Swan Delta Conference 2019
Full Programme

Monday 25 November 2019
First Day

8:30am Abrolhos Room Registration Desk Open
Come and chat with fellow delegates while enjoying a morning coffee before the talks

9:00am Island Suite Opening of Conference
9:30am Welcome to Country

9:45am Island Suite Plenary Speaker

Tanya Evans
Who Is Tasked with Modernisation of Mathematics Education?
Research Mathematicians and Mathematics Education Researchers Bridging the Disciplinary Gap

10:45am – 11:10 Abrolhos Room Morning Tea

	Carnac Room Chair: Anita Campbell	**Garden Room** Chair: Simon James	**Rottnest Room** Chair: Michael Jennings
11:15 – 11:35am	**Bridgette Yani** Academic Maturity of Students in an Extended Programme in Mathematics	**Sepideh Stewart** Leading Students Towards the Formal World of Mathematical Thinking: A Mathematician's Reflections on Teaching Eigentheory	**Don Shearman** Analysis Using Natural Language Processing of Feedback Data from Two Mathematics Support Centres
11:40 – 12:00pm	**Emily Cilli-Turner** Tertiary Students' Changing Views on Mathematical Creativity	**Juan Ponce Campuzano** The Use of Phase Portraits to Visualise and Investigate Isolated Singular Points of Complex Functions	**David Hartley** Reflecting on The Role Of Centralised Mathematics Support
12:05 – 12:25pm	**Ching-Ching Yang** Design-Based Instruction Integrating Computational Thinking for University Statistics - From Probability to Errors in Hypothesis Testing	**Paul Hernandez-Martinez** Mathematics Lecturers' Views on Their Mathematical Modelling Practices	**Jeff Waldock** The Potential of Recreational Mathematics to Support the Development of Mathematical Learning

12:30 – 1:30pm Atrium Garden Restaurant Lunch

	Carnac Room Chair: Phil Kane	**Garden Room** Chair: Anthony Morphett	**Rottnest Room** Chair: Jeff Nijsse
1:35 – 1:55pm	**Bathi Kasturiarachi** Tight Connection to Success: The College Credit Plus Program	**Ant Edwards** Student Polling – It's the Taking Part That Counts	**Renu Choudhary** Reflection, Introspection, & Transformation
2:00 – 2:20pm	**Catherine Hassell Sweatman** Reflections on the Value of Teaching Mathematics to Art and Design Students	**Carole Birrell** A Project for Teaching Sample Surveys Using a Virtual Population	**Zanele Ngcobo** Transformation of Pre-Service Mathematics Teacher Specialised Content Knowledge Through Error Analysis
2:25 – 2:45pm	**Rachel Passmore** Capstone Courses - What, Why and How?	**Antony Dekkers** Changing Attitudes – How Academics Utilise Tablets in Mathematics	**Anthony Cronin** Supporting Lecturers to Monitor Their Students' Learning and Develop Their Practice Through Engaging with Mathematics Support Centre Feedback

2:50 – 3:15pm Abrolhos Room Afternoon Tea.

3:20pm Island Suite Plenary Speaker

Chris Sangwin
Playfulness and a Mathematics Education for the Twenty First Century

4:30pm Island Suite Catch up time at the end of the first day with Maths Crafts with Julia Collins and Katherine Seaton.

Swan Delta Conference 2019
Full Programme

Tuesday 26 November 2019
Conference Dinner

8:30am Abrolhos Room Registration Desk Open Come and chat with fellow delegates while enjoying a morning coffee before the talks			
8:45am Island Suite Conference housekeeping.			
9:00am Island Suite Plenary Speaker **Harry Wiggins** The Catch-22 of Teaching			
10:05am Island Suite Workshop: **Jessica Libertini** Designing Math That Matters: Leveraging Interdisciplinary Partnerships on Your Campus to Motivate Learning			
10:50 – 11:10am Abrolhos Room Morning Tea			
	Carnac Room Chair: Renu Choudary	**Garden Room** Chair: Emily Cilli-Turner	**Rottnest Room** Chair: Ant Edwards
11:15 – 11:35am	**Igor Kontorovich** Why do Students Not Check Their Solutions to Mathematical Problems? A Field-based Hypothesis on Epistemological Status	**Jeff Waldock** Custom Software Tools for Active Learning in Higher Education Mathematics	**Poh Hillock** A Support Learning Programme for First-year Mathematics
11:40 – 12:00pm	**Elsabe Weyer** The Influence of Student Engagement and Self-Regulation on the Performance of First-Year Natural Science Mathematics Students	**Jeff Nijsse** Coding Tutorials: Using Jupyter and Scratch to Teach Concepts in Blockchain	**Anita Campbell** Design-based Research principles for Successful Peer Tutoring on Social Media
12:05 – 12:25pm	**Deb King** Making the Grade: Do Maths Marks Matter for Women in STEM?	**Owen Jepps** Group-Work Video Presentations as an Effective Linear Algebra Assessment Task	**Phil Kane** Support for Students with Mathematics Learning Disabilities (MLDs) on Bridging or Foundation Programmes at New Zealand Universities
12:30 – 1:30pm Atrium Garden Restaurant Lunch			
	Carnac Room Chair: Greg Oates	**Garden Room** Chair: Rachel Passmore	**Rottnest Room** Chair: Cami Sawyer
1:35 – 1:55pm	**Kaitlin Riegel** Student-Lecturer Partnerships in Undergraduate Mathematics Question Design	**Christine Mangelsdorf** Developing Skills and Enhancing Feedback Through Online Assessment with Webwork	**Simon James** Anonymous Peer Feedback for Problem-Solving Portfolios
2:00 – 2:20pm	**Tracy Craig** The Twente Educational Model in Theory and Practice: Two Modules as Exemplars	**Karsten Schmidt** Teaching Decision Theory in the PC Lab	**Leanne Rylands** Workshop Support for First-Year Mathematics and Statistics
2:25 – 2:45pm	**Nazim Khan** Who Uses Online Material?	**Kevin McLeod** Project-Based Learning in an Introductory Algebra Class at the University of Wisconsin-Milwaukee	**Anthony Morphett** Modelling Mathematics Support Uptake with Rumour Models

Swan Delta Conference 2019
Full Programme

Tuesday 26 November 2019
Conference Dinner
(continued)

2:50 – 3:15pm Abrolhos Room Afternoon Tea.

	Carnac Room Chair: Harry Wiggins	**Garden Room** Chair: Johann Engelbrecht	**Rottnest Room** Chair: Barbara Miller-Reilly
3:20 – 3:40pm	**Trudie Benadé** The Relevance of a Mathematics Course for Computer Science Students	**Kerri Spooner** How Do We Teach Mathematical Modelling?	**Katherine Seaton** Laying Groundwork for an Understanding of Academic Integrity in Mathematics Tasks
3:45 – 4:05pm	**Maureen Edwards** Aspects of a Transitional Subject for First Year Mathematics Students: A Reflection	**David Easdown** Reaching Out: Introduction to Calculus	**Diana White** Using Primary Source Projects to Teach Undergarduate Mathematics Content: Analysis of Instructor Implementations and Perceptions
4:10 – 4:30pm	**Deon Solomons & Honjiswa Conana** Reflections of Change: Addressing Challenges in the Transition to Second Year Mathematics	**Robyn Reaburn** Crossing the Liminal Space: Students' Understanding of Confidence Intervals	**Anne D'Arcy-Warmington** Even in This Digital World, Simple Concrete Manipulatives May Be Used to Cement Mathematical and Statistical Understanding

4:30pm Abrolhos Room Catch up time at the end of the second day.

From 6:00pm for a 7:00pm start, until late
Conference Dinner
Fremantle Sailing Club
151 Marine Terrace, Fremantle

There will be activities during the dinner.

Wednesday 27 November 2019
Excursion Day

Excursion Day

There is a tradition of having an excursion day in the middle of the Delta Conference, to give participants the opportunity to explore the local area and form stronger networks.

For Swan Delta, this will occur on Wednesday 27 November.

Note that the excursion cost is not included in the conference registration and will be payable separately to the tour operator. A selection of possible excursions is available on the website and a link is provided to help facilitate excursion bookings. Please use the link before the start of the conference or organise your trip at the Registration Desk on the Sunday or Monday.

Swan Delta Conference 2019
Full Programme

Thursday 28 November 2019
Teachers Day

In addition to Swan Delta delegates we welcome Teachers Day delegates from Secondary Schools, Curtin College and Further Education Colleges. On Teachers Day we particularly encourage participation from colleagues engaged with secondary school and with the transition between school and university.

8:30am Abrolhos Room Registration Desk Open
Come and chat with fellow delegates while enjoying a morning coffee before the talks

8:45am Island Suite Conference housekeeping.

9:00am Island Suite Plenary Speaker
Janine Sprakel
Why CHOOSEMATHS?

