits conceptual understanding and improved conceptual understanding helps in the development of procedural fluency. This does not imply, however, that these crossover effects are symmetrical in their importance and there is debate about where the emphasis in teaching should lie [1]. In addition, being conscious of the desired level of abstraction does not necessarily mean it is a simple matter to instill this awareness in students. Construal Level Theory (CLT) resonates with both of these dualisms.

A third substantial section of the mathematics education literature which can be viewed through the lens of Construal Level Theory is the use of heuristic strategies in mathematical problem-solving. The use of heuristics or "rules of thumb" in mathematical problem-solving has seen a great deal of support yet the classroom evidence suggests that students often find the simply-expressed strategies challenging to use in mathematical contexts [2,3]. Considering the archetypal heuristic strategies from the viewpoint of Construal Level Theory it seems apparent that different strategies require different construals, potentially at odds with the construal called for by the problem within which the heuristic strategy is being used.

In this article we shall argue that Construal Level Theory not only sheds light on the challenging nature of mathematics as a subject of study (prompting suggestions for successful pedagogy), but that it knits together and deepens insight into several major areas of the mathematics education literature. The duality apparent in much of the mathematics education literature on problem-solving is echoed in CLT's notion of near and far psychological distance.

## 2. Construal Level Theory

A theory which is emerging from the psychosocial work on "priming" promises to shed new light on the sometimes uneasy truce between abstraction and concreteness in mathematics. Construal Level Theory [4,5] is a theory in social psychology that suggests a relation between psychological distance and the level of mental abstraction a person will adopt. Greater psychological distance typically primes people to adopt an abstract mental mindset while psychological closeness causes people to adopt a concrete mental mindset. The converse is also true and the level of abstraction adopted has a similar effect on perception of psychological distance. In general people tend to operate in one of two mental modes, "near mode" or "far mode" [6], with closeness and concrete thinking comprising near mode and distance and abstract thinking comprising far mode.

People only ever experience the here and now. All events and objects removed from our direct experience can be said to be psychologically distant. There are four basic dimensions of psychological distance; these include the common sense dimensions of time and space but also the more abstract notions of social distance and hypotheticality [5]. Social distance can refer to how familiar people are with each other but also to other social differences. For example, employers and their employees are often socially distant from one another even if they spend a great deal of time together. The dimension of hypotheticality involves the ability of people to form counterfactuals of past events, imagine hypothetical future scenarios and imagine different ways the world could be. The more improbable an imagined event, the greater the psychological distance involved. Psychological distance is thus egocentric and subjective; it is measured by perceived distance from the self.

Construal Level Theory suggests that we cope with psychological distance by forming abstract construals of distant events and objects and the more distant the object the more abstract the construal [4]. Abstraction necessarily involves omitting details which are considered irrelevant and focusing on the core important features. Psychologically distant objects are typically lacking in context-specific detail but nonetheless often guide action based on the core abstract features of the object. For example, people are often required to interact with socially distant strangers where the only information we have about them is the social space they occupy which plays the dominant role in determining appropriate behaviour. Our actions will be different if the stranger is our new boss, a shop assistant or our doctor. Our behaviour will largely be determined by the relatively stable, abstract norms

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governing the social role the stranger is fulfilling and not on idiosyncratic details of the individual.

The different distance dimensions and level of mental construal are all interrelated [4]. An event distant in one dimension is more likely to be perceived as distant on the others as well. People thinking about an event in the distant future are more likely to think it will be far away and are more likely to consider more improbable scenarios. Similarly, adopting an abstract mental mindset increases the likelihood of viewing other objects abstractly. Importantly for our purposes in this paper, psychological distance impacts on the level of mental construal people will adopt. In general, greater psychological distance primes more abstract thinking and psychological proximity primes more concrete thinking. The converse is also true so thinking of an object abstractly causes it to be viewed as more psychologically distant.

To the extent that encouraging students to adopt the appropriate level of abstraction is important, CLT has significant implications for mathematics education. Teachers have a substantial level of control over the classroom environment and CLT implies that students can be induced to think abstractly or concretely through increasing or decreased psychological distance.

## 3. Abstraction

Abstraction is used in a philosophical and mathematical sense, as a topic of metaphysics or ontology and it is also used in a psychological sense, as something that humans, and to a lesser degree other animals, do naturally and automatically [7]. The philosophical and psychological senses of abstraction are related because humans have to use their powers of abstraction to think, however imperfectly, about abstract mathematical objects and if we are to be able to solve mathematical problems.

Mathematics deals with abstract objects and processes. Abstract objects, as opposed to concrete objects, have no tangible existence in the natural world. Philosophers debate whether abstract objects can meaningfully be said to exist but mathematicians take their existence for granted and so will we in this article. While all mathematical objects are equally abstract in the sense that they have no tangible existence, there are superordinate and subordinate objects.

As a psychological process, we classify physical objects and actions with varying degrees of abstraction and levels of abstraction form a hierarchy so it makes sense to speak of higher and lower levels of abstraction; a physical object can be considered a concrete representation of a purely mathematical object. Much of mathematics and science involves developing and systematising our natural ability to conceptualise things abstractly. This enables us to separate powerful ideas from their concrete context in the here and now and apply them in novel circumstances and far away in space and time.

Actions and events also can be viewed more or less abstractly. Viewing an action abstractly typically involves considering the high level goal of the action. A concrete view of the action focuses more on how the action is performed. Broadly speaking, high-level abstract construals consider why an action is performed and a low-level construal considers how it is performed [4]. Often, evidence that a person has developed an abstract conception of a mathematical notion is that she can call to mind an appropriate concrete representation of that notion [8]

Finally it is important to note that thinking about abstract objects does not necessarily mean thinking in a highly abstract way. The numbers 3 and $i$ are purely abstract objects, but once we have developed proficiency with using and manipulating these objects, they will be

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viewed at a low level of cognitive abstraction because they are merely building blocks for thinking about higher order objects or solving problems which requires thinking about relationships at a much higher level of abstraction.

## 4. Procedural and conceptual approaches

Most, if not all, of the articles in the edited volume of [9] stress simultaneously the distinction between procedural and conceptual knowledge, the reliance of the development of one on the development of the other, and the importance of links between them. Hiebert and Lefevre [1] define conceptual knowledge as "knowledge that is rich in relationships" [1,p.3] and procedural knowledge as "the formal language, or symbol system, of mathematics" as well as the "algorithms, or rules, for completing mathematical tasks" [1,p.6]. Both types of knowledge are necessary for competent mathematical engagement, neither can categorically be considered more important than the other, and both types of knowledge are often inextricably entwined in a mathematical task. Sinclair and Sinclair [10] similarly draw a distinction between 'knowing-how-to" and understanding. Carpenter [11] argues that "distinctions between levels of problem solving reflect differences in how problems are represented and how these representations incorporate different kinds of relations [between conceptual and procedural knowledge]" [11,p.121]. Hiebert and Wearne [12] argue that competence is characterised by the links and connections between procedural and conceptual knowledge.

Rittle-Johnson, Siegler and Alibali have argued that procedural and conceptual knowledge influence each other in an iterative fashion, with improvements in one domain allowing and spurring developments in the other [13]. Evidence suggests that in the case of certain mathematical concepts, procedural fluency precedes conceptual understanding of the concept, but in other cases the reverse is true [14]. Rittle-Johnson and Siegler suggest that the relative familiarity of concepts or procedures from everyday experience can have an effect on which competency students develop first. Jon Star has argued that conceptual knowledge has received too much attention in the mathematics education research literature and argues for a reconceptualization of procedural knowledge that should receive more research focus [15]. He argues that procedural knowledge can be of a more flexible and robust nature than that implied by the definitions typically used without deserving the classification of conceptual knowledge. Typically procedural is taken to mean superficial and conceptual is taken to mean deep but this is not always the case. Star argues for classifications that would include deep procedural knowledge and shallow conceptual knowledge. In a similar way to Star, de Jong and Ferguson-Hessler suggest distinctions between types of knowledge on the one hand and qualities of knowledge on the other. In this classification, types of knowledge are either procedural or conceptual and qualities are deep or shallow [16].

Construal Level Theory displays obvious parallels with the debate around procedural versus conceptual approaches to mathematical engagement yet adds depth by arguing that such approaches are innate to the human brain's construal as influenced by contextual priming. Mathematics education always takes place within a broader (non-mathematical) context over which the teacher has some control. We are primed to think abstractly or concretely in surprising and unintentional ways; for example, thinking about distant times, places and socially distant individuals primes abstract thinking. These non-mathematical cues can be used along with explicit mathematical instruction to aid students. The relationship between distance and construal level means that communication can unintentionally be at odds with the appropriate level of abstraction helping to explain how easily communication can break down in the classroom.

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Research investigating student performance in procedural and conceptual mathematics involving first year calculus students [17] suggests that while students do not necessarily perform better on either procedural or conceptual problems they typically are more confident of their ability to handle conceptual problems. Relative to their ability to cope with procedural and conceptual problems, students tend to be more accurate in their assessment of their procedural abilities and overconfident of their conceptual understanding. Construal Level Theory provides a potential theoretical explanation of this empirical finding. In other research Construal Level Theory has been invoked to explain a phenomenon known as the illusion of explanatory depth (IOED). IOEDs occur when people believe they understand a concept better than they do in reality. Alter et al [18] argue that IOEDs occur when people adopt an inappropriately abstract construal when assessing their own understanding of concrete concepts. Studies have shown that people who naturally adopt a concrete construal style and people who are primed to adopt a more concrete construal style experience diminished IOEDs.

We argue that Construal Level Theory provides an alternate language for addressing the distinction between procedural and conceptual knowledge as well as providing a theoretical framework for understanding existing empirical work in this area.

## 5. Cognition and meta-cognition

Metacognition, distinct from cognition itself, has seen a variety of definitions. When cognition is defined explicitly in contrast to metacognition (rather than being used as a catch-all for anything memory or problem-solving related) it invites descriptions such as applying knowledge or using skills; in mathematics, cognition is the manipulation of symbols, perceptions of physical space, application of learned algorithms. In bald, simplistic terms – easily challenged – cognition is thinking and metacognition is thinking about thinking. Schoenfeld [19] divides metacognitive processes into three categories: "(1) individuals' declarative knowledge about their cognitive processes, (2) self-regulatory procedures, including monitoring and "on-line" decision making, and (3) beliefs and affects and their effects on performance" (p. 347). Often it is only the first two categories, that is, individuals' awareness of their own cognitive processes, and the monitoring and regulation of these processes which are referred to in discussions on metacognition [20,21,22]. It is these definitions of metacognition as distinct from cognition which embody dualism and are amenable to analysis through Construal Level Theory.

It has been argued that successful mathematical problem-solving involves interplay between both cognitive and metacognitive strategies [21,23], although it has been observed that telling them apart can be difficult. Artzt and Armour-Thomas [23], acknowledging this difficulty of separating cognitive from metacognitive, divide the problem-solving process into episodes characterised as either cognitive or metacognitive or both. In the (perhaps somewhat artificial) distinction between cognition and metacognition, one can see the utility of Construal Level Theory, shedding illumination on the reasons why it is challenging to practice successful metacognition when in a construal mode suited to more concrete cognitive action.