10:00am Island Suite Speed Networking

10:30am – 11:10 Abrolhos Room Morning Tea

	Carnac Room Chair: Tracy Craig	**Garden Room** Chair: Paul Hernandez Martinez	**Rottnest Room** Chair: Kevin McLeod
11:15 – 11:35am	**Carolyn Kennett** Examining Pre-Service Teachers' Mathematical Confidence	**John Lamerand** Ratios, Geometry and Health Messages	**Margaret Marshman** Making Mathematics Teachers: The Beliefs About Mathematics and Mathematics Teaching and Learning Held by Mathematics Teacher Educators and Pre-Service Teachers
11:40 – 12:00pm	**Jyoti Jhagroo** Preservice Primary Mathematics Teachers' Professional Learning Within a University-School Partnership	**Michael Jennings** Teacher and Lecturer Perspectives on Secondary School Students' Understanding of the Limit Definition of the Derivative	**Jane Kirkham** Are Advanced Mathematics Courses Valued by Western Australian Male and Female Senior Secondary School Students?
12:05 – 12:50pm	Workshop Chair: Cristina Varsavsky **Cami Sawyer** Engaging Mathematicians in Addressing the Needs of the 21st Century Learner	Workshop Chair: Jeff Waldock **Michael Rutkay** Algebra Using Post-It Notes	Workshop Chair: Tanya Evans **Mary Jane O'Callaghan** Statistics and Origami

12:50 – 1:55pm Atrium Garden Restaurant Lunch

	Carnac Room Chair: David Rodda	**Garden Room** Chair: Chris Sangwin	**Rottnest Room** Chair: Sepideh Stewart
2:00 – 2:20pm	**Norm Hoffman** Randomness: Its Importance in Statistical Inference and How To Teach Students About It	**Bruce May** Mathematical Text Comprehension: The Case of a Cohort of Pre-Service Mathematics Teachers	Workshop **Helen Middleton** Are You Ready to Play the Science Game? Melding Maths Skills for Transition into University Science
2:25 – 2:45pm	**Harry Wiggins** Five Ways Complex Numbers Give Insight Into Real Numbers	**Ng Wee Leng** Pre-Class Tasks Used in a Flipped Linear Algebra Course for Student Teachers	

2:45 – 3:25pm Abrolhos Room Posters Session
Extended Afternoon Tea.

3:30pm Island Suite Plenary Speaker
Chris Matthews
Teaching Culture = Deep Learning

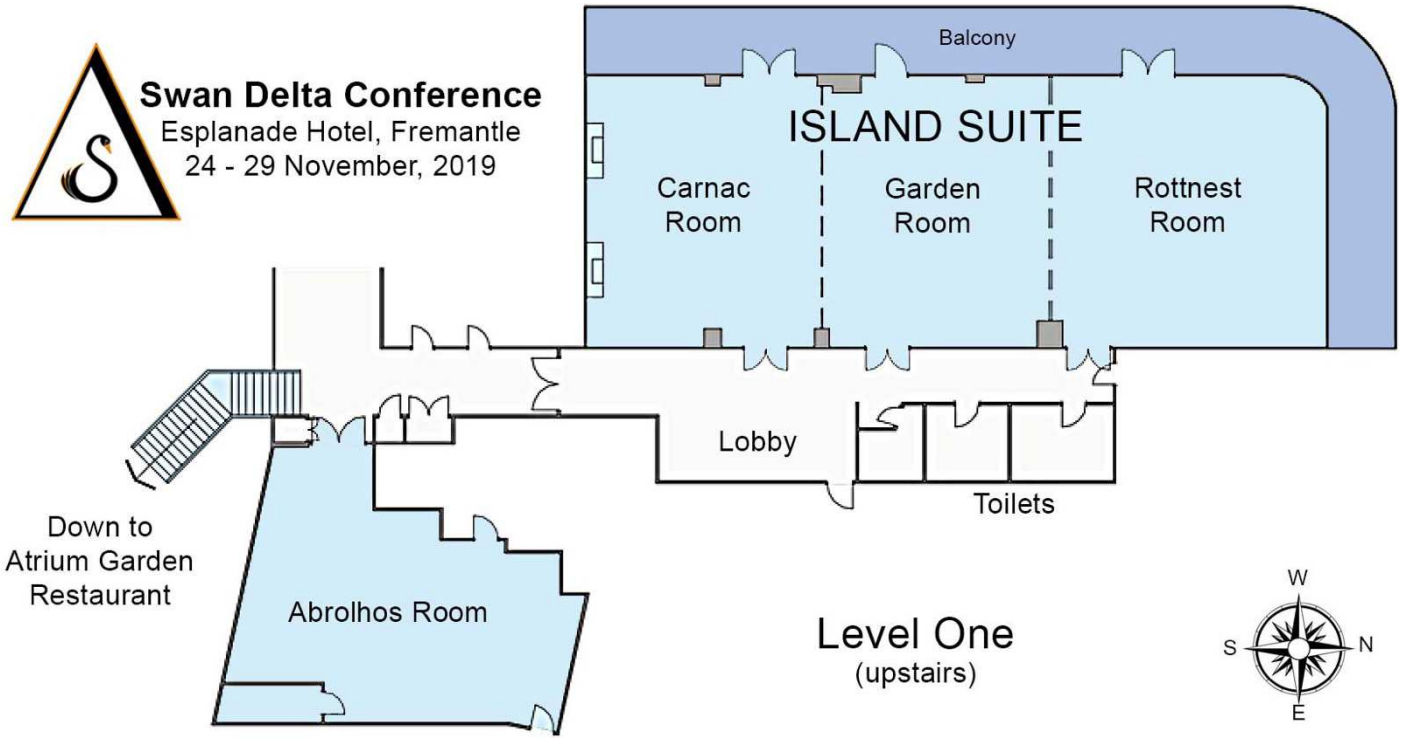
4:30pm Abrolhos Room Catch up time at the end of the third day.

Swan Delta Conference 2019
Full Programme

Friday 28 November 2019
Final Day

8:30am Abrolhos Room Come and chat with fellow delegates while enjoying a morning coffee before the talks.
8:45am Island Suite Conference housekeeping.
9:00am Island Suite Plenary Speaker **Cristina Varsavsky** The History of Delta: 22 Years and Counting
10:00am Island Suite Delta Brainstorm
10:30am – 11:10am Abrolhos Room Morning Tea
11:15am – 11:30 Island Suite Next Delta's presentation by Phil Kane and Rachel Passmore
11:30am Island Suite Closing Ceremony

FLOOR PLAN



THE 12TH DELTA CONFERENCE ON THE TEACHING AND LEARNING OF UNDERGRADUATE MATHEMATICS AND STATISTICS

REFLECTIONS OF CHANGE PROCEEDINGS

EDITORS: JIM PETTIGREW, LEANNE RYLANDS, DON SHEARMAN, ALEXANDRA YEUNG

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Proceedings of the 12th Delta conference on the teaching and learning of undergraduate mathematics and statistics

Conference held 24–29 November 2019, Fremantle (Western Australia, Australia)

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Editors: Jim Pettigrew, Leanne Rylands, Don Shearman, Alexandra Yeung

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FOREWORD

The first of the Delta series of conferences on the teaching and learning of undergraduate mathematics and statistics was held in Brisbane, Australia, in 1997. A Delta conference has been held every second year since then. All are held in the southern hemisphere and so far the host countries have been Australia, South Africa, New Zealand, Argentina and Brazil.

Swan Delta is the twelfth conference in the Delta series, running from 24 to 29 November 2019, at the Esplanade Hotel in Fremantle on the west coast of Australia.

The theme of the 2019 conference is Reflections of Change. In a sense all papers have a connection to this theme as some outline change and some will inform and inspire change. The papers included in the proceedings touch on many topics, some directly addressing discipline issues, others taking a more universal or oblique perspective. One paper looks at a classically misunderstood concept in foundational statistics. Another tackles the tension between discipline rigor and relevance in service teaching. A standout contribution considers the persistent gender imbalance in STEM disciplines and its bearing on female students' perceptions of their performance and willingness to continue with mathematical learning pathways. Also reviewed are differences, symmetries and productive possibilities in the relationship between teachers and students, curriculum innovations in engineering, the need for an injection of creativity in the learning of proof, program evaluation drawing on evident connections between extracurricular learning support and student outcomes, and varying interpretations among numerate and non-numerate readers of mathematically-infused text.

These proceedings contain the papers to be presented at Swan Delta. The review and editing process for the proceedings was carried out fairly and diligently over a period of weeks, with each submission run through plagiarism detection software and double-blind peer-reviewed by a minimum of two reviewers. Of the 25 submissions, 10 were accepted and are included here.

Alongside the presentations of each of the included papers, the conference will feature a number of short oral presentations, poster presentations and workshops. Abstracts for each of these is included in the proceedings. Presentations will also be given by the authors of the papers in the Special Delta Issue of the *International Journal of Mathematical Education in Science and Technology*.

We give our heartfelt thanks to the various committees who made Swan Delta happen and to Nazim Khan, in particular, who oversaw and coordinated it all. We thank the International Steering committee for keeping Delta going and growing, and reserve special thanks for those attentive educators who reviewed papers and not only provided thoughtful, constructive and richly informative feedback to authors, but also enabled the synthesis of high quality writing and research in the proceedings.

We hope that you find Swan Delta to be enjoyable and stimulating.

Swan Delta Proceedings Editorial Team
Jim Pettigrew, Leanne Rylands, Don Shearman and Alexandra Yeung

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PAPERS

THE RELEVANCE OF A MATHEMATICS COURSE FOR COMPUTER SCIENCE STUDENTS

Trudie Benadé[a], Janet Liebenberg[a]

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KEYWORDS: mathematics as service subject, mathematics for computer science, service teaching, mathematics for non-mathematicians

ABSTRACT

Computer science students tend to lack mathematics knowledge. For these students taking a mathematics course that is not their main field of study, the importance of the subject is often given a low priority and for lecturers teaching a service subject it is sometimes considered as a matter of lesser importance. These students may not be able to draw the connection through to the application in their discipline. On the other hand, for mathematics lecturers teaching their mainstream students is a priority. Therefore, the successful design of mathematics as a service subject is faced with many challenges. There is a need to reconsider what mathematics should be taught and how this mathematics should be taught to these students. This study aims to scrutinize the learning outcomes of the present mathematics courses at a specific university and to determine whether this contributes to the mathematical needs as indicated by the computer science lecturers. A qualitative study was done in which interviews were held with mathematics, as well as computer science lecturers regarding the content of the mathematics courses. The responses from the interviews were compared with the learning outcomes of the two relevant mathematics courses. It was found that the mathematics done at present, is not sufficient. New topics should be included and the appropriate level of detail of the relevant subdivisions in the topics, should be addressed. It is also clear that there is a need to reconsider the way that mathematics is taught to these students.

INTRODUCTION

We live in a rapidly changing world; science and technology are advancing, and societies are developing with new needs, paving the way for new possibilities. In this changing world, mathematics is taught to a diverse range of students at universities. Students taking a mathematics course because they are interested in the subject are normally motivated to study the course but if mathematics is a service subject for fields such as computer science, architecture, engineering and economic sciences, it may be necessary for educators to create or foster motivation. This gives rise to new possibilities for and new demands on the teaching of mathematics for the students in the service subject (Baldwin, Walker, & Henderson, 2013; Beaubouef, 2002).

The ACM (Association for Computing Machinery), is the largest and oldest international scientific and industrial computer society that fosters research and communication in a broad range of computing areas through special interest groups. At the recent ACM Special Interest Group on Computer Science Education (SIGCSE 2019) conference, a discussion on “Modernizing mathematics in computer science”, was attended by a surprisingly large number of delegates. The discussion centered around the mathematical foundations included in the curriculum for computer science students (Anthony, Liben-Nowell, Minnes, & Osera, 2019).

Numerous problems were raised in this regard: What mathematics is required in a foundation course? Who is responsible for teaching the course? How is mathematics related to the computer science goal? What is the purpose of mathematical rigor in foundational mathematics? What concepts should be covered? How specialized should the different courses be? Should the mathematics class for computer science students differ from that of mathematics students? During this discussion, consensus could not be reached regarding most of the questions raised and therefore it was decided to put together a survey regarding this topic, which is still in progress. Apart from the SIGCSE discussion, there appears to be little other recent literature on service mathematics for computer science students, and therefore some of the literature used in this study is to some extent outdated. Therefore, there is a need for a better understanding of the relevance of a mathematics course for computer science students.

Kent and Noss (2001) and Grove, Croft, Kyle, and Lawson (2015) pointed out that the following questions regarding the teaching of service mathematics are essential: “What is its purpose? What are the fundamental objects and relationships of study?”. They suggested that teaching mathematics using the context of the applicable disciplines may enhance students’ potential to know how, where and when to apply their mathematical knowledge as most students are unable to connect their mathematical knowledge to other disciplines without support.

The research question that guided this study was: What are the requirements of mathematics as a service subject for computer science courses?

LITERATURE BACKGROUND

According to Simons (1988), mathematics as a service subject arose out of a specific need, and therefore in this literature study, attention will be given to the following questions:

- Why do we teach mathematics to these students?
- What mathematics should be taught to these students?
- How should these mathematics courses be taught?

Why do we teach mathematics to these students?

A solid mathematical background is fundamental to the study of some computer science majors as well as other disciplines (Beaubouef, 2002; Hodgen, McAlinden, & Tomei, 2014). In the curriculum guidelines for undergraduate degree programs in computer science, compiled by a joint task force of the ACM and IEEE (Institute of Electrical and Electronics Engineers) Computer Society, the connection between mathematics and many areas of computer science is viewed as important and they recommend that computer science programs should provide students with a level of “mathematical maturity” (Sahami et al., 2013). Bruce, Drysdale, Kelemen, and Tucker (2003) commented that learning the right kind of mathematics is essential to the understanding and practice of computer science and indicated that mathematical thinking is valuable in computing – some applications in computer science involve computations and others rely on mathematical reasoning.

According to Henderson (2018), the amount of mathematics you know will determine what mathematics you will use and therefore the implication is that the more mathematics you know, the better it will be for you.

What mathematics should be taught to these students?

One of the myths in the teaching of mathematics at university level is the existence of context-free universal content (Alsina, 2001). The content of first-year mathematics courses tends to be generic and consists of basic knowledge and skills learned in a mathematical context devoid of applications to real-life situations. It is taken for granted that some core elements of

mathematics need to be learned by all students before applications can be made. However, results show that these approaches suppress students' interest and hamper interdisciplinary applications (Alsina, 2001). As mentioned by the joint task force of the ACM that compiled the curriculum guidelines for undergraduate degree programs, it is difficult to indicate specific topics to be taught to computer science students. They found that an understanding of linear algebra plays a critical role in some areas of computing; however, in other areas, linear algebra would not necessarily be a requirement, which is also the case with calculus and differential equations. More generally, they believe that it is important for these students to understand arithmetic manipulations, summations and basic arithmetic.

According to Henderson (2018) an understanding of

- combinatorics and calculus is helpful to code efficiently with less redundant loops;
- set theory and groups will help to better targeting and filtering of data to acquire improved structured data;
- linear algebra will aid in better 3-D programming used in modelling materials, such as video games;
- statistics can help to be better with analytics, machine learning and financial tasks;
- basic arithmetic helps to accomplish basic problems.

Thus, the more mathematics you know, the more tools you have to solve problems. The more tools you have to solve problems, the greater your potential is as a software developer.

In a quantitative part of a study conducted by Liebenberg, Huisman, and Mentz (2015) it was found that mathematics is overemphasized at university, while in the qualitative part of the same study it was found that mathematics, and especially the topics calculus, algebra and matrices are important in the workplace. These contradicting findings indicated the need for further study in this regard.

Durán and Marshall (2018) found that the needs of students vary, depending on their disciplines and they suggested the necessity of a flexible mathematics curriculum after a thorough needs assessment has been done.

How should these mathematics courses be taught?

In a study done by Schäfer et al. (2013), it was found that the primary motive for students changing their major subject of computer science (or even leaving university without a degree) is the high cognitive demand of the mathematics courses. Students fail to overcome the highly abstract knowledge in mathematics and cannot combine, on their own, the mathematical problem-solving approach with concrete, real-life experiences. Matic (2014) advocates for the emphasis on applications in meaningful contexts that would enable improved use of mathematical knowledge in a particular discipline.

Bingolbali and Ozmantar (2009) explored the teaching of mathematics to non-mathematics students in their first year at university. They focused on the lecturers' approaches and the amendments they made in their instruction when teaching students from different departments. They raised two important issues regarding service training:

- lecturers mostly teach what they want to teach and decide for themselves how they want to teach it;
- regarding the role of mathematics: is it a mental discipline or a tool for students' professions?

The view of lecturers, therefore, has a great effect on determining what to teach and how it should be taught to students who are not majoring in mathematics.

Schäfer et al. (2013), Simons (1988), Murakami (1988) and Nardi (2016) identified problems regarding the teaching of mathematics as a service subject.

- Attitude of staff lecturing these subjects: In most mathematics departments, research and the lecturing of students who major in mathematics have priority and service teaching is of less importance. When mathematics is taught by mathematicians, they tend to teach it with too much stress on the mathematical rigor instead of showing the appropriateness of the mathematics in the discipline (Murakami, 1988; Schäfer et al., 2013).
- Motivation of students in the course: Since the students did not choose mathematics as their main field of study, for them it is a low priority and they are therefore not motivated. A reason for this attitude may be that they do not see the reason for certain topics to be taught because of a lack of integration between mathematics and applications in their field. In order to improve the motivation of students, more examples from the students' disciplines can be incorporated into the course. This is not always easy, as the mathematics lecturers are not always fluent in the service subject's area (Murakami, 1988; Nardi, 2016).

Hence, for many teachers who first found mathematics magnetic for its own sake, the acts of teaching and having to justify the service function of their courses are often a challenge.

A study done by the Mathematical Association of America indicated that students lack conceptual mastery of mathematics and could not apply what they had learned (Siegel, 1988). Faculties of engineering, chemistry, physics and biology asked for more integrated mathematics with an emphasis on problem-solving techniques. Matic (2014) recommends that mathematics courses should be adapted to the particular study program where the material would be presented through the practical applications in that discipline.