## 6. Problem-solving heuristics

The term *heuristic* is usually used as an adjective (Oxford English Dictionary), such as in heuristic process, heuristic technique, and so on, to the point where the adjective form is used to define the noun form. *Heuristic (adjective): serving to find out or discover. Heuristic*

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*(noun): A heuristic process or method for attempting the solution of a problem; a rule or item of information used in such a process* (Oxford English Dictionary). Georg Pólya's [24] widely cited heuristic strategies include *draw a diagram*, *solve a related problem*, *think of a theorem with a similar conclusion*, and several more. Heuristics can be understood as being rules of thumb, or rough guides to how to respond to particular situations.

Those who have made a study of heuristic strategies agree on at least two important features: they are useful problem-solving tools and they are difficult to master [2,25] . Scrutiny of specific heuristic strategies suggests that there is a duality inherent in them. The heuristic strategy of *draw a diagram* requires attention to the specific requirements of the problem, such as shapes, variables, quantities, relative positions or motions. In contrast, the heuristic strategy of *think of a theorem with a similar conclusion* requires the student to consider clusters of theorems, to explicitly back away from the minutiae of the problem and consider a more general area of mathematics within which the problem is located. Further, that strategy requires the student to already have recognized links between theorems, a demand for abstraction.

Schoenfeld [2] has made major contributions to the study of heuristics in problem-solving by breaking many of Pólya's strategies into sub-strategies, specific to particular types of problems, and experienced considerable success in teaching a problem-solving course based on those sub-strategies. At the broadest level the strategy suggests students approach each new problem by attempting to progress through a series of stages: Analysis, design, exploration, implementation and verification.

Design is not a distinct stage to be implemented in sequence but is rather intended to remind students to maintain a global, goal oriented mind set. In terms of Construal Level Theory, this can be viewed as an explicit suggestion to begin the solution process by adopting an abstract construal and then proceeding to more concrete complex, detailed calculation. Schoenfeld's description of design also suggests that no matter how demanding the concrete details become it is important to maintain a global perspective so that unproductive lines of calculation can be terminated and other avenues explored. This suggests that even though focus on concrete details may be required it is always necessary to maintain higher level awareness, in other words it is essential that a problem solver continuously shift between both abstract and concrete construals while working on a problem.

Schoenfeld elaborates more fully the analysis, exploration and verification stages. The first two steps in the analysis stage involve drawing a diagram or rough sketch and examining special cases. Both of these steps involve attempts to make the problem take a more concrete form. The third suggested step in the analysis stage involves simplifying the problem by exploiting symmetry, scaling or using "without loss of generality" arguments before getting immersed in details. This involves the use of abstract mathematical techniques and involves an abstract mind set.

The exploration stage involves three steps to be attempted sequentially and represents a progression from more concrete to more abstract and speculative. In terms of Construal Level Theory, the exploration stage moves progressively more abstract and distant on the hypotheticality dimension.

The implementation stage primarily involves procedural proficiency and is the only stage to primarily invoke a concrete mind set. The final stage of verification is broken into two steps; specific tests and general tests. The specific tests involve a relatively low level of construal, but Schoenfeld stresses that at the general level, verifying the problem solution often suggests alternate solutions and connections and can promote conscious awareness of which parts of the process were useful and can be used in future. In other words, global verification helps to develop deeper conceptual understanding of the principles involved.

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Most stages of implementing this heuristic strategy involve both high and low level construals. Construal Level Theory indicates that high and low level construals involve adopting fundamentally different mind sets and thus suggests that this continuous shifting between different modes of thinking is intrinsically unintuitive and challenging.

This duality, which we argue is inherent within the lists of heuristic strategies we have encountered, constitutes a challenge for the student who attempts to use such a list as a problem-solving tool. Certain strategies require cognitive engagement with detailed problem features whereas others require disengagement from problem features and engagement with abstract concepts, theorems and higher level theoretical networks.

## 7. Pedagogic Implications of Construal Level Theory

The level of mental construal has been shown to have an effect on mathematical performance. Perhaps the best known example of this is the phenomenon known as stereotype threat [26]. There is a negative stereotype of women not having mathematical aptitude and when reminded of this stereotype, women typically perform worse on mathematical tasks. This effect has been replicated in the context of Construal Level Theory where "near" and "far" mode priming had different effects on men and women. In the "far" priming condition, women performed worse on mathematical problems relative that when primed to think in "near" mode. People more readily access stereotypes in "far" mode and it is suggested that this negative effect is induced by increased tendency to access a negative self-stereotype in "far" mode [27].

There is also evidence to suggest that task consistent priming is associated with improved task performance and task inconsistent priming is associated negatively with performance. A simple example of this is that subjects' performance on gestalt completion tests, a high-level task, is improved when they anticipated working on a task in the more distant future [28]. Other experiments show that people generate more creative solutions on abstract tasks when primed for psychological distance. On the other hand, performance logic puzzles designed to test analytical thinking was impeded by psychologically distant priming. It should be stressed that the tasks testing creative, abstract reasoning were not mathematical in nature.

These results have particular relevance for procedural and conceptual knowledge in mathematics. Solving mathematical problems requires creative use of mathematical resources and this process is likely facilitated by psychological distance. On the other hand, many mathematical tasks are more procedural and place heavy demands on analytical reasoning processes. Thus it is plausible that when the focus is on problem solving or conceptual understanding a "far" mental mode will facilitate acquisition and when procedural fluency is the goal a "near" mental mode is more beneficial.

Construal level theory also suggests ways in which students can be primed to adopt different levels of psychological distance. Referring to objects distant in time and space is the most obvious and easily implemented example of affecting psychological distance. Social distance is a major component of most classroom environments. Formal contexts and speech is associated with psychological distance while informality induces psychological closeness. Group work is a common pedagogic technique and allowing groups of friends to work together will is psychologically closer than being required to work with less familiar classmates. Psychological distance can even be influence by using shadow effects in printed fonts, by creating the illusion of depth. There is no limit to the variety of ways psychological distance can be manipulated by a teacher in a classroom context and this could be a valuable tool for helping students adopt the appropriate level of abstraction.

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We argue that a teacher who is aware of the potential psychological effects of priming could influence student learning in multiple ways. Given a mathematical task's level of abstraction, the teacher can prime "near" or "far" mode with pre-task activities, task wording, level and type of interaction (group/individual, written/verbal) and possibly many more options. Similarly, a teacher aware of the different cognitive roles of "near" and "far" construal levels could recognize when certain demands on the learner, such as needing to shift between modes when applying heuristic techniques, are unrealistic and unlikely to be met.

## 8. Conclusions

Procedural and conceptual knowledge distinctions lend themselves easily to analysis through the lens of Construal Level Theory. Procedural knowledge is clearly related to the concrete construal level associated with "near" psychological distance and conceptual knowledge is related to the abstract construal level associated with "far" psychological distance. While a switch of language used to refer to these two phenomena might not be seen as valuable, what is valuable is CLT's insistence that the student's mind is primed to work in one of those two modes by many other contextual factors associated with a specific mathematical problem. Simply the wording of the problem can prime the student to construe the problem in different ways. A teacher who is aware of this potential priming effect and one who wishes the student to operate at a certain construal level, can prime the student in useful and productive ways.

Cognition and metacognition, we argue, can similarly be addressed using CLT. Cognition is associated with concrete construal and metacognition with abstract construal. No doubt there are subtleties which can be argued and definitions of the two phenomena which are hard to categorise through CLT, however there is certainly a high degree of agreement between the two definition sets. Successful problem-solving usually requires the two processes of cognition and metacognition to occur for a sustained period simultaneously, or at least with frequent shifting between them. Construal Level Theory suggests that this is more easily demanded of the student than can be enacted with ease.

Interpreting heuristic strategies for problem-solving through the lens of CLT suggests that one reason for students' difficulty in using heuristic strategies is the near-far dichotomy between strategies often regarded as otherwise similar. Many environmental and contextual characteristics contribute to a student's priming for construal level, not least of which is the context and working of the mathematical problem itself. To require students to cognitively draw upon each of a list of heuristic strategies with equal emphasis is unrealistic when viewed in this light. A question for further enquiry would be to investigate whether heuristic strategies chosen for construal level similar to one another and to the specific problem construal requirements might achieve greater problem-solving success than "mixed" lists such as found in [24] and [2].

In this article we suggest that CLT provides a useful framework for analyzing students' approach to and interpretation of problems. The innate duality underpinning CLT echoes the duality inherent in procedural/conceptual approaches, cognitive/metacognitive monitoring and near/far heuristic demands.

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# Undergraduate mathematics and statistics assessment practices in Australia

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**Keywords:** Mathematics and statistics assessment, undergraduate, graduate attributes, threshold learning outcomes

Within the higher education sector, the current focus is on graduate outcomes and the development of threshold learning outcomes for the various disciplines, including mathematics and statistics. We ask the question, how can these outcomes be demonstrated through the assessment of units of study students undertake as part of their undergraduate program? Our study focusses on the types of assessment used across Australia, using a web search methodology to analyse information publically available on the institutions' websites. The findings confirm that mathematics and statistics students in Australia are still largely assessed through closed book in-semester tests and final examinations, and in-semester assignments, contributing 70% and 30% respectively to the final grade of their units of study. Our results broadly match those of a similar study done in the UK, with some differences, particularly in the percentage contribution of closed book assessment to final grade in statistical units and across year levels. Finally, implications of assessment practices for certification of student achievements are discussed.

## 1. Introduction

A focus on graduate learning outcomes is the current trend in higher education around the world [1, 2, 3], and Australia is no exception. The establishment of the Tertiary Education Quality and Standards Agency (TEQSA) in 2012 heralds significant changes in the Australian higher education sector with institutions now required to go through a periodic re-accreditation cycle [4]. The implementation phase of the regulatory requirements has been based on defined higher education provider standards. The next stage will also involve course design and learning outcomes standards, and these will soon form a normal part of the re-accreditation process.

Prior to this shift to a regulated higher education sector in Australia, and following similar international developments, the Australian Learning and Teaching Council (ALTC) (now called the Office of Learning and Teaching (OLT)) has invested in developing the Learning and Teaching Academic Standards (LTAS) [5]. As a result of this project, the various discipline groups developed the Threshold Learning Outcomes (TLO) for their disciplines. The mathematics and statistics TLOs [6] have been developed to align with the Science TLOs [7]. Although the learning outcomes standards are still under development, and no final decision has been made on how TEQSA will monitor attainment of these

standards, there are strong indications that institutions will be able to use the discipline TLOs as reference points for demonstrating the quality of their graduates.

The mathematics and statistics TLOs articulate the minimum characteristics a mathematics or statistics graduate should have at the point of graduation. The TLO graduate characteristics are grouped under the following broad headings [6]:

- *Understanding*: Demonstrating a coherent understanding of the mathematical sciences.
- *Knowledge:* Exhibiting depth and breadth of knowledge in the mathematical sciences.
- *Inquiry and problem solving:* Investigating and solving problems using mathematical and statistical methods.
- *Communication:* Communicate mathematical and statistical information, arguments, or results for a range of purposes using a variety of means.
- *Responsibility:* Demonstrate personal, professional and social responsibility.

The question is then, how would institutions demonstrate that their mathematics and statistics graduates have these qualities? We argue that assessment is central to this development. The role of assessment in shaping student learning has been widely advocated for many years [see for example, 8, 9, 10]. Assessment tasks signal to students what they must learn, and formative assessment provides students with feedback as they develop understanding of new concepts. In this new regulated environment, assessment as certification for what students learn, is gaining new importance. If assessment is designed carefully, then students' assessed work would be the most effective way to produce evidence for attainment of learning outcomes standards.

Within the context of the standards agenda, this paper provides a level of understanding of the assessment practices in undergraduate mathematics and statistics across Australian institutions, which will help to inform the debate around effective and defensible approaches for demonstrating graduate outcomes. This study focuses on the certification aspect of assessment. It extends to Australia the study conducted in the UK by Iannone and Simpson [11, 12]. It asks the questions

(1) What are the types of assessment tasks used in mathematics and statistics units of study (subjects) in Australia?
(2) What is the contribution of each type of assessment towards the marks and/or grades awarded to students on completion of the units of study?
(3) Are Australian assessment practices in undergraduate mathematics and statistics different to those used in the UK?

## 2. Methodology

A search through institutional websites was the methodology followed to find out what types of assessment tasks are used in Australian institutions in undergraduate mathematics and statistics units of study. This methodology mirrors the approach used in the UK by Iannone and Simpson [11, 12]. In Australian universities it is common practice to make basic information about each unit offering available publically, including learning outcomes, synopsis, prerequisite studies and assessment tasks, together with their contribution to the final marks and grades for the units.

Thirty Australian higher education institutions offer a degree in mathematics or statistics; in most cases mathematics and statistics are offered as a major study within the Bachelor of Science. Our web search resulted in finding information about assessment breakdown for 16 of these institutions, including six of the Group of Eight research intensive universities which produce the majority of mathematics and statistics graduates in Australia.

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Only units offered and taught by a mathematics or statistics department were considered. A total of 395 undergraduate units were found, of which 88 were first year units, 124 second year units, and 183 third year units.

## 3. Findings and discussion

### 3.1. Closed Book Assessment

Closed book assessment in the form of final exam and test was found to be the most dominant form of assessment in the majority of undergraduate mathematics and statistics units in Australian universities. Just under 90% of the units (345) included a final examination at the end of the semester; of these 38% (128 units) also had an invigilated test during the semester. Final examinations and in-semester tests are taken under supervision, and students are given a limited time to complete them. Figure 1 shows the distribution of the percentage contribution closed book assessment (final examination and in-semester test(s)) makes to the final grade of students undertaking the units. It shows that undergraduate mathematics and statistics units across Australia are largely assessed through a closed book assessment, amounting to between 60 and 80 per cent of the students' grade for the units. Project units were the only units that did not have invigilated closed book assessment.



**Figure 1.** Percentage contribution of closed book assessment to final grade, in 395 undergraduate mathematics and statistics units at 16 Australian universities.

When unit assessments from all universities are pooled and examined by year level, we observe a general trend for the mean percentage contribution of closed book exams to decrease with increasing year level. This trend is not just seen in individual universities, but also across all universities (Figure 2). The mean percentage of closed book assessment decreases by 7% between first year and third year. There are a number of universities which have a final year project unit (6 units in total). These project units have no closed book assessment and hence the inclusion of these units may skew the mean values. Following the methodology of Iannone and Simpson (2011), the six units specifically labelled as a project were removed. Even without these units, the general trend is still decreasing, and the mean percentage of closed book assessment decreases by 8.5% between first year and third year.

This drop through year levels is different to the trend found by Iannone and Simpson (2012). They found that closed book assessment increases with increasing year level in UK

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universities. Between first and third year, the mean percentage of closed book assessment increased by 10%. When projects were excluded, the mean percentage of closed book assessment tasks increased by 15% from first year to third year. This is an interesting difference which is difficult to explain with the available information; in the Australian context this may be due to the significant difference between first year and higher year class sizes which make in-semester assessment more manageable.
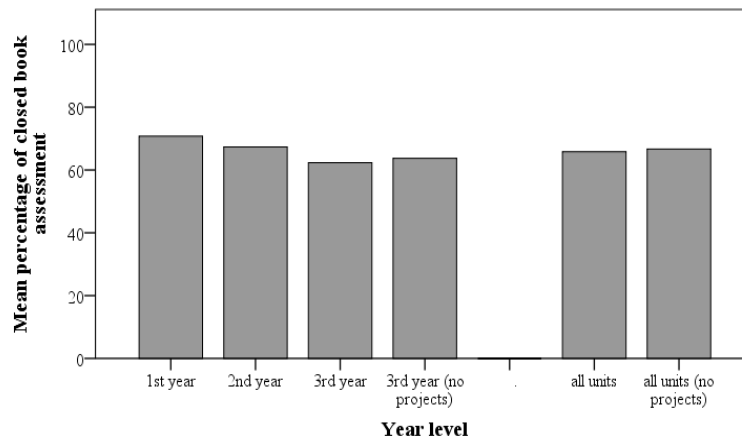


**Figure 2.** Mean percentage contribution of closed book assessment to final grade in undergraduate mathematics and statistics units in Australia, by year level.

The medians, on the other hand, are remarkably consistent across all year levels (70% for each of the three year levels and for all year levels combined). The spread, as measured by the interquartile range (the middle 50 per cent) increases from first to second year, and then decreases strongly in third year (Figure 3).

### 3.2. The assessment of statistics

Iannone & Simpson [11, 12] found a different pattern of assessment in statistics units as compared to non-statistics units. The observed difference in assessment practices may be due to their definition as to what constitutes an "open" exam whereby "students are allowed access to supporting materials", which includes the use of statistical tables. They also found that there is a greater use of computer packages in many of the statistical units in their study.

No evidence of a different pattern in assessment practices can be discerned in Australian universities. The study found and examined 72 statistics units, covering all levels of undergraduate statistics. The proportion of closed book assessment is consistent with other non-statistics units. The mean of percentage contribution of closed book assessment to the grade in statistics units is 65.3% and for non statistics units it is 65.9%. Both groups have the same median (70%). The interquartile range of the statistical units shows less spread than non-statistical units. Also, the range of the closed book assessment of statistical units is entirely contained within the range of non-statistical units. (Figure 4). All these features indicate no discernible difference in closed book assessment patterns between statistical and non statistical units. Further, there is no difference in the use of computer based assessment in statistical units when compared to non-statistical units in Australia.

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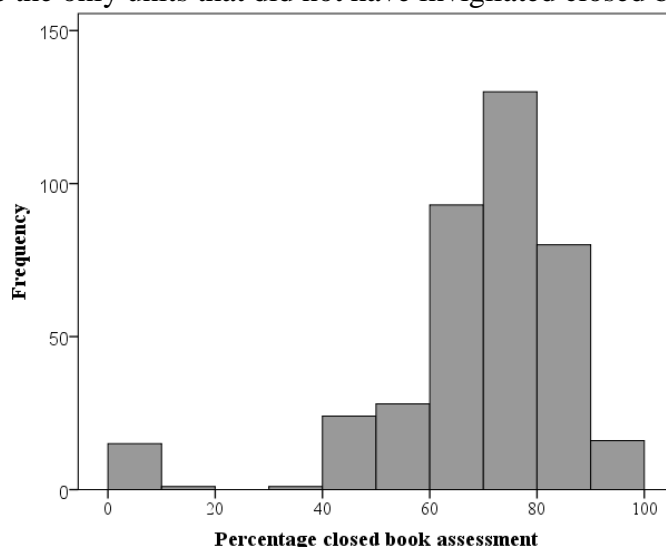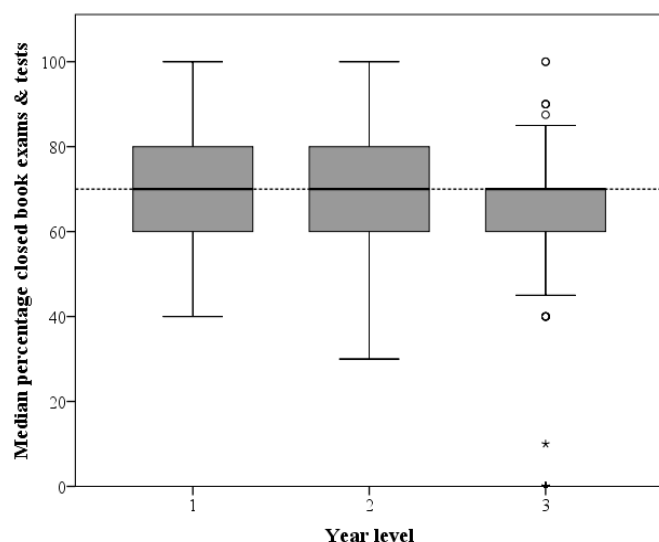**Figure 3.** Median percentage contribution of closed book assessment to final grade in undergraduate mathematics and statistics units in Australia, by year level.
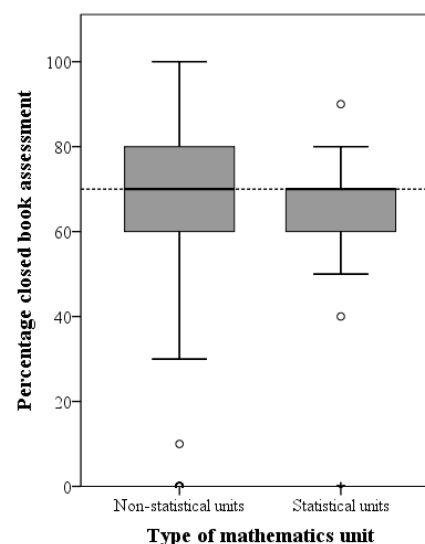
**Figure 4.** Percentage contribution to final grade of closed book assessment in 324 non statistical units and 72 statistical units in Australia.

### 3.3. University rankings versus proportion of closed book assessment

Plotting the mean percentage contribution of closed book assessment for each university against their ranking within Australia according to The Higher Education rank [13] showed no relationship between the average percentage of closed book assessment and a university's league table position (Figure 5).

This is in contrast with UK universities, where Iannone and Simpson [11, 12] found a direct relationship between university rank and the contribution of closed book exam to the final grade for the unit. That is, the higher the university ranked, the higher the proportion that closed book assessment contributed to the unit grade. Again, the scope of this study does not allow to ellaborate on these differences; further investigation is required.

### 3.4. Assignments

Assignments were the second most common form of assessment found in our web search. In the Australian context, mathematics and statistics assignments usually consist of a number of problems or exercises. Of the 395 units in the study, 264 (67%) have assignments in some form declared in their assessment schedule. Another 59 units list a "continuous" assessment component, which could include assignments. The mean percentage contribution to assessment, when averaged over all undergraduate mathematics and statistics units is 27.5%. The average contribution of assignments increases by year level, matching the decrease in closed book assessment with a 22.4% in first year, 26.1% in second year and 30.6% in third year.