As commented by Kitchenham, Budgen, Brereton, and Woodall (2005), there is a need to rethink the way in which mathematics is taught to software engineering students, as they cannot see the importance of mathematics in the course. Henderson (2003) noted that many software engineers have not been taught to use mathematics as a useful tool. Therefore, there is a need to reconsider how mathematics is taught to these students.

Macbean (2004) suggested that if students saw the significance of mathematics in their courses, they would be more motivated to study mathematics. This should happen through the faculty to promote the importance of mathematics and to adapt the teaching of it appropriately.

METHODOLOGY

In this section, the research design, demographics of the participants, data collection and the analysis of the data are explained.

A qualitative study was conducted and interviews, as well as document analysis, were used to collect data. Three mathematics and seven computer science lecturers from a university in South Africa took part. One-on-one semi-structured interviews were used to collect qualitative data. The mathematics lecturers involved were those teaching the service mathematics courses. The purpose of the interviews was to acquire information about the lecturers' experiences, perceptions and opinions about the mathematics required for courses serving computer science students. The interviews with the lecturers were recorded and notes were made while listening to the recordings and then summarizing them (see Table 1).

The guiding questions used in the interviews were:

- Which mathematics do you think computer science students need?
- Do you think that the current level of mathematics is sufficient for computer science students?

The computer science students take two mathematics courses during the course of their studies: Mathematical techniques (MTHS113) in the first semester of their first year, and Discrete mathematics (MTHS225) in the second semester of their second year. The topics in the curricula of these two mathematics courses were scrutinized and compared with the outcomes of the interviews (see Table 1).

RESULTS AND DISCUSSION

From Table 1 it is clear that there is a difference between the needs of the computer science lecturers and the reality of what is covered in the mathematics service courses. In the view of the lecturers, certain topics are not addressed in the two mathematics courses that are essential for the computer science students (marked with * in Table 1). These topics are:

- Boolean algebra
- Linear algebra
- Integration
- Trigonometric functions
- Dot product
- Binary and Hexadecimal notation
- Fourier analysis

Table 1: Comparison between needs and reality

Topics needed as indicated by the computer science lecturers	**Number of comments regarding topics needed**	**Actual topics in MTHS113**	**Actual topics in MTHS225**
Matrices	5	√	√
Integration*	4		
Differentiation and optimizing of functions*	4	√	
Basic mathematics concepts	3	√	√
Linear algebra*	3		
Functions	3	√	√
Summations	2		√
Boolean algebra*	2		
Number systems	2	√	√
Sequences and series	2		√

Economical mathematics	2	√	
Logical thinking	2		√
Trigonometric functions*	1		
Probability	1		√
Logarithms	1	√	
Dot product*	1		
Coordinate geometry	1		
Proposition and predicate logic	1		√
Decision Tables*	1		
Binary and Hexadecimal notation*	1		
Fourier analysis*	1		

Four of the seven computer science lecturers indicated that integration is important for computer science students and it is not covered in either of these courses. Boolean algebra, binary and hexadecimal notation, linear algebra, Fourier analysis, as well as the dot product are important topics for the computer science lecturers. Although the lecturers indicated the need for trigonometric functions and coordinate geometry, these topics are part of the secondary school curriculum and students should be familiar with them and they are therefore not included in the two mathematics courses in question.

According to the lecturers an understanding of:

- matrices will aid in image processing, data structures, decision support systems and data analytics;
- integration and differentiation will be helpful in networks, decision support systems and image processing;
- linear algebra will improve 3-D programming, data structuring and designing and application of algorithms;
- dot product is necessary for data structures and algorithms;
- Boolean algebra and the binary and hexadecimal number systems will improve students' comprehension of the computer's internal processing system;
- basic arithmetic helps to solve basic programming problems.

A theme that emerged from the interviews was that some of the topics are included in the curriculum of the specific course, but the focus of the mathematics lecturer is different from what the computer science lecturer expects. An example is the economical mathematics topic. This topic is included in one of the courses, but the specific subdivisions are not necessarily part of the learning outcomes. A computer science lecturer commented that the following subdivisions of economical mathematics are necessary for the specific course offered, but is not covered in the mathematics courses: Net present value; Return on investment; Payback analysis. Therefore, not only should the main topics be revised, but the subdivisions should also be taken into consideration.

Bingolbali and Ozmantar (2009) also found in their study that lecturers teaching mathematics to non-mathematics students decide for themselves what they want to teach and how they are going to teach it. One of the computer science lecturers commented:

> *"I was and am still very upset when I learned that what I thought was taught to our computer science students, was not the case. Although some of the topics that I think are important are listed in the course outcomes, I now came to the realization that the mathematics lecturer does his own thing".*

According to Murakami (1988), mathematics lecturers tend to be too formal when teaching mathematics to non-mathematics students. Howson and Kahane (1988) believe that rigor is necessary, but formal proofs are not crucial for service teaching. In this study, the one mathematics lecturer who teaches the service course was of the opinion that more applications than theory should be included in the course. However, the same lecturer noted that proofs of theorems are important, as it improves logical thinking. These opposing statements reiterate the findings of Bingolbali and Ozmantar (2009) and Schäfer et al. (2013) that the excessive occurrence of abstract mathematics is one of the reasons for the high dropout rate in computer science programs.

It was found that most of the content in the analyzed courses is not presented in a computer science context. Alsina (2001) stated that students cannot automatically apply their mathematical knowledge in a specific context without assistance. The courses should therefore be contextualized specifically for computer science students. A mathematics lecturer proposed that

> "*a course should be developed specifically for computer science students so that the focus will be on the necessary techniques in context with a focus on specific skills*".

Sometimes mathematics lecturers do not have the background of the service subject to be able to teach the mathematics content in a specific context (Murakami, 1988).
One of the computer science lecturers suggested that

> "*the mathematics should be taught in context by a computer science lecturer so that applications relevant to various computer science courses can be used*".

However as noted by another computer science lecturer, a shortage of staff in the computer science department at this university, is posing a challenge.

Some of the computer science lecturers expressed their concern about the students' mathematical knowledge. They are of the opinion that the focus should be on deeper understanding instead of only procedural knowledge. As two computer science lecturers commented:

> "*The problem is not necessarily the content of the curriculum*. *Students are definitely lacking mathematical knowledge and skills".*
> *"Mathematics currently being done is not sufficient for computer science and specifically for neural networks and image processing"*

A computer science lecturer suggested that lecturers should adjust their teaching according to what is offered in the mathematics courses. However, it appeared that these computer science lecturers are not necessarily up to date with the contents of the various mathematics service courses:

> "*The problem is that computer science lecturers are not fully aware of the content of the curricula of the different mathematics courses, and can therefore not link to the students' existing mathematical knowledge".*

CONCLUSION AND RECOMMENDATION

For students taking a mathematics course that is not their main field of study, the importance of the subject is often given a low priority. Furthermore, for lecturers teaching a service subject, it is sometimes considered as a matter of lesser importance. Students may not see the importance of the mathematics being taught and may not be able to make the connection through to the application in their discipline. For mathematics lecturers, their own research and teaching their mainstream students are often a priority. Therefore, the successful design of mathematics as a service subject faces many challenges. It requires a high level of understanding and cooperation between the mathematics lecturers and those in the computer science department. It also requires a careful selection of examples and applications to enhance the motivation of these non-mathematics students.

It is clear that the mathematics done at present, is not sufficient and computer science students' lack of applicable mathematics knowledge needs attention. New topics should be included and the appropriate level of detail of the relevant subdivisions in the topics should be addressed. It is also clear that there is a need to reconsider the way mathematics is taught to these students. More attention should be paid to real-world applications in the computer science field. Lecturers should make sure that they know what mathematics knowledge their students possess and teach accordingly.

Although the present mathematics seems to be insufficient, the computer science students are still succeeding since they avoid certain mathematics intensive courses or pass their courses by concentrating on sections not using the mathematics topics as indicated in this study. The fact of the matter is, that they are not prepared for certain careers in the computer science field that require the specific mathematics knowledge.

In conclusion, it would appear that the topic of service mathematics is once again gaining attention and therefore we suggest a reconsideration of teaching mathematics to computer science students.

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TERTIARY STUDENTS' CHANGING VIEWS ON MATHEMATICAL CREATIVITY

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ABSTRACT

Tertiary students represent the near-future leaders and employees of science and mathematics. In a world that is in increasing demand for creativity, our mathematics courses and programs need to shift from more routine and computational to more creative and problem-solving focused. In this paper, we present preliminary results of a qualitative research study in which we examined students' perceptions of mathematical creativity in a transition to proof course. In our investigation, we conducted interviews with students as well as collected their reflection assignments at the end of the semester. Using a definition of creativity from a relativistic perspective, we analysed interview data to describe students' perspectives of mathematical creativity by the end of the semester and how their reported views evolved. Our findings indicate that undergraduate students have robust views of creativity and showed numerous shifts in how they felt about creativity or how they saw themselves as a creative person.