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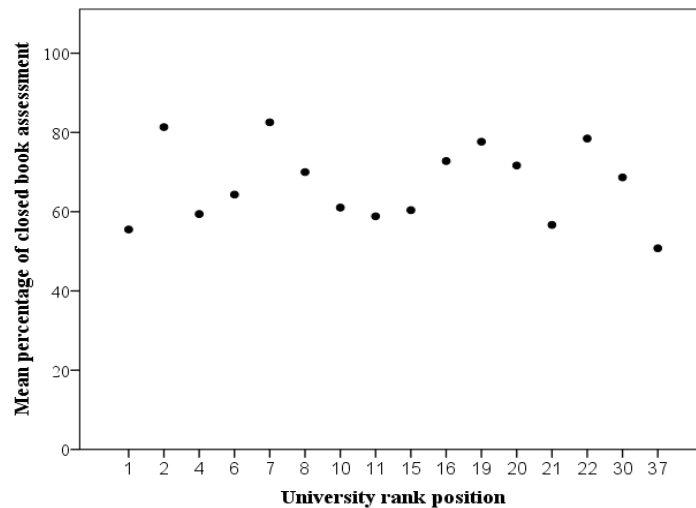
**Figure 5.** University rank position within Australia and mean percentage contribution to final grade of closed book assessment for undergraduate mathematics and statistics units.

### 3.5. Other assessment types

In addition to assignments and closed book exams, we found a diverse range of assessment types being used for the assessment of undergraduate mathematics and statistics units, but each of these appeared only a small number of times.

In addition to six third-year project only units where the entire mark for the unit is based on a project, we found research projects in 45 units (11%) across all year levels. The majority of these also included a presentation element whose contribution to the final mark ranged from 10% to 40%. We also found 13 units not related to a research project that required oral presentations; these were mostly in third year and contributing from 5% to 35% to the final grade (median 10%).

Given the readily available mathematics and statistics software, online testing facilities, and sophisticated learning management systems, it was surprising to find out that only 17 units (4%) explicitly state computer based assessment, either in the form of online quizzes or in the use of software packages. The majority of online assessment occurs in first year units (15 units) with contribution to overall unit assessment ranging from 5% to 30%.

Group projects appeared in only four first year units, contributing from 10% to 30% to the final grade. Twenty units have portfolios; one university alone is responsible for 19 of these portfolios, with 10% to 15% contribution towards the final grade. The remaining assessment task types included essays (5 units, 10 to 30%), oral exam (1 unit, 40%), lab book record keeping (2 units, 20 and 25%), literature reviews (3 units, 40 to 50%), case studies (5 units, 20%), guided discussion (1 unit, 5%), reflection (1 unit, 10%).

### 3.6. Are TLOs being assessed?

Our study shows that assignments (problem sets), hour-long tests and examinations, which form part of a century long tradition in mathematics and statistics assessment, seem to remain the norm. Can these practices be defensible, considering that "in general, students consider important those aspects of instruction that their teacher emphasizes and assesses regularly" [14]?
Mathematicians are used to formulating the learning outcomes for the students in terms of procedural knowledge (performing correct steps in a valid sequence) rather than student

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achievements in using mathematics and statistics (mathematical thinking and analysis) [10]. This results in idiosyncratic assessment practices that favour routine mastery, leaving a gap between what students "know" and what they "can do". Madison [10] explains this as "tensions between what is practical and is effective in judging student performance and understanding", and that "being unable to see evidence of value in the hard work of effective assessment, [mathematicians] often rely on the results of practical methods, believing they are measuring similar or highly correlated results". Bergqvist, on the other hand, leaves open the possibility that "there may be a gap between what the teachers believe is tested [in closed book exams] and what is actually tested" [15].

How can these assessment practices focussing largely on closed book examinations align with the current higher education agenda towards greater accountability and the requirements to demonstrate learning outcomes? Are the mathematics and statistics TLOs assessed in any meaningful way throughout the units students take in completing their program of study? Closed book tests and exams, undertaken within a set limited time, may be appropriate for assessing procedural knowledge, and could, to some minor extent address the "knowledge" and "inquiry and problem solving" dimensions of the TLOs. It is hard to see how these types of assessment could effectively measure performance in the "communication" and "responsibility" aspects. If these are assessed at all, the only place would be through the relatively low stakes assignments. Furthermore, as stated by Houston, "learning mathematics for the principal purpose of passing examinations often leads to surface learning, to memory learning alone, to learning that can only see small parts and not the whole of a subject, to learning wherein many of the skills and much of the knowledge required to be a working mathematician are overlooked" [9].

Universities have been addressing the skills agenda in various ways, including integration of skills in key units at each year level, or the so called "capstone units" at the end of the program where students draw together what they learn from the point of admission [16]. However, in the context of mathematics and statistics, the only units found in our search that could fit the "capstone" definition were research project units. Not all students undertake project units where available; such units are offered only to the very high achieving students, and not all take up this opportunity.

## 4. Conclusion

A wide range of assessment types are used across Australian mathematics and statistics departments for assessing their undergraduate students. However, the predominant forms of assessment used are closed book exams and in-semester tests and assignments, and the typical contributions these make to the student final grade are 70% and 30% respectively. While assessment in higher education has taken some prominence in recent years, in undergraduate mathematics assessment is still a minority culture backed by centuries of tradition [15]. Our findings largely align with practices in the UK reported by Iannone and Simpson. Although there is evidence of less traditional assessment approaches such as reflection, essay writing and portfolios, these appear to correspond to small pockets of innovation. Finally, it should be noted that this study only looks broadly at the types of assessment tasks students are asked to complete towards the mathematics and statistics units they enrol in, and it does not reveal the specific tasks each of these involves.

Our investigation of assessment types was initiated around the question of whether attainment of the nationally agreed threshold learning outcomes could be demonstrated through assessment tasks our students complete as part of their undergraduate studies. Given the strong focus on final examinations which are likely to test only procedural knowledge, there seems to be a gap between what mathematicians and statisticians think their graduates

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should be able to do (as articulated in the nationally endorsed threshold learning outcomes) and the types of assessment they use to drive learning and to grade their achievements. Conclusions, however, cannot be made based only on the assessment types used. A comprehensive review of the assessment tasks (that is, what students are actually asked to do in the different assessment tasks) is needed to decide if current assessment practices are fit for this new purpose or if a radical change is required.

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# Abstracts

# The role of modeling mathematics teaching in improving mathematical teaching skills for student teachers

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**Keywords:** Teacher preparation; mathematics instruction; teaching theory; modeling teaching

This qualitative study used action research methodology, in five stages, to discover the effectiveness of a teaching model for teaching basic mathematical teaching skills to students training to be mathematics teachers at the College of Education, Taibah University, Saudi Arabia. In Stage 1, the researcher reviewed studies and research on effective mathematical teaching skills, and constructed a five step model (orientation, concept, generalizing, practice and closure). In Stage 2, 15 student teachers participated in a teaching methods course for seven weeks of intensive professional development. This related to understanding and implementing the five-step model, and exploring practical examples. In Stage 3 the student teachers implemented the model using a collaborative action research approach. The process involved student teachers visiting each other and coding evidence of model implementation. In Stage 4, the student teachers and researcher met again. This time, the participants spent two lectures involved in reflection and learning to preparing their final reflection project. In Stage 5, the student teachers and the researcher held a two-day conference during which they presented their reflections and findings. Student teachers emphasized that the model can be applied to the conventional syllabus, which developed their teaching skills, besides gaining positive attitudes towards teaching.

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# The mathematics of juggling

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**Keywords:** Mathematics; juggling; workshop; siteswap

A surprising number of mathematicians are jugglers (and vice versa). Upon reflection, the connection is perhaps not quite as odd as it seems. Mathematics is the science of patterns, while juggling is concerned with patterns that can be woven in three dimensions, via manipulation of various objects.

This hands on workshop will introduce you to the art of juggling, as well as the mathematics used to describe and generate new juggling patterns. It will also provide ideas on how you can use juggling to create a more memorable learning experience.

Come along and learn how to juggle three balls.

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# Is there a relationship between learning space and satisfaction with learning experience in a first year statistics tutorial class?

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**Keywords:** Learning space; active and collaborative learning; first year statistics

Over the last decade, from primary schools to higher education institutions, there has been an emphasis on the role of learning spaces in encouraging or debilitating students' learning experiences. Many new classrooms are built with public funds to create active and collaborative learning spaces which, it has been argued, enable better learning experiences.

Students' perceived learning experience obtained through a quantitative survey in their last tutorial class in 2012 (n=226) in a large first year statistics unit at Macquarie University supports such claims. We found that active and collaborative learning is significantly correlated with students' satisfaction in their tutorial classes. Students felt comfortable solving problems after their active and collaborative learning, since they believed that they had a deeper understanding of the concepts. We did not find enough evidence to conclude that the in-class activities had prepared students to tackle the unit's assessment tasks more effectively or enhanced their critical thinking ability.

# The changing notion of a mathematics problem in the internet age

## MARCELO C. BORBA
*State University of Sao Paulo, Brazil*

> *"If production of knowledge is understood in this way, what constitutes a ''problem'' will depend on the nature of the humans-with-media collective. A problem that needs to be solved, or that puzzles someone, may not be a problem when a search software tool like Google is available. Similarly, a real problem for collectives of humans-with-orality may not constitute a problem for a collective of humans-with-paper-and-pencil." (p. 804, Borba (2012)"*

In this talk I will unpack the above quote from a paper recently published in ZDM. I will discuss first the way internet and mobile phones in particular and digital technology in general, are changing the nature of what it means to be a human being (Castells, 2009; Borba, 2012). In order to do this, I will present the notion of production of knowledge that emphasizes the role of different technologies throughout history (Levy, 1993; Villareal, Borba, 2010). I will discuss how different media reorganize mathematical thinking (Tikhomirov, 1981, Borba and Villarreal, 2005). I will briefly discuss how different artifacts such as the blackboard are important for the development of education and how notions such as demonstrations are embedded in media such as paper-and-pencil. Examples related to the way simulation has been gaining importance – in mathematics and mathematics education - after computers became available will also be presented.

I will next discuss a perspective regarding the notion of "problem" in which problem is seen as having an objective and subjective aspect, respectively an obstacle to be overcome and an interest in overcoming such an obstacle. In doing so, I will argue that a problem changes depending on the media that is available; in other words, I will discuss how different collectives of human-with-media (Borba and Villarreal, 2005) relate to different obstacles, and how they may become a problem or not to such a collective.

Having discussed the notions of humans-with-media and problem, I will bring this a discussion to the classroom. I will show how a simple function activity may be a problem for collectives of humans-with-paper-and-pencil, but not for a collective of humans-with-Geogebra. Could "calculate the integral of $x^2/(1-x^2)^{-1/2}$ or of $\ln(x)$" be a problem for some students in the math classroom? I will argue that this depends if *wolphram alpha* is admitted or not.

I will show these different examples showing how digital technology has evolved, how it has introduced different possibilities of problems, and how it has transformed "old problems" into mere "exercises".

Finally I will discuss whether or not the Internet should be admitted into the classroom (Borba, 2009) and I will show how I am admitting Internet in the classroom where I teach basic notions of Calculus. I will show how video, Facebook and regular software can create different possibilities for learning in the regular classroom and in the "online classroom". Open questions will be presented and discussion will be presented regarding whether the Internet will be admitted in the classroom, or whether the classroom will be dissolved in the Internet.

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## Changing assessment: Shifting the emphasis to learning and use

DAVID BOUD

*University of Technology Sydney, Australia*

The dominance of grading and certification is being replaced by a view that emphasises learning and how assessment should foster it and represent it. A new agenda is emerging that frames assessment not in terms of knowledge acquired (assessment of learning), but as building students' capacity to make judgements (assessment for learning). The presentation will outline how the agenda has been changing and consider implications for how assessment is conducted in higher education.

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# Curriculum design, assessment and technology in mathematics for biomedical sciences: a case study

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*University of Melbourne, Australia*

**Keywords:** Assessment; biomedical science; curriculum design; technology

We will discuss the principles of student-centred curriculum design, outcome-based assessment design, and the use of interactive technology in the development of a new mathematics subject for undergraduate biomedical science students. The subject is intended to teach quantitative skills as well as an appreciation of the role of mathematics in biomedical science. We will describe the principles informing the design of the curriculum, such as the choice of topics to complement other biochemistry and genetics subjects in the course and based around themes such as equilibrium and stability. We will discuss the use of various interactive applets and similar technology in the course, and their effect on student learning. We will also discuss the design of assessment, including the use of oral presentations to develop and assess oral communication skills, with reference to the Threshold Learning Outcomes for Mathematical Sciences.

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# Roving mathematics assistance

CARMEL COADY, LYN ARMSTRONG, HARKIRAT DHINDSA, JOHN NICHOLLS, JIM PETTIGREW AND DON SHEARMAN

*University of Western Sydney, Australia*

**Keywords:** Mathematics support; library roving; university students

In recent years, universities have experienced an influx of students enrolling in courses that require a level of mathematical competency which many students lack. Hence these institutions have been under increasing pressure to provide a variety of support mechanisms to assist students, not only with the content of the mathematical units undertaken, but also to bridge any gaps in the assumed mathematical knowledge of students. Students' interaction with many of these services occurs at times and places dictated by the providers, with the result that many students who require assistance, do not avail themselves of it. With the changing role of the library to a "library commons" and in an attempt to reverse this trend, it was decided to trial a service whereby "we come to the students", by way of Library Roving. This presentation will outline the results of the piloting of this type of assistance in a university setting, with preliminary results indicating that students view such support very favourably and that students in all years, from a variety of courses, receive assistance when and where they decide. The staff involved have also been very receptive despite at times, being challenged by the diversity of the questions asked.

# Do highly mathematics-anxious students fear mathematics or the thought of attempting a mathematics question?

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*Curtin University, Australia*

**Keywords:** Mathematics Anxiety; Brain Activity; Tertiary Mathematics

Commonly heard phrases in any mathematics class to students are 'use your brain' or 'just have a go, write something'. What if the brain reacts to hinder the mathematical thought process before it even has a chance to start? Do students fear the thought of attempting a mathematics question more than the actual mathematics itself? It could be this fear that prevents students from obtaining a better grade in tests rather than lack of mathematical knowledge.

The work of Beilock and Lyons (2011) investigated how the brain reacted to knowing a mathematical rather than word question was to be asked. The highly mathematics-anxious students who did not activate the associated part of the brain proceeded to answer only 68% of the questions correctly. So the answer to helping highly mathematics-anxious students may be utilising this control of fear rather than extra exercises. This presentation will discuss simple activities that may aid our weaker students to improve their score, recognise and control their fear and decrease mathematical anxiety.

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# Summer school versus term-time for fundamental mathematics at the tertiary level

DAVID EASDOWN, GEORGE PAPADOPOULOS AND COLLIN ZHENG

*University of Sydney, Australia*

**Keywords:** Summer school versus term-time; fundamental first year mathematics; students with learning difficulties

At the University of Sydney, there exists evidence that students undertaking first year mathematics units achieve superior teaching and learning outcomes and experience higher overall course satisfaction by completing these units at Summer School rather than during standard term-time. We discuss relevant issues, with far-reaching implications for intensive, short-duration type teaching and learning in general, and with the particular aim of improving teaching practices and quality of learning during term-time. This is especially important due to the mandatory nature of first year mathematics for Science degrees at the University of Sydney, which becomes problematic for students with an inadequate background in mathematics from high school, with learning difficulties or phobias, or returning to study after long absences. Many such students enrol into Fundamental Level units, which form the focus of this research. We make qualitative observations about students' relative rates of progress and development of mathematical skills and maturity, inspect quantitative data about relative performances in term-time and in Summer School, discuss possible reasons for the differences, and attempt to place the findings within the contexts of modern theories of learning, such as the theory of threshold concepts, constructive alignment and the SOLO taxonomy.

# An elementary statistical workshop course for postgraduate research students

ANT EDWARDS[1] AND CHRIS MELLOR[2]

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**Keywords:** Statistics support; workshops; postgraduate; graduate; statistical confidence

Postgraduate students from Arts, Humanities and Social Science subjects increasingly need a firm grounding in school- and college-level statistics. This need is only partially met by departmental research methods courses, and Mathematics and Statistics Support Centres are increasingly relied upon to provide additional statistics support for postgraduate students. This presentation reports on a workshop series designed to introduce students to the topics of descriptive statistics, graphs, hypothesis testing, and the computer package SPSS. It provides evidence that the workshops met their aims and that they were popular both in terms of student attendance and feedback. It demonstrates that such a workshop series may (a) attract attendees from a wider variety of departments than might be expected, (b) be of interest for both lectured Masters and research PhD students and (c) that it can be a good way of increasing knowledge of other forms of support available from Mathematics and Statistics Support Centres.

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# Modified Japanese lesson study for post-secondary mathematics education

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[2] *University of Northern Colorado, United States*

**Keywords:** Pre-calculus courses; lesson study; curriculum development

For several years Japanese educators have engaged in a method known as *lesson study* to develop elementary curriculum. *Lesson study* is a practice in which teachers consider long term goals of education, determine the goals of their particular subject area, plan specific lessons and then systematically study the impact of these lessons through classroom testing. Although *lesson study* has appeared almost exclusively among teachers at the elementary level of education – and most educational research comes from this level – there is evidence that this practice can be applied at the post-secondary educational level. In this presentation we will share our experience of implementing *modified lesson study* at the college level. We will discuss how we implemented these practices to develop mathematics lessons that would address a lack of student success in a multi-section course (College Algebra) infamous in the United States for its high failure rate. Indeed, the difficulties students encounter in this course have been acknowledged as one of the leading causes of student drop-out at the post-secondary level. In our presentation we will demonstrate our *modified lesson study* framework by discussing *Blood Alcohol Level,* a lesson designed as an introduction to the concept of mathematical function and their models.

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# Practising engineers' conceptions about how mathematics should be taught to engineering students

JOHANN ENGELBRECHT[1], CHRISTER BERGSTEN[1] AND OWE KÅGESTEN[2]

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**Keywords**: Engineering students; procedural mathematics; conceptual mathematics

Demands from engineering faculties to mathematics departments have traditionally mainly been for teaching computational mathematical skills to engineering students. At the same time high expectations exist in terms of analytic and creative knowledge based skills for engineers as a work force. We report from a collaboration project between South Africa and Sweden with the aim to investigate whether the emphasis in undergraduate mathematics courses for engineering students would benefit from being more conceptually oriented than the traditional more procedurally oriented way of teaching. In this presentation we report on conceptions of practising engineers about this issue.

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# How should we treat repeating students?

JOHN HANNAH AND ALEX JAMES

*University of Canterbury, New Zealand*

Usually repeating students simply join the next occurrence of the failed course. They attend lectures and read handouts which assume they have never seen the material before. Often they are set the same homework, tutorial exercises and assignments that they were supposed to do the first time round. Unfortunately, by repeating the entire course experience like this, such students often drift towards a repeat of the failure too. This talk looks at a small group of students repeating a first-year engineering mathematics course. For political reasons, rather than educational ones, the repeat version of the course was restricted to students who had failed the course in the previous semester. But educationally, this meant that the course could be tailored to their common background. Thus the lecturer could take advantage of their prior experience of the course content while at the same time focusing attention on generic and transferable skills, such as time management, study habits and applying mathematical ideas in new contexts. All the students passed the repeated course, and the lessons learned seem to have persisted since, as a group, they have performed significantly better in their second year mathematics course than might have otherwise been expected.

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# Attitudes of pre-service primary teachers to mathematics

DILSHARA HILL AND CAROLYN KENNETT

*Macquarie University, Australia*

**Keywords**: Mathematics; attitudes; pre-service primary teachers

In this presentation we look at the attitudes that pre-service primary teachers have towards mathematics, and in particular we look at the cohort of these students at Macquarie University. Our study involved surveying students regarding their mathematical background and their attitudes to mathematics. This presentation looks at the results, which were analysed using both a quantitative and qualitative approach. We will be focusing on the areas that affect attitude such as maths anxiety and students' mathematical background, together with discussing our own reflections about teaching mathematics.

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# Threshold concepts in finance: the role of mathematics

SUSAN HOADLEY, LEIGH WOOD, LEONIE TICKLE AND TIM KYNG

*Macquarie University, Australia*

**Keywords:** Threshold concepts; finance; mathematics; curriculum; pedagogy

Threshold concept theory proposes that there are a limited number of transformative concepts that are central to the mastery of any discipline (Meyer & Land, 2003, Cousin, 2006). Research into threshold concepts in finance is limited to the work of Diamond (2011, 2013) and Diamond and Smith (2011) in relation to quantitative finance and business statistics.

We report on a broad project to investigate staff and student perceptions of threshold concepts in finance, with the aim of improving curriculum design and identifying specific pedagogical practices for teaching threshold concepts in order to improve student engagement and outcomes. The particular focus of this part of the project is the investigation of mathematical and statistical threshold concepts in finance programs.

The project combines qualitative and quantitative methods in the collection and analysis of primary data from finance staff and students. Initial findings indicate a degree of ambiguity in relation to the role of mathematics in the finance curriculum. Threshold concepts offer a way to be specific and explicit about the role of mathematics in finance and address this ambiguity.

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# Assignment submission via video in a large first year calculus class

DAVID HOLGATE, JUSTIN MUNYAKAZI, LEILA ADAMS AND DUNCAN SMITH
*University of the Western Cape, South Africa*

**Keywords**: Technology; mobile phone; video; assessment; group work

A face to face explanation of the solution to a mathematics problem demonstrates understanding far more than a written report. This assertion was the core motivation behind us asking our first-year calculus students to submit one of their weekly assignments by video.

Students in our course work in groups of five throughout the year and submit weekly assignments, usually as a group. These assignments form part of the ongoing assessment for the course. There are a number of difficulties in evaluating group submissions. Obvious concerns include judging who contributed to the solution, how much was copied and which group members understood. More subtly, such written submissions cannot always expose where misunderstanding occurs nor reflect the degree of group engagement.

In this talk we will report on the success of requiring groups to submit a mobile phone recorded video of them explaining the weekly assignment. This was exploratory on our part and proved to be more successful than we anticipated! Student engagement was very high and we were certainly able to examine the assignment with deeper insight. The intention is to include such assignments in our course more regularly next year.

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# The presence of mathematics anxiety in future primary school teachers and factors affecting abatement

SIMON JAMES, LYNN BATTEN AND MICHELLE CYGANOWSKI

*Deakin University, Australia*

**Keywords:** Mathematics anxiety; whole teacher approach; primary teaching

This study reflects on the implementation of various teaching initiatives for reducing anxiety toward mathematics in students studying to become primary school teachers. We highlight similarities between these practices and those promoted by the 'Whole Teacher' approach – in particular, the aim to develop attitudes along with knowledge and skills. Here, the negative past associations with mathematics and anxiety toward mathematics that students bring with them have been a key consideration when designing the subject content and delivery. Given the important role these students will have in shaping mathematics education in the future, we suggest frameworks such as that of the 'Whole Teacher' could be extended to the university setting. We investigate four years of student feedback pertaining to a first year undergraduate mathematics unit, contending that the teaching initiatives introduced over time have helped students develop a positive attitude toward mathematics. We note, however, that the student-teacher relationship was still the most prominent factor directly identified by students who previously had a fear or negative attitude toward mathematics.