INTRODUCTION

Creativity has become one of the most sought-after skills for academia and industry employers (World Economic Forum, 2016). Additionally, the importance of creativity is highlighted in curriculum-standard documents internationally (Askew, 2013). Cropley (2015) summarized these points: "[t]eaching engineers (and other STEM disciplines) to think creatively is absolutely essential to a society's ability to generate wealth, and as a result provide a stable, safe, healthy and productive environment for its citizens" (p.140). While difficult to define (Mann, 2006), mathematical creativity may even be more critical in science, technology, engineering, and mathematics (STEM), since mathematics is so prevalent and acts as a gatekeeper in STEM fields (Carnevale, Smith & Strohl, 2013). The number of studies examining students' mathematical creativity at the tertiary level and how to enhance it is slowly growing, but compared to the number of studies at primary and secondary school mathematics level, it is still sparse.

To address this particular need we, as the creativity research group, have been conducting studies at the tertiary level mathematics courses (Karakok, Savic, Tang & El Turkey, 2015; Tang, El Turkey, Savic & Karakok, 2015; Savic, Karakok, Tang, El Turkey & Naccarato, 2017; El Turkey, Tang, Savic, Karakok, Cilli-Turner & Plaxco, 2018; Omar, Karakok, Savic & El Turkey, 2019). In this paper, we share preliminary results of a research study that we conducted in a introduction-to-proofs course. In this qualitative study, we explored students' perceptions of mathematical creativity and how their perspectives evolved over the period of the course.

THEORETICAL PERSPECTIVE AND BACKGROUND LITERATURE

Our research projects on mathematical creativity can be grounded in the *Developmental* perspective of creativity (Kozbelt, Beghetto & Runco, 2010). The developmental perspective asserts that creativity develops over time and emphasizes the role of the environment in which students are provided authentic tasks and opportunities to interact with others.

We operationalize mathematical creativity as "a process of offering new solutions or insights that are unexpected for the student, with respect to their mathematical background or the problems [they've] seen before" (Savic et al., 2017; p.1419). Our focus in this definition is on the *process* (Pelczer & Rodriguez, 2011) of creation, rather than the *product* that is created at the end of a process (Runco & Jaeger, 2012). This particular orientation allows us to keep a dynamic view rather than a static one to capture nuances in the individual's thinking. Furthermore, the definition takes a relativistic perspective—creativity relative to the student—in contrast to absolute creativity for the field of mathematics (Leikin, 2009; Beghetto & Kaufman, 2013).

The process and relativistic perspectives are particularly important in exploring how to enhance students' mathematical creativity. For example, Levenson (2013), using a similar view point, focused on the discussion of ideas put forth by individual students and how these ideas helped in developing a product of collective mathematical creativity in fifth and sixth grade mathematics classrooms. Levenson also emphasized the teachers' roles in facilitating these discussions. Moore-Russo and Demler (2018) examined the perceptions of U.S. faculty and staff participants from gifted mathematics programs and found that, through counts of coding using several creativity frameworks, mathematical creativity in education was more of a process than "a subjective experience" (p.23).

Nevertheless, students' mathematical creativity has been explored using different perspectives in other studies (e.g., Leikin, 2013; Torrance, 1966; Dehaene, Spelke, Pinel, Stanescu & Tsivkin, 1999). Focusing on quantitative measures, researchers have been implementing three Torrance (1966) categories in their studies:

- Fluency ("the number of appropriate ways produced for solving a problem" (p.391)),
- Flexibility ("different groups of ways of solving" (p.391)), and
- Originality ("conventionality of a solution in a particular group of students with a similar educational history" (p.392))

Leikin (2009), for example, used a point system to evaluate these categories in students' work. In this system, the "originality" measurement is given a high score if the solution produced is only prevalent among 15% or less of all solutions produced with a group of students. While Leikin acknowledged that solutions must be "appropriate" – "The notion of appropriateness has replaced the notion of correctness" (p.391), it seemed that an expert (e.g., an instructor or a researcher) was the one who made the judgment on what is or should be appropriate or original. With our perspective on mathematical creativity, we problematize such instances and aim to shift our focus to the producers of such solutions – the students. Our aim is to understand students' perspectives on their own mathematical creativity. However, we notice that there is a need to first explore students' perceptions of mathematical creativity, particularly at the tertiary level.

While there is research on mathematicians' and mathematics instructors' perceptions on mathematical creativity (Karakok et al., 2015; Borwein, Liljedahl & Zhai, 2014; Hadamard, 1945; Sriraman, 2009), research on students' perceptions on mathematical creativity has received less attention. In one of our earlier studies, we examined students' and

mathematicians' definitions of mathematical creativity using three process categories: taking risks, making connections, and creating ideas (Tang et al., 2015). We found that students rarely associated making connections with mathematical creativity (9% of responses), whereas the mathematicians' responses associated with making connections was about 38%. This study alerted us to think about explicitly valuing and discussing the processes that are deemed to be important in developing mathematical creativity in a classroom setting.

In this paper, we share how we approached this objective while exploring the following research questions:

- What are tertiary students' perceptions of mathematical creativity?
- In what ways do these views evolve in an introduction-to-proofs course which emphasized mathematical creativity?

METHODS

To address the research question, we collected data in an introduction-to-proofs course at a small liberal arts college in southwestern United States. This course is typically taken by mathematics majors or minors in their second or third year and includes topics such as sets, logic and various proof techniques (e.g., direct proof, contradiction, contraposition and induction). The course was taught using an inquiry-based learning (IBL) pedagogical approach - students often worked in small groups and gave presentations to the class on their proofs.

The course explicitly valued creativity by making use of the Creativity-in-Progress Rubric (CPR) on Proving (Savic et al., 2107; Omar et al., 2019; Karakok et al., 2016), which is a formative assessment tool that students can use to persevere in proving and encourage creative processes. The rubric has two main categories: making connections and taking risks. The instructor gave assignments and exam questions where students had to use the rubric to assess their own or other's work. Students also used reflection assignments to think about their definitions of mathematical creativity. Students used their responses to discuss their definitions with their groups.

At the end of the semester, 4 female and 3 male students agreed to be interviewed. Each student participated in a 60 – 90 minute semi-structured interview where they were asked to describe the course, discuss their views on creativity, and discuss the use of the CPR in the course. During the interview, students were also asked to compare their current views of mathematical creativity to the previous ones they shared on reflection assignments or pre-survey data and discuss, if possible, reasons for such changes. The interview protocol can be found in the appendix.

Interviews were coded using hypothesis coding (Saldaña, 2013) and five categories were extracted from the research questions of a larger project; one of the categories being creativity. Three of the seven participants' transcripts were coded separately by the first and second author with 97% agreement. Because of this high degree of interrater reliability, the rest of the transcripts were coded by the first author only. A second-level coding was then done, where all utterances coded for creativity were coded with *views*, if a student was espousing their view of mathematical creativity, or *evolution*, if a student was speaking to how their views had evolved or changed after the course.

RESULTS

Students' Views on Mathematical Creativity

Many of the students in this study expressed views on creativity that were strikingly similar to the three Torrance categories that we discussed in the previous section. For example, three of the students often talked about creativity as having a component of originality, as highlighted by the following quotes:

> *What I think is, it's mostly being able to, I guess bring your own… ideas to the table. Like, um, kind of doing something that no one else has done, or like figuring something out in a way someone else probably didn't figure it out, or like working off another person's ideas. (Cargo[1])*

> *Being creative in mathematics is the same as being creative in anything else. It's taking the road less traveled. It's not just doing what the herd is doing but finding your own way to get to where you need to be. (Stephanie)*

> *I would say to be creative in mathematics is basically anything, if you were ever faced with a problem you don't really wanna stick with the generic, or you don't want to find the generic way to answer it. You wanna find a way to solve the problem on your own by whatever means you can, as long as it works… and consistently works. (Peyton)*

Stephanie demonstrated how her search for something original when writing a mathematical proof turned into a moment of surprise and enjoyment:

> *There were a lot of moments where you just almost stumble across something and you work through it and it ends up working and it's completely different than what the other students had done. And it's exciting. (Stephanie)*

We coded Olivia's perspective to include flexibility and originality when asked about her views on creativity:

> *[M]y personal definition of creativity, and I guess to just really sum it up in one statement is just really thinking outside of the box and being able to be comfortable or at least willing to take risks. And, um, not just follow a standard format or a standard procedure, but being willing to be flexible and try different approaches, something you wouldn't normally try, and, um, ya I guess that's pretty much how I would describe it is just being able to be flexible and think abstract, think of something out of the ordinary.*

Whereas, Stephanie's perspective seems to relate to fluency as she spoke about solving a problem in multiple ways:

> *So, in most lecture-based classes you're taught this is how you do it. But as you get into the higher mathematics I've found that you can make connections from one to the other and you can solve things in different ways. Instead of using a calc trick to solve a problem, I might use a trig trick or just the geometric equations to solve something rather than doing a whole integral.*

The students' perspectives also included making connections as an important piece of mathematical creativity. For example, Olivia said:

> *So, in that I think it [referring to IBL] forces you to really try to make connections and it forces you to get creative because you have, um, very little like understanding of the right way to do it, so it kind of throws that out of a student's mind, out of my mind. And so it makes anything possible.*

[1] The names used in this paper are pseudonyms chosen by the students.