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# Linear algebra for engineering: Using rectangular systems

DAVID J. JEFFREY AND ROBERT M. CORLESS

*Western University, Canada*

**Keywords**: Gaussian elimination; engineering linear algebra; computer algebra

This presentation addresses two themes. The first theme is the giving of greater emphasis to the analysis of rectangular systems when teaching linear algebra. The argument is made that, especially when teaching engineers, for whom the computational aspects of the subject are typically preferred over abstract concepts, there are advantages to the shifted emphasis. The second theme is the teaching of linear algebra with technology, especially with symbolic (computer algebra) systems. The connection between the themes is that the first theme leads inevitably to many questions requiring the row reduction of matrices. This activity is dull and highly error prone, and yet critical to the analysis of problems. Hence, the relegation of row reduction to a computer is desirable.

If a computer-algebra system such as Maple is used for the row-reduction of a matrix, the intermediate steps are not shown. If a matrix contains one or more symbolic elements, there is a danger that the final result will not be valid for all values of the parameter. One solution is to get the computer to return the "Turing factors" of the matrix. Then, in addition to obtaining the desired reduced row-echelon form, the user gets a warning of special cases.

# Factors influencing student decision on senior secondary school subjects

MICHAEL JENNINGS

*The University of Queensland, Australia*

**Keywords:** Transition; first-year; attitudes

There are substantial and ongoing concerns in the Australian and international secondary and tertiary education sectors about students' transition from secondary to tertiary mathematics. Declining enrolments in university mathematics and increasing failure rates in first year are often attributed to falling participation in advanced mathematics in secondary school and less stringent university entry requirements, which have adversely affected students' mathematical preparedness for university study.

In this presentation I will present data collected on three topics: reasons for choosing/not choosing advanced mathematics in secondary school, attitudes towards learning mathematics at school, and attitudes towards learning mathematics at university. This data was collected from four separate groups of people: secondary school mathematics students, secondary school mathematics teachers, university mathematics academics, and university mathematics education academics. The results suggest that there are distinct differences in students' thoughts depending on which mathematics they study in the last two years of secondary school. There are also differences between what students say are the reasons for their subject choice and what mathematics academics think are the reasons. The data also sheds light on subject choice myths. This presentation is part of a two-year state-wide longitudinal project that is investigating the transition from secondary to tertiary mathematics.

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# Future state of the evolving classroom in mathematics

BATHI KASTURIARACHI

*Kent State University at Stark, United States*

**Keywords**: Blended courses; cooperative learning; experiential learner

The renewal movement in undergraduate mathematics in the United States has roots that trace back to the 1986 Tulane Conference that outlined a vision for change in calculus along with suggestions for implementation. The movement, with its bold and innovative approaches focused on student-centered learning, was able to uncover the richness in undergraduate mathematics by supporting reform that was curriculum driven at the bow and technologically backed at the stern. As we approach the three decade mark of the renewal movement we see a rapid evolution that is occurring, which without proper streamlining, could have a profound impact on all of undergraduate mathematics education. Outstanding pedagogical practices take into account the environment in which learning occurs as well as the background of the student body, making the finest practices, institution dependent. This presentation reports on three models of delivery that are currently being used and have worked successfully to varying degrees. We will emphasize one of these – the blended model, and explain how it could be used to motivate students to excel in mathematics. Details of these pedagogical practices, as well as appropriate evidence of success, are presented.

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# Effectiveness of online lectures in a first year mathematics unit

CAROLYN KENNETT AND DILSHARA HILL

*Macquarie University, Australia*

**Keywords:** Online lectures; mathematics; first year students

There has been a considerable amount of literature examining the student response to online lectures in mathematics. Fewer papers, however, consider the effectiveness of online lectures as a vehicle for enhancing student success in mathematics. In this presentation we examine the results of a study done to determine the effectiveness on online lectures in a first year mathematics unit. The study involved surveying the students regarding their use of the online lecture material and an analysis of the results of different cohorts of students. Our sample included students who only accessed the lectures online as well as a cohort who attended both the "live" lectures and used the online material as necessary. Using the results of this study, student feedback, and our own reflections we will discuss some conclusions that can be drawn from this investigation.

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# The role of first year coordinators of mathematics programs

DEBORAH KING

*University of Melbourne, Australia*

**Keywords:** First year; transition; mathematics

In this talk we discuss our progress on the OLT funded project Building Leadership Capacity in University First Year Learning and Teaching in the Mathematical Sciences, which seeks to build a vibrant national network of scholars who provide leadership in first year learning and teaching in the mathematical sciences. In addition, it aims to examine and promote the unique leadership role of first year coordinators and educators.

We have now interviewed sixteen academics who have a significant role in first year mathematics teaching. We asked these academics what their responsibilities are in relation to curriculum design, administration, student support, student learning and transition activities.

In addition, for those with designated leadership roles, we asked how they and others viewed their role, why did they want to hold such a position, did they have a high level of job satisfaction, and did they see that there was a career pathway for them once they had left their position.

In our June workshop, around 40 academics gathered to discuss the main challenges of their roles as mathematics educators. Student preparedness, diversity of incoming students, student engagement and high academic workloads were common examples given. We give a brief report of these findings.

# Leading students to creatively develop mathematical models via leaky buckets

BRYNJA KOHLER AND JAMES POWELL

*Utah State University, United States*

**Keywords:** Problem-based learning; mathematical modelling; instructional strategies

Suppose you have a bucket full of water with a hole in it, and you want to predict how long it will take for the bucket to empty. One can derive a slick first order differential equation for the height of the fluid as a function of time based on elementary physics as done by Torricelli in the 17th century – but how useful is such a model for practical purposes? In our experience this classical model is actually quite poor at capturing real data, creating ripe ground for problem-based learning. We have developed a laboratory exercise and modeling problem for undergraduate students in which they are challenged to come up with a better model than the classic. But a good problem doesn't teach itself. We have also studied practical instructional strategies that mathematics educators have endorsed for maintaining a productive and challenging classroom atmosphere while groups of students work on cognitively demanding tasks at the elementary level. In our study, we observed that analogous instructional approaches are effective for keeping university students engaged at high levels of cognition while developing original mathematical models.

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# Dealing with engagement issues in engineering Mathematics

CHRISTINE MANGELSDORF

*University of Melbourne, Australia*

**Keywords:** Engineering mathematics; engagement issues; assessment; tutorials

Engineering Mathematics is a core second year level subject for students majoring in all branches of engineering at the University of Melbourne. The subject has been taught three times a year since 2009. From the outset, teaching staff had difficulty engaging the students in Engineering Mathematics. Every semester, there were issues with poor lecture and tutorial attendance, substandard performance on continual assessment and on the final exam, and poor pass rates. To deal with these engagement and performance issues, in 2013, teaching staff introduced a hurdle on continual assessment and changed the format of tutorials. In this talk, I will discuss the challenges faced, the changes made to assessment and tutorials, and the effect of the changes on student engagement and performance in assessment

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# Aiding conceptualisation in first-year statistics with interactive applets

ANTHONY MORPHETT

*University of Melbourne, Australia*

**Keywords:** Statistics; applets; Mathematica

Visualisation is an important part of conceptualisation, and interactive computer applets are a medium well-suited for aiding visualisation of statistical concepts. Interactive applets have been used for this purpose in statistics education for many years, however there are still important statistical concepts for which no suitable interactive resources exist. We will report on a series of online interactive demonstration applets currently being developed which explore fundamental concepts of statistics in ways that existing resources do not. The applets are written largely in Mathematica and freely accessible in a browser via the Wolfram Demonstrations Project. We will discuss the pedagogical and technical principles informing the design of the applets and their implementation in first-year statistics classes, and discuss our experiences with Mathematica as a platform for such resources.

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# Film: developing a new resource for teaching undergraduate biometry

MIRANDA MORTLOCK

*University of Queensland, Australia*

**Keywords:** Film; biometry; experimental design; video; visualisation

A 15 min pilot video made by the speaker became a valuable teaching tool in a large biometry course (STAT2701) in the School of Agriculture and Food Science at the University of Queensland.

This video was motivated by the need to bring the relevance of experimental design and analysis into a large class of undergraduates who had little experience of research or experimentation. It aimed to bring the field, glasshouse and paddock into the lecture theatre.

The video covered biological variation as the background to biometry and its important use in experimental design. A script was developed for the filming of experiments in the research fields and glasshouses. Researchers and graduate students assisted in production.

Students appreciated the value of biometry as a relevant tool to assist them in meaningful scientific research. The final cut incorporated student and tutor feedback.

The success of this has led to another project in 2013 involving students participating in the visualisation of concepts in biometry and two more films under production.

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# The significance of designing a course in mathematics focusing on fundamental mathematical concepts

SHO NIITSUMA AND RYOSUKE NAGAOKA

*Meiji University, Japan*

**Keywords:** New approach toward modern mathematics; the theory of real numbers; tertiary mathematics education

Very regrettably, most students become discouraged in the earliest stages of fundamental mathematics courses and only do enough to ensure that they pass. Ultimately, some fail to master advanced mathematical concepts. To point out plausible reasons why they fail is easy, but it is of no use to decide which is the most critical. I propose a change in the approach towards tertiary education, specifically within mathematics departments. The most essential point is to not rush the exposition of foundational theories to students. Instead, we should try to pique the students' interest in mathematics by teaching historical, cultural, and philosophical aspects of modern mathematics. By experiencing a deeper understanding of mathematics, young students can become more enthusiastic in learning and become braver to explore a new world of mathematics. In order to achieve this goal, it is believed that university mathematics teachers should refrain from teaching as many mathematical topics as possible and as fast as possible. Instead, they should become prudent, and put some emphasis upon the non-technical value of university mathematics. Not all mathematics lessons should be preparation for further development of "advanced" or "applied" theory.

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# Assessing students using multi-choice tests and exams: How to examine skills, processes and in-depth mathematical understanding

JULIA NOVAK AND TANYA EVANS

*The University of Auckland, New Zealand*

**Keywords:** Assessment; multi-choice questions (MCQ); learning outcomes; concept maps

Several years ago we decided to move to a MCQ format for tests and exams for our large first year mathematics courses. A major consideration has always been whether we are assessing in ways comparable with long answer pen and paper tests. We will discuss how our questions are created to test a wide range of learning outcomes. The possible use of expert concept maps to inform identification of key outcomes will also be discussed. Our exploration of ways in which a wider spectrum of learning outcomes might be assessed using MCQs forms part of the Learning in Undergraduate Mathematics, Outcomes Spectra Project (LUMOS) funded by Ako Aotearoa (National Centre For Tertiary Teaching Excellence) and TLRI (Teaching & Learning Research Initiative).