It is very possible that the student perspectives on creativity outlined above were highly influenced by the course and the use of the CPR on Proving. While there was no explicit mention of definitions of creativity in this course, the CPR on Proving was developed through a review of the literature as well as by asking mathematicians and students what mathematical creativity meant to them (Karakok et al., 2016). Therefore, these research-based ideas were present in the rubric either explicitly or implicitly. For example, one of the main categories of the rubric is called making connections and another subcategory was flexibility; so students may have been adopting the rubric language to describe their ideas on mathematical creativity at the time of the interview.

Students also had views on creativity that differed greatly from the definitions given in the literature. Several students spoke about the inherency of creativity, taking the perspective that it is an ability that you are born with and is fixed.

> *Um, to me creativity, that's kind of like, born with—is like being able to come up with like a nifty idea for like a creative like art project that will make it like simple or like being able to- I know art takes like a lot of practice and a lot of work, like, itself to do—to be able to like draw or like paint. (Alice)*

Vladmir also explained that recognising mistakes and evaluating them could lead to creativity:

> *I think that like when you mess up, you know when you mess up, the first thing you wanna do is you find out why, why or how, right? And you go back, and sometimes I think it's when you go back and you're forced to look the second or third time, that's usually when you find like that separate path you know that might lead to like a creative path to get to your answer. (Vladmir)*

Finally, three of the students saw creativity as akin to efficiency; that is, the shorter the proof was, the more they saw it as creative. In fact, two students spoke of another student in the class as being the most creative, since his proofs were the shortest and often made use of tricks that others had not thought of.

> *So, the one guy I was telling you about before, he was very efficient. He would make these algebraic tricks up, and then another person would come up with an algebraic trick to use. So, his creative moment, I could then use to expand on and do something a little different with to have my own creative moment. (Stephanie)*

> *[I]n our class we used 'more efficient' to be able to create like a shorter proof. Um, or like in any case um just having, being able to find like um a technique that works that doesn't necessarily make everything longer. It kind of just makes it more, like easier to understand too. (Alice)*

> *That's interesting too. [laughs] That's very, it seems so simple to come up with the n plus 1 squared is obviously less than n plus 1 squared times something else that's positive. And by that, just by that simple first step they were able to come up with the proof. But it really only took that one little thing… Ya that's very cool. That's very short too, very efficient. (Peyton)*

Evolution of Students' Creativity Perspectives

Three of the students reported an explicit shift in how they thought about creativity or how they saw themselves as creative people. These changing perspectives stemmed from different sources for different students. For Stephanie, a change in her view of mathematical creativity was due to having more tools to work with now that she had taken a class on proofs:

> *I think I started to look at creativity a little bit different through this course...Prior to this it's been all very applied mathematics...So before, just using the trig equations to solve geometry was creative for me. Whereas now, this has just opened up a whole new door*

of opportunities for it because I can solve a proof using a contradiction, while somebody else used a contrapositive and somebody else used a direct proof and somebody else used induction, and we all do it completely different.

For other students, shifts were attributed to the classroom community and the way that the course was structured. Olivia spoke of this when she mentioned:

We kind of all went in with kind of not really feeling confident in our abilities to be creative, so it was really interesting to see students that were quiet, reserved early on like show their work later in the semester and they had done something like totally cool and amazing... And seeing their involvement increase as the semester went on. So I feel you know their ability, like their confidence levels went up and I could say that's true of me as well. So I wanna say that it's, you know it wasn't that like all the creative people took this course because I didn't consider myself creative and I took the course, and I would say that that's probably true of other students as well.

Stephanie echoed Olivia's comment almost exactly:

At the beginning of the semester, I think a lot of people in that class were very shy and quiet, and so it was kind of hard to judge where their creativity was because they weren't sharing it as much. Um, by the end of the course you had everybody speaking, you had everybody giving their opinions and how to work on things together, and you saw everyone grow. You saw everyone coming up with their own tools and tricks. And everyone was posing questions, not just the few of us that were outspoken to begin with. So you definitely saw growth in the class, um not only with the shyness but with the creativity, and coming up with their own ideas to change things and make them better.

Since the course was taught using IBL, students were encouraged to present their work to each other, especially if they approached a problem using a different method, thus some of these shifts seemed to be a result of seeing others' as creative and reflecting it back on themselves. For instance, Peyton said:

I really, I really did not feel like I was being creative at all throughout the course. It really was just things in my head, it makes sense that led to a conclusion that made sense. But, considering that I thought other people were exceptionally creative, I kind of thought that maybe they though that about me too.

The most striking change is evinced by Peyton who went from not seeing mathematics as a creative subject to enjoying the creativity in mathematics.

Interviewer: *And in your reflections you said something about, um, 'I think I am on the spectrum that generally believes that, believes there is no need for creativity in mathematics. That's been a key reason why I enjoy math. I know, I know if I get the answer then I have done it correct. There is a set process and if I learn the process then I'll be able, I'll' – what's that – 'I'll be successful'. So, do you wanna comment on that part?*

Peyton: *I… should have made that more in the past tense, because I believed that prior to taking this course. Um, but ya generally in past I figured, 'cause math has always been lecture-based. There has been, you can figure out problems and it's creative in the sense that you can figure out how, where you wanna start with the problem. But I like being able to know that if I am doing it correctly, the process correctly, then I will get to the answer. If I just repeat the process over and over then I know I'm going to learn it, which I do enjoy. I enjoy knowing when I'm gonna do something correctly as opposed to just spending a lot of time and then not even knowing if it's gonna yield good results. But this course changed that quite a bit, because there really was no assurance that anything would be correct, but it still… required me to use different thought processes to get to a result hoping for the best, which was stressful to say the least, but still, it was fun.*

DISCUSSION

In this study, we noticed that students' view on mathematical creativity were centred around categories such as originality, flexibility and fluency as well as processes such as making connections, recognizing and evaluating mistakes, and mathematically observing other's approaches. There were some students who still viewed creativity as an ability that a person has from birth, even though other students pointed out the possibility of development of creativity. Overall, students had a variety of views on creativity and, for many of the students, these views changed over the semester as they saw the instructor and other students focus on creativity and demonstrate creative mathematical practices. While Moore-Russo and Demler (2018) did examine students' views of mathematical creativity at the tertiary level, their study was conducted with pre-service teachers. Our study is the first one to observe undergraduate students views of creativity and to determine how these views change throughout a semester.

These results also suggest that it is possible to affect students views on mathematical creativity through teaching practices. In our study (Karakok et al., 2015), we found that mathematicians believed creativity to be essential to their work yet didn't teach it in their courses or feel that their students particularly saw mathematics as a creative discipline. This study, while small, has implications for teaching as, it seems, that this particular instructor's course design aimed to explicitly value and foster students' mathematical creativity and facilitated the evolvement of students' perspective on mathematical creativity. We believe that this particular observation, namely, the connection between course design and teachers' actions and ever-changing students' perspectives on mathematical creativity requires additional exploration. In particular, which teacher actions are more fruitful to afford such changes and what other course design features contribute to these changes are important questions to explore at the tertiary level mathematics courses.

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APPENDIX

Interview Questions:

1. **Class-Intro Question** *to get students talking, keep it short*:
   What other courses did you take this semester?
   a. Can you tell me more about how each course was taught?
   b. Did those courses influence your work in M305?
   c. Can you explain in what ways it did or did not influence?
   d. Did this course influence your work in those classes? In what ways?

2. **Definition of creativity** *to get students to think about creativity and trying to capture their "authentic" definition*:
   - Can you tell us what it means to you to be creative in mathematics?
   - In your reflection you said, "" Can you expand on that?
   - Did you feel creative in this course?
     - Can you tell me about it?
     - Why do think you were creative?

   This question has two levels- their definition and if they feel they are creative.

3. **Creativity in this course** *to elicit a) some teacher actions that they felt fostered creativity or b) advice on how to build a classroom to foster creativity*
   - (Use their moment of creativity from previous question: which aspects of the course contributed this particular "creative moment" or other times you felt creative and how?)
   - In your opinion, which other aspect(s) could contribute to a student's mathematical creativity?

4. **Examine other students' proofs** *trying to triangulate their definition in action with 3 students' proof product:*
   In this question I would like for you to read this theorem, please.
   a. I don't want you to prove this question, but look at other students' proofs and tell me what you think about their proofs. What do you think about this proof, which was constructed by another student? (*keep it open ended like this first to get everything they can say-proofs will be emailed at the time of the interview. After they some stuff focus it to creativity*)
   b. In particular, do you think it is creative? Why?
   c. How does this match your definition/perspective of creativity?

5. **Rubric** *to see how students use the rubric, to tease out some product and process of creativity definitions/ideas from students' perspective*
   How would you evaluate these students' proofs using the rubric?

6. **Rubric use in their proof process/course** *to have them focus on their use of the rubric if they haven't already*
   How did you personally used the rubric while you were working on a proof?
   a. Which aspects/categories were useful to you and why?
   b. Which aspects/categories were challenging for you to use and why?
   c. Did you use the rubric or ideas from this rubric in another courses?
   d. *If needed*: How can we improve it for students' use?
   e. *If needed*: How can we improve its use in classroom?