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# Immersive technology in an undergraduate mathematics course

GREG OATES, LOUISE SHERYN AND MIKE THOMAS

*The University of Auckland, New Zealand*

**Keywords**: Technology; first year courses; pedagogy

A team led by Bill Barton and Judy Paterson at The University of Auckland is investigating different approaches to the teaching and learning of undergraduate mathematics, in a project funded by Ako Aoteoroa called Capturing Learning in Undergraduate Mathematics. A large part of the study centres on the development of three innovative approaches, one of which is "Intensive Student Use of Technology". This technology component of the study is being developed by Greg Oates, Louise Sheryn and Mike Thomas, in a first-year foundation course at Auckland called Functioning in Mathematics, which essentially covers basic Calculus and Linear Algebra topics at approximately the same level as senior secondary courses. This presentation will briefly outline the immersive use of technology being trialled in this project, and the nature of the research being conducted. Some preliminary results from the pilot phase of the study will also be presented.

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# Using Moodle for assessments of large groups: Mathematics students' experiences

PRAGASHNI PADAYACHEE

*Nelson Mandela Metropolitan University, South Africa*

**Keywords:** Online assessment; Moodle; blended learning; mathematics students

With the rapid growth in higher education in South Africa, student numbers have grown considerably in many undergraduate courses. Teaching large groups seems to have become the norm at many higher education institutions. Assessments of these large groups is one of the biggest challenges facing educators. To enhance student learning consideration should be given to efficient assessment and effective feedback of these assessments. In response many are considering online assessment.

This action research study describes the experiences of a of group of 469 students taking the mathematics for accounting course and taking their assessments via Moodle at the Nelson Mandela Metropolitan University in the Eastern Cape of South Africa.

Preliminary analysis of the data revealed that students maintained that Moodle provided a supportive learning environment in which to learn mathematics. Students indicated that assessments were more effective and efficient, providing immediate feedback and that this approach to assessment impacted favourably on their mathematics learning.

Knowledge gained from this study may contribute to an improvement of the assessment practices of the mathematics for accounting course by presenting its pitfalls and successes.

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# Calculus unlimited

JOHN WILLIAM RICE
*The University of Sydney, Australia*

Curriculum reform is a major preoccupation at all levels of education around the world. Science and Mathematics have received particular attention because they are fundamental to 21$^{st}$ century society, they are considered abstract and difficult, and students dislike them, often with a passion. I focus on reform of the calculus curriculum because it is the quintessential example of this problem.

I propose that the calculus curriculum could be improved considerably by avoiding the concept of a limit. I argue that the ideas preceding Newton and Leibnitz, such as expressed in the work of Galileo or Fermat for example, are actually quite precise in their context, more intuitive and develop better the threshold concepts of function, and derivative along with the rules of calculus.

The direct interpretation of infinite decimals, which does not involve limits, provides the threshold understanding of real numbers, integrals and sums of series. This understanding promotes simple numerical methods, particularly the decimation method, into methods for exact solution of equations rather than mere approximations. In fact, the decimation method becomes a proof of the intermediate value theorem. The fundamental theorem of calculus likewise becomes more transparent via the threshold understanding of real numbers and function values.

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# Mathematics exams as a learning process to build skills, knowledge and confidence

CAROLYN. E. SANDISON AND CAROLE. L. BIRRELL

*University of Wollongong, Australia*

**Keywords:** Mathematics confidence; feedback; mastery exams; pre-service teachers

This paper focuses on the challenges of inspiring mathematics learning in pre-service primary teachers and the subsequent modifications made to a pair of mathematics content subjects to address these issues. Many of the student cohort have poor basic mathematical skills, little confidence in their mathematical ability and as a result often exhibit avoidance behaviour, yet need an in-depth understanding of certain mathematical concepts to be successful in their chosen career. To motivate learning, modifications were made to the implementation of the subjects which include the introduction of mastery exams into the assessment and additional face-to-face class time providing support for those students deemed to be at risk. We present our particular context, a review of the literature and details of our mastery exam system. We reflect on the implementation of this system, the outcomes in student performance, and student reaction towards the assessment structure.

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# Motivating students in an introductory matrix algebra course

KARSTEN SCHMIDT

*Schmalkalden University of Applied Sciences, Germany*

**Keywords:** Matrix algebra; magic square; eigenvalue; inverse

Linear algebra is certainly an area of mathematics of increasing importance. Unfortunately, most business and economics students do not appreciate courses in mathematics and statistics. This is particularly true for the (introductory) matrix algebra course because most topics are rather abstract and easy-to-apply examples are difficult to find.

However, magic squares can be very helpful in stimulating students' interest in matrix algebra and are easy to apply. A magic square of order n is a square arrangement of $n^2$ real numbers, such that the sum of the elements in each row, column, and diagonal is equal to a constant s, its magic sum.

Many interesting activities can be carried out in class at very different stages of the course, using a Computer Algebra System to facilitate computations.

Any 3x3 magic square M can be written as the sum of two matrices, $M = sG + N$, where $G = 1/3J$ (J denotes the matrix of ones), and also N has a simple structure defined by only two real numbers. This allows additional interesting activities.

Magic squares from different times and regions, like the ancient Chinese 3x3 Lo-Shu (4,9,2;3,5,7;8,1,6), will be used as examples in the presentation.

# Teaching foundations of finite group theory via programming

ILYA SHILIN

*Sholokhov Moscow State University for the Humanities, and Moscow Aviation Institute, Russia*

This presentation contains some programming problems which can be suggested for first-year mathematics students starting to learn foundations of group theory. These problems are related to important notions such as subgroup, coset, normal divisor, symmetric group, normalizer, centralizer, homomorphism, and automorphism. To translate a problem with mathematical language to a program, students should thoroughly understand the essence of the problem, by considering the problem in detail. When translating the results obtained on the computer back into mathematical language, the same happens. Thus, the using of programming contributes to a successful, non-formal learning theory and promotes programming skills. The above problems are suggested for students of the Faculty of Science at Sholokhov Moscow State University for the Humanities. More details of our approach are available in I. A. Shilin, Some Programming Problems for Learning Foundations of Group Theory, Int J Math Educ Sci Technol, **DOI:**10.1080/0020739X.2013.837523.

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## Views of mathematics assessment in the UK

ADRIAN SIMPSON
*Durham University, United Kingdom*

This talk will outline the key findings of the Higher Education Academy funded project to explore assessment in mathematics degrees in the UK. It will outline current assessment practices and less well known alternative forms of assessment. However, it will focus on three different, but inter-related views on assessment: the view of students on how they are currently assessed, the views of staff on assessment practices and the views of students on encountering a form of assessment which was quite new to them.

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# An interactive online calculus text for the iPad (and other browsers)

DAVID SMITH AND LAWRENCE MOORE

*Duke University, United States*

**Keywords**: Differential calculus; integral calculus; interactive text; internet; sage computer algebra system; iPad; standards-compliant browser

The online textbook, Calculus: Modeling and Application, 2nd edition, published by the Mathematical Association of America (MAA), has been adapted for reading and interacting on the iPad. The entire text and its interactions are accessible in any standards-compliant browser (e.g., Firefox, Safari) on any platform. The major changes in this adaptation include replacing XHTML pages with HTML, recasting mathematical symbols in MathJax, and replacing use of commercial computer algebra systems (CAS) with embedded "interacts" that are processed by the open-source CAS Sage. Features of the book will be demonstrated directly from an iPad, and attendees are welcome to use their own iPads, notebooks, or other Internet-capable devices.

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# Study process for mathematics: Disjointed or embedded?

MARITZ SNYDERS AND SARIE SNYDERS

*Nelson Mandela Metropolitan University, South Africa*

**Keywords:** Learning development; embedding skills; study strategies; mathematics teaching

Are our students underprepared or is our institution unprepared for the students that enter higher education? Do students have the necessary skills to cope with the demands of tertiary studies? Are the needed competencies offered in a disjointed or an embedded manner? These are important questions when we consider mathematics pass rates, particularly in South Africa with our huge shortage of qualified teachers. Lecturers are challenged to embed skills development in the classroom and thereby offer students learning development opportunities.

The collaboration between a Student Academic Developer and mathematics lecturer led to integrating study strategies into the mathematics classroom. Students were offered a generic Study Process workshop, the study strategies were then adapted and applied to studying mathematics. Action research methodology was used and students wrote reflective journals on how they studied mathematics in the past and how they study mathematics now.  These reflections, together with regular meetings between the researcher, a Student Academic Developer, and the mathematics lecturer resulted in a study process model for Mathematics.

In this presentation the research process that was followed, as well as some of the results, will be shared.

# Preliminaries for a first year course on modelling

KERRI SPOONER

*Auckland University of Technology, New Zealand*

**Keywords:** Mathematical modelling; transition secondary to tertiary

I will present the results of my 2012 research project on outcomes achieved by secondary school students who were given an authentic mathematical modelling experience. This research has impact on the potential understanding of modelling by secondary school students entering university. I will discuss what aspects of the modelling experience were possible at secondary school and therefore should be easily achievable at undergraduate level; what aspects students enjoyed about the experience and are therefore useful to include; and what aspects they struggled with, providing a sign that these things need good instruction and relevant time.

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# Mathematical knowledge of the lecturer and mathematical knowledge for lecturing at the undergraduate level: An attempt to distinguish between the two

DHANYA SURITH

*University of Auckland, New Zealand*

**Keywords:** Mathematical knowledge; mathematical knowledge of teacher; mathematical knowledge for teaching

This paper is an attempt to distinguish between mathematical knowledge possessed by a lecturer and mathematical knowledge that is used for lecturing. For that, existing theoretical models on mathematical knowledge for primary and secondary teaching were examined. The key idea is the understanding that lecturing contexts mostly reveal mathematical knowledge used for lecturing, but not the entirety of mathematical knowledge possessed by the lecturer. This broadens the notion of mathematical knowledge for lecturing (MKfLg) by embedding it in the mathematical knowledge of lecturer (MKoLr). Acknowledging the difficulty in predicting exact boundaries between these two mathematical knowledge bases, working definitions are constructed. A major research question that may arise on the acceptance of these definitions is what particular component of MKoLr is the main contributor to MKfLg. To this end, my research is to develop a theoretical model of mathematical knowledge required for lecturing at the tertiary level, based on models constructed for secondary teaching. The model will be adapted to include research knowledge and other tertiary factors. Of particular interest will be the unique and personal knowledge lecturers gain from their own research work.

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# Differential equations and handheld CAS technology

PATRICK TOBIN AND VIDA WEISS

*Australian Catholic University, Australia*

**Keywords:** Technology; differential equation; computer algebra systems

The use of handheld technology for computer algebra systems (CAS) in test and examination assessment within undergraduate mathematics courses in Australia and elsewhere has proved controversial. In this talk we present some results on the use of the CAS calculators in a service mathematics course focusing on the teaching of differential equations. We will examine some features of using the CAS and draw on experience from elsewhere. The results suggest that the time to master the tool is not a problem, though this has been flagged elsewhere as an issue. Our results also show that students don't necessarily use the CAS to full benefit, even though they are generally positive about the learning tools. As with a CAS-free course there is still difficulty in expanding the role of conceptual learning over procedural learning. However, there are prospects for expanding students' conceptual learning through use of multiple representations, which combined with other CAS options in learning can free up time to look at applications, formulations and the interpretation of solutions.