7. **General Evaluation of Creativity in Math Courses**
   You know in courses instructors evaluate students' learning in various ways, such as exams, in class participation, homework and such. Do you think students' creativity should be or could be graded or evaluated?
   a. Why/why not?
   b. Do you think this rubric would help?

THE TWENTE EDUCATIONAL MODEL IN THEORY AND PRACTICE: TWO MODULES AS EXEMPLARS

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ABSTRACT

Teaching for an unknown future in rapidly changing times is a challenge with which all educators grapple, not least amongst them engineering educators. The University of Twente's eponymous educational model addresses this challenge, reflecting their "High Tech - Human Touch" motto by blending the technical needs of the degree with current research in engineering education. In this paper I will present and describe the Twente Educational Model. As illustration of the model in practice I present two exemplar modules in the departments of Advanced Technology and Electrical Engineering. While challenges to the model remain, it represents a mature model of curriculum reform involving project-based learning and adds to the literature on successful engineering education reform.

INTRODUCTION

Educating university students in a rapidly changing world is a widely discussed challenge recognised globally (Graham, 2012; Graham, 2018; Belski, Adunka and Mayer, 2016). Approaches related to meeting this challenge employ a variety of descriptive terms, amongst them future-proofing education (Meijers and den Brok, 2013; Cornejo, O'Hara, Tarazona-Vasquez, Barrios and Power, 2018), teaching 21st century skills (Jang, 2016), or teaching for an unknown or uncertain future (Barnett, 2004; Stein, 2017). In engineering, teaching disciplinary skills alone may once have been sufficient to prepare a graduate for the workplace (although perhaps it never was altogether), but certainly today there is increased need for transferable skills such as communication, problem solving and project management (King, Varsavsky, Belward and Matthews, 2017; Jollands, Jolly and Molyneaux, 2012), as well as an orientation towards lifelong learning (Graham, 2012; Cornejo et al., 2018).

Graham, in her 2012 report on successful curriculum change to address 21st century challenges, observes that successful engineering education change almost without exception involves an interconnected and redesigned curriculum. In order to take a course or programme that was once taught traditionally and to change it to meet the needs of the student of today, it is not enough to simply manipulate content such as updating the topics or to add non-technical skills on top of the existing technical programme (Graham, 2012; Edström and Kolmos, 2014). Fundamental curriculum-wide change is needed. One such frequently adopted change by current and emerging leaders in engineering education (Graham, 2018) is towards a curriculum driven by project-based learning (PBL). Project-based learning, alternately project-led education (Centre of Expertise in Learning and Teaching, 2017), in essence involves having projects be the platform for students to develop process skills such as project management, self-directed learning, communication and collaboration as well as the traditional technical and disciplinary skills and knowledge. Projects can further allow for analysis and identification of problems in addition to problem solving itself (Edström and

Kolmos, 2014). PBL, done well, is recognised as a form of teaching and learning preparing students well for the workplace. For instance, Edström and Kolmos (2014) cite research indicating that employers value graduates who have been through a PBL curriculum highly, saying that they can work "from day one" (p. 542), are motivated and have well developed skills. Jollands et al. (2012) in their study comparing the work readiness of PBL and non-PBL graduates find that certain skills are equivalently developed through project-based learning as through the vacation work of non-PBL graduates, however their research suggests that communication skills and the ability to systematically apply engineering knowledge in design are better developed through a PBL curriculum than through a traditional programme with vacation work.

There is no single definition of project-based learning on which everyone agrees, however there are broad principles which focus on the teaching and learning process. Edström and Kolmos (2014) suggest three principles underpinning effective project-based learning, namely (1) an orientation towards defining and analysing problems, (2) interdisciplinary curriculum content and (3) a social approach to learning. Edström and Kolmos's third principle, that of a social approach to learning, resonates with Schoenfeld's (1992) view of thinking mathematically. Mathematics is "an act of sense-making that is socially constructed and socially transmitted" (p. 339) and "classroom mathematics must mirror this sense of mathematics as a sense-making activity, if students are to come to understand and use mathematics in meaningful ways" (pp. 339-340). Project-based learning provides links between classroom mathematics and real world applications, encouraging intrinsic motivation to master the work as well as developing the skill of thinking mathematically (and as an engineer) through active sense making. The philosophy behind PBL therefore places as much value on how students learn than on what students learn. Recognising the need for curriculum change to address the challenges of a rapidly changing world, changes that perhaps a technical university is particularly well positioned to carry out, in 2013 the University of Twente (UT) in the Netherlands rolled out an innovative institution-wide curriculum with project-based learning in thematic modules at its core.

THE TWENTE EDUCATIONAL MODEL

The undergraduate curriculum at the University of Twente was redesigned in the period 2010-2013 and the design was rolled out across all faculties in September 2013. The redesigned curriculum hoped to sustain and renew the university's profile as an entrepreneurial university developing sustainable solutions to societal problems, increase student retention, and improve the educational offerings through research-driven innovations (CELT, 2017; Visscher-Voerman and Muller, 2017; Warmerdam, 2017). The drivers behind the redesign of the curriculum have been discussed elsewhere (Visscher-Voerman and Muller, 2017; ter Braack, Rouwenhorst and Slotman, 2015; Bollen, van der Meij, Leemkuil and McKenney, 2015; Venner, 2018; van den Berg, Steens and Oude Alink, 2015); now in 2019 the Twente Educational Model (TEM) has matured and is understood to have two primary foci.

The first focus of TEM is the incorporation of interdisciplinarity into the undergraduate curriculum. The University of Twente, as a technical research and entrepreneurial university, engages in interdisciplinary research. By clustering undergraduate studies in modules designed to integrate disciplinary units with a common interest in a central project, TEM seeks to reflect the interdisciplinary nature of the institution's research in the classroom (Damgrave and Lutters, 2016). The educational model is designed to avoid the "silo" effect, which creates apparent barriers between disciplines, barriers which are not there in research nor in the technical careers the students may follow (CELT, 2017).

The second focus of TEM is the challenge of teaching and learning in a rapidly changing world. Society and its demands of graduates change fast. Many careers, including within the field of engineering, exist today which did not exist twenty years ago and it is reasonable to assume that the same will be true of twenty years in the future (Belski et al., 2016; CELT, 2017). What TEM hopes to achieve is to provide students with the opportunities to develop skills as communicative problem solvers who can respond to rapid change and continue to learn (ter Braack et al., 2015). The value of a technical degree is strengthened by well-developed skills in organisation and communication (CELT, 2017). The influences on TEM therefore derive from the technical world of interdisciplinary research and industry as well as from the world of educational and social science research related to transferable non-technical skills.

The undergraduate programme is designed as a series of (potentially interrelated) modules, each with a theme. Each module lasts one quarter, so a three year degree is comprised of twelve modules, as shown in Figure 1. At the core of each module is a team project; as far as is possible the project is based on something in the real world, “an activity that challenges students to independently gain knowledge and skills” (CELT, 2017, p. 7). The rest of the module consists of units that (ideally) cohere with one another as well as with the project. An example is the first module in the department of Advanced Technology where projects related to dynamic systems (roller coaster design, for instance) are at the core of the module, supported by units (sometimes called courses) in calculus, mechanics and laboratory practice. Knowledge and skills offered in the module’s units are all necessary for successful completion of the project and success at all the parts of the module are necessary for a passing grade for the module. The projects are designed to be appealing and thereby create an intrinsic interest in developing the necessary skills (see also Cornejo et al., 2018). The ideal pedagogic context is one of student-centred teaching and active learning (CELT, 2017; Venner, 2018; Damgrave and Lutters, 2016).

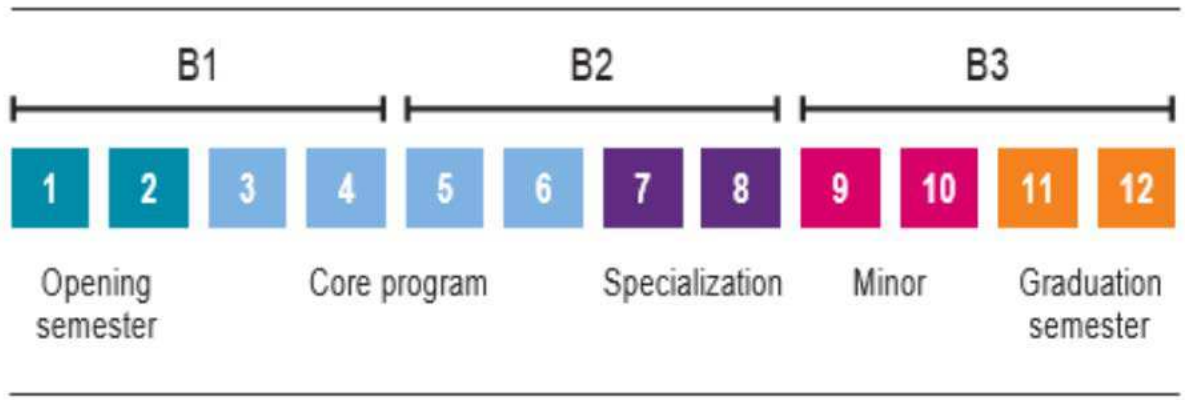

Figure 1: Module structure within Bachelor’s Programme
Figure source: Visscher-Voerman and Muller, 2017. Reproduced with permission.