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# Development of a summated scale for measuring approaches to assessment practice of undergraduate mathematics lecturers

SVEN TRENHOLM [1], LARA ALCOCK[2] AND CAROL ROBINSON[2]

[1] *University of South Australia, Australia and Loughborough University, United Kingdom*
[2] *Loughborough University, United Kingdom*

**Keywords:** Assessment; deep learning; approaches to teaching; psychometric instrument

Findings associate undergraduate mathematics lecturers with surface and transmissive approaches to teaching. Apart from the recognized dominance of summative instruments, little is known about these lecturers' approaches to assessment practice.

Early work in undergraduate science education led to the creation of a two-scale Approaches to Teaching Inventory (ATI) that was to be used to explore the relationship between approaches to teaching and other aspects of the teaching-learning environment. For similar purposes, basing our work on Samuelowicz and Bain's (2002) framework for identifying lecturers' approaches to assessment practice, we sought to develop a measure (S&B) that quantifies mathematics lecturers' approaches to assessment practice.

An online survey was created using both the ATI and a newly created S&B questionnaire. Seventy mathematics lecturers responded. Results show that the S&B measure was significantly ($p<0.01$) correlated, in the expected directions, with both ATI measures. With the ATI as an established psychometric instrument, findings suggest the S&B measure has some validity and is measuring a similar underlying construct. We discuss these findings along with some of the limitations and problems identified by participants.

Samuelowicz, K., & Bain, J. D. (2002). Identifying academics' orientations to assessment practice. *Higher Education, 43*(2), 173-201.

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# Mathematics and statistics for life science students: Discussing the contribution of mathematics and statistics departments

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**Keywords:** Quantitative skills; curriculum; life sciences

In the past decade, undergraduate science education, particularly the life sciences, have come under scrutiny for their lack of quantitative rigour. While some have argued that life scientists should teach quantitative skills (QS) within discipline context, recent consensus suggests that mathematicians and scientists should work together to build QS of science students. However, communication between scientists and mathematicians on matters of education has proven difficult. Building on the recent Australian Quantitative Skills in Science project, workshops aiming to facilitate communication and consensus between life scientists and mathematicians were conducted. These focused on several questions: What are the specific QS graduate learning outcomes for life sciences majors? What mathematical and statistical knowledge and skills are needed in the life sciences major? Where and how are these skills being taught across the life sciences major?

We will present data of workshop outcomes drawn from three sources: (1) participant sourced responses using standard templates provided before and during the workshops; (2) records of the workshop proceedings; and (3) documents obtained from participating institutions. This session will be informative but informal, inviting participants to engage in discussion on how mathematicians can, and should, contribute to the teaching of QS to life science students.

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# Using a classroom response system to transform student engagement

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**Keywords:** In-class response system; student engagement; formative assessment; team work

Student engagement with course material can be variable. Lectures are often didactic and in group tutorials it is often difficult to get everyone to make productive use of the time.

In-class response systems promote cooperative learning with "students becoming active participants in their learning" (Beatty, 2006). 'Socrative' allows any web-enabled device to respond to either ad-hoc or prepared tests, and has been used weekly with one final year group in 2012. This introduced team-work and an element of competition and engagement improved dramatically. Each session also included group discussion evaluating what had been learned and how 'Socrative' had helped, informing the delivery of the following class and involving students closely in the module design and delivery.

'Socrative' has also been used in lectures with a first year group to provoke discussion and for formative assessment. In this session I will demonstrate the system, giving examples of its use in a classroom setting, and discuss its effective use.

If you plan to attend, and own a smartphone or tablet device, please download and install the free 'Socrative Student' app first.

Beatty, I.D et. al., "*Designing Effective Questions for Classroom Response System Teaching*", Am. J Phys., 74, pp31-39, 2006.

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# The statistical anxiety rating scale: a review and a new application

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**Keywords:** Statistics anxiety; student demographics; first year teaching

The Statistical Anxiety Rating Scale (STARS) is a 51 item, 5 point Likert scale developed by Cruise and Wilkins in 1980 to measure levels of statistics anxiety in students. Despite its age, and some American-centric and out of date items, the STARS is still commonly used as a measure of student attitudes towards statistics. In this presentation I will cover three aspects of the scale, and some experiences with administering it to a cohort of first year online students. Firstly, I will examine the relevance of the scale to 21st century statistics education and discuss whether collecting such information is useful for statistics educators. Secondly, I will discuss some STARS results from a sample of Australian students completing an online first year statistics unit. Finally, I will present an example of where the STARS was administered to students, but it inadvertently became a useful teaching example and created some insightful, student-lead discussions about bias and data collection.

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# Student confidence in mathematics – pre- and post-support

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**Keywords:** Mathematics confidence; mathematics support; success

Learning Development university lecturers are charged with supporting students who find mathematics "challenging" but who need to study mathematics – and pass it – as part of the requirements of their non-specialist-mathematics courses. In order to measure both the success of the students who seek assistance through Learning Development and the possible effects of the support, the students' levels of confidence in mathematics were studied, comparing their confidence pre- and post-support. Students' mathematics confidence was studied in two different aspects: firstly, their levels of confidence in mathematics overall and secondly, their levels of confidence in the various topics for which they were seeking support through Learning Development. Did their level of confidence in a topic change after a support session and, perhaps more importantly, did their overall level of confidence in mathematics change over a period of support?

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# Practice to academy transitions

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**Keywords:** Professional development; mathematics; statistics; faculty

Macquarie Lighthouse was the first marine lighthouse built in Australia. Following the arrival of the First Fleet in 1788, the Macquarie flagstaff was erected to signal the approach of supply ships destined for Sydney Cove. Through the fog of an aging workforce, increased casualisation, student diversity and changes to modes, methods and media in teaching and learning, universities have been compelled to look elsewhere to recruit a substantial portion of their academic staff. In short, the demand for academics and academic capability is far exceeding supply. The sirens call radiating from universities promises new recruits greater work/life balance, increased flexibility and intellectual prestige. While affecting all disciplines, these broader sectoral trends directly affect the capacity of universities to provide quality teaching and learning; in particular, for undergraduate mathematics and statistics education. For many industry-practitioners, the above reasons are sufficient to compel them to 'make the move' from industry to academia. However, the everyday realities of life in higher education institutions come with a series of significant and unexpected challenges. Drawing on qualitative interview data with transitioned 'pracademics', this presentation explores the teaching and learning implications of these transitions and the personal/professional challenges experienced by these individuals.

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# A longitudinal study of students undertaking a mathematics major: Changes in attitudes, learning behaviours and achievement

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**Keywords:** Mathematics education; higher education; attitudes; learning behaviours; achievement; longitudinal study

How do a student's attitudes, learning behaviours and achievement in mathematics or statistics inter-relate with each other and how do these change during the course of their undergraduate degree program? In this talk I report on a longitudinal study of students through the three years of a mathematics degree. The two questions I investigated were "What are students' attitudes and learning behaviours towards mathematics and to what extent do these attitudes and learning behaviours change as students' progress in mathematics?" and "Are these attitudes and learning behaviours related to achievement?" Students were surveyed after each semester on a maximum of two mathematics or statistics courses they had taken. Responses from a small group of 21 students who had completed both first and third level course surveys are discussed in this talk as are the attitudes and learning behaviours that have changed over this period.

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# On an "essential" difficulty in bridging the gap between school and university mathematics

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Many authors have pointed out the "gaps" between school and university mathematics and much has been written on this since Klein's Elementarmathematik vom höheren Standpunkte aus. In the field of geometry we have a serious problem in bridging the gap. In Japan, where there was, and still is, a lot of emphasis on geometry in secondary schools, the scope is limited to poorly systemized elementary geometry (from a logical point of view) and to "common sense" analytic geometry. Meanwhile, in universities, only modern geometry with no active concern for "traditional" or "classical" geometry is taught. As a result, young university students feel much embarrassed with their first encounter with modern geometry and most of them are urged to "memorize" the important theorems and their proofs without any theoretical understanding. When they become school teachers they cannot teach geometry in any depth or with real understanding. I dare to point out the importance of giving lectures on a little old fashioned geometry in universities, especially where not all students are expected to be research mathematicians.

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# Posters

# Can instructors accurately describe their classroom practice? Assessing the impact of professional development on inquiry-based learning and teaching in undergraduate mathematics

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**Keywords**: Inquiry-based learning; professional development; evaluation

Professional development (PD) is an important tool for changing instructional practice in higher education. Yet in order to design and deliver effective PD programs, we must be able to measure the instructional changes that instructors do or do not put into practice in response to PD. We will present findings on the outcomes of multi-day workshops on inquiry-based learning (IBL) in college mathematics that offer insight on instructors' teaching practices and beliefs and how these may evolve in response to PD experiences.

Participating instructors had strong pre-existing beliefs in the value of inquiry, but their initial classroom practices emphasized lecture and solving examples. Pre/post surveys for 3 cohorts show that, during the workshop, faculty made strong gains in IBL knowledge and moderate gains in IBL skills. Their beliefs in IBL's effectiveness and motivation to use it—already strong—became even stronger. Follow-up results for the first cohort suggest that over 50% of respondents have made significant shifts toward greater use of student-centered teaching in the year following the workshop. Moreover, the evaluation instruments appear to be sensitive to differences in emphasis from workshop to workshop. Ongoing work will seek to validate items that self-report instructional practice against classroom observation.

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# Teaching and learning differential equations with graphical, numerical and analytical approaches

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**Keywords**: Teaching and learning; differential equations; computational resources; graphic numerical and analytical approach

The work we present was carried out to improve the differential equations teaching-learning process and explore the potential of computers to provide favourable conditions for meaningful learning. The research involves the development of a pedagogical practice with students of Engineering and Industrial Chemistry courses. The practice teaching proposals are focused on problem solving in situations with the use of computer which initially explore the equation solutions obtained using software and later use analytical techniques. The methodology of classes follows assumptions from the Vygotsky's Theory and the instructional materials are based on Ausubel's Meaningful Learning Theory. We used specially designed tools for information collection about the students' learning during the lessons: a questionnaire, interviews, guides to activities, initial and final knowledge tests and field notes. Our results indicate that the use of computational resources can be an important tool in the differential equations teaching-learning process. However, it is also important to emphasize that when we offer a differentiated approach we often create discomfort and dissatisfaction among the students, and despite working with numeric and graphic analytical approaches the students still prioritize the analytical techniques.

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# 'Who put the maths in chemistry?' Raising student awareness and self-efficacy in mathematical process skills in first-year chemistry

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**Keywords**: Quantitative skills; confidence; self-regulation; chemistry

The diversity of backgrounds, abilities and aspirations of students in large-enrolment first-year chemistry courses presents a challenge to instructors who aim to identify the nature of the remedial support necessary to underpin assumed mathematical skills. Many students enrol in chemistry without intermediate level senior secondary mathematics despite it being a prerequisite. Several of the basic skills that appear to be missing derive from even earlier in their high school experience including proportional reasoning and unit conversions. A separate but well known issue is that students, who may be very competent in basic mathematical process skills, struggle to recognise a mathematical relationship underlying a specific problem framed in a chemistry context (Finlayson, 2013). A diagnostic test delivered in 2012 to a large first-year chemistry cohort revealed that while many students had competence in senior high school mathematics, they possessed low confidence in their ability to apply their existing knowledge to chemistry problems. Supporting resources were released to the students through the online course learning management system (Blackboard) to encourage self-regulated study, this enabled application of analytics to students' access. A post-test (N = 806) and subsequent focus groups explored students' self-awareness of their own mathematical skills and their relation to chemistry problem-solving.

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