The interrelated system of the projects and the supporting units is in line with the interdisciplinary aim of TEM, while the roles the students are called upon to play in their project teams speak to the aim of producing graduates who are flexible problem solvers. Within the teams, students are expected to take on multiple roles, specifically researcher, designer and organiser (CELT, 2017; Visscher-Voerman and Muller, 2017; ter Braack et al.2015). Not only does taking on these roles help develop the corresponding skills, but it allows students to follow their own specific interests and recognise and develop individual talents.

Each module's total grade of 15 European Credits (ECs) is contributed to by all the parts of the module. Some modules are graded as one unit. Others have the units graded separately, for instance each first-year mathematics unit usually has its own test with the grade contributing to the final single grade for the module.

The shift to the Twente Educational Model with its focus on project-based learning undoubtedly came with challenges. An obvious one was the need to design a suitable project for each module. In some cases existing projects could be adapted but in many cases entirely new projects needed to be created. A challenge experienced by the mathematics department was to continue teaching primarily the same content while living up to the potential of TEM by relating to non-mathematics units in the modules. In certain cases this was done by re-ordering the topics strategically (such as in my Calculus 1 course where differential equations are taught unusually early) or by interleaving two similar courses to create one course which can serve the needs of different groups of students.

In 2017, Visscher-Voerman and Muller reported on a suite of quantitative and qualitative evaluations on the success of TEM and concluded that the curriculum restructuring at UT has been successful, although work is still to be done. Visscher-Voerman and Muller's measures of success of TEM included increased student appreciation, increased student success rates and increase in innovative methods of teaching and assessing. Van den Berg et al. (2015) posit that a fully introduced TEM would take at least seven years. Below I present two modules as examples of the Twente Educational Model in practice, some six years after initial implementation, module 1 of Advanced Technology and module 2 of Electrical Engineering. The Advanced Technology module is a good example of a module that is self contained, while the Electrical Engineering module is a good example of a module that is interrelated with other first-year modules.

TWO MODULES AS EXEMPLARS OF THE TWENTE EDUCATIONAL MODEL

Advanced Technology Module 1: Mechanics

The department of Advanced Technology in the faculty of Science and Technology offers a bachelor's programme that aligns well with the Twente Educational Model's two aims of providing interdisciplinary education and producing graduates who are prepared to deal with a rapidly changing technological world. The programme combines knowledge and skills from "electrical engineering, chemical engineering, applied physics, mathematics, and mechanical engineering in a context that is both commercial and society-conscious" (Department of Advanced Technology, UT - Bachelor's Programme in Advanced Technology). The first module in the first year of the programme is called Mechanics and is designed to provide a first encounter with the world of engineering (Department of Advanced Technology, UT - The First Year of Advanced Technology). Over a period of ten weeks, students learn to model dynamic systems through engaging in one of a large number of offered projects. Their work in the project is supported by a unit on calculus, a unit on mechanics and training in laboratory work and experimental procedures.

The projects all involve a story and a problem, a research question, an experiment and a goal. The projects all involve having to develop a model using differential equations. The fifteen topics at time of writing include archery, car suspension, golf, gyroscopic spacecraft control, pole vaulting, rocket propulsion, roller coaster design, seismometers and swings. The students are also provided with some keywords relating to their model. For example, for pole vaulting the keywords are mass and spring systems, oscillations, bending, inertial versus non-inertial reference frame. "Rather than being separate parts, the intent is to have coherence between the various subjects. To this end the project integrates mathematics and mechanics and forms

the playground for achieving a deeper understanding of the subjects as well as developing the academic skills" (Department of Advanced Technology, UT – Bachelor's Programme Advanced Technology: Information Guide 2019, p. 10).

The mathematics unit is Calculus 1 and covers differential equations, differentiation, functions and limits, introductory vector analysis, complex numbers, logic, sets and proofs. In order to meet the needs of the Mechanics module, differential equations are covered in the first three weeks of the mathematics course. Solutions to second order differential equations require complex numbers, so that topic is covered in some detail in the second week.

The unit on mechanics covers the topics of Newton's laws of motion in translational and rotational domains, conservation of momentum, angular momentum and energy, rotation and static equilibrium. Free body diagrams are used to analyse static and dynamic motion and mechanical second order systems, such as springs and dampers, are studied.

The laboratory practice unit aims to develop basic skills for carrying out experimental work, such as formulating hypotheses, planning an experiment and laboratory safety. Data acquisition, data processing and error handling are covered as is the importance of keeping a systematic journal. A basic course in Matlab is offered within the laboratory practice unit for programming.

A sequence of one-off workshops is also offered as general support for the module and the project. These include a review of school mathematics, *LaTeX*, use of *Mathematica*, presentation skills and academic writing in English.

In groups of about eight, the students work on their projects throughout the module. The supporting units are presented in different ways depending on their nature; for instance calculus is presented relatively traditionally in the form of lectures and a combination of traditional tutorials and interactive small classes called "guided self study". Mechanics, in contrast, is presented in a sequence of blocks of preparation session, short lecture and tutorial on chosen problems.

The entire module is graded with a single result out of ten (with a passing grade of 5.5) which is a weighted average of the assessment grades of all the supporting units; calculus and mechanics grades are determined through tests, lab practice through journals and hand-in assignments on lab assignments, error analysis and programming skills, and the project itself is graded on group work, the final submitted report and the project presentation and ensuing discussion.

Electrical Engineering Module 2: Electric Circuits

The Department of Electrical Engineering offers a bachelor's degree designed to equip graduates with knowledge and skills applicable to a wide range of technical fields. With the types of research carried out at the university, the students get the opportunity during their degrees to work on cutting edge high-tech applications, such as robot-supported surgery. The bachelor's programme begins with a module introducing the students to electrical engineering and electronics and then continues with the second module called Electric Circuits. The focus of the second module is on learning to systematically analyse electrical circuits of passive elements. The students learn how to model a circuit using ideal circuit elements and ideal element equations in order to analyse both dynamic and static behaviour of the system (Department of Electrical Engineering, UT - The First Year of Electrical Engineering; Spreeuwers, 2018a).

The single project which forms the core of the module requires the students to “design and build a so-called solar inverter, which is used to convert the DC power of a solar panel into AC power and feed it into the power grid with maximum efficiency” (Spreeuwers, 2018a, p. 8). Early in the module the students begin to prepare for the project which is fully realised in an intensive two week period at the end of the module. All materials, such as solar panels, are provided. A small prize is awarded to the group which the most efficient solar inverter (Spreeuwers, 2018a, 2018b).

The mathematics unit completed by the students during module 2 is Calculus 2. This unit deals with sequences and series, integration theory and sundry techniques for solving integrals, as well as an introduction to multivariable calculus. Certain skills are employed immediately in the second module (such as integration) while others are preparation for the vector calculus unit included in the third module.

The circuit analysis unit addresses the behaviour of passive analog circuits and presents methods for analysing circuit models. It emphasises systematic analysis methods such as the node voltage method, convolutions, Fourier series, Bode diagrams and 2-port circuits.

In the unit on laboratory work, the students test the ideas of circuit analysis in practice and deepen their understanding of the concepts. The students are taught to use a journal during the course of the lab assignments for purposes of validity and replicability and thereafter to write a scientific report. Core to the lab work is generating hypotheses based on models of systems and then testing those hypotheses through observation (Spreeuwers, 2018a, 2018c).

From the point of view of mathematics, the first four modules of electrical engineering are more easily understood as a connected unit, rather than four individual modules. The second module is a good example of this across-module network. The second module requires skills encountered in module 1 (solving differential equations and working with complex numbers) and module 2 (integration, partial derivatives, series) and alludes to topics only to be encountered in module 4 (solving systems of linear equations).

Calculus 2 is taught through lectures, interactive small classes (called guided self study) and tutorials. Circuit analysis is similarly taught, however is assessed through multiple small tests rather than the single large test in calculus. Laboratory practice consists of eight weekly assignments which are individually graded.

Commonalities and differences

Other than certain large scale constraints, such as each module being worth 15 ECs, the teaching team and module coordinator of each thematic module have freedom to choose how each disciplinary unit is taught and assessed and how the different parts of the module can be designed to work together for a coherent purpose. The two modules described here are similarly structured but do have some striking differences, such as a wide variety of projects in Advanced Technology module 1 and only one project in Electrical Engineering module 2. The institution-wide nature of TEM imposes a structure across every department and every faculty, which has advantages such as students in modules 9 and 10 being able to choose “minor” modules from anywhere else across the university and have them seamlessly fit into their own degree structure, but simultaneously allows freedom to structure each module in ways which might differ markedly from other modules even in the same department.

An important part of the success of the system relies on the teaching team working together. For the two modules described above, each module’s team consists of the module coordinator, lecturers for the relevant units, the laboratory manager, the project manager, the study advisor (for guidance and counseling of first-year students (CELT, 2017)) and possibly

senior tutors. Effective teamwork among the teaching staff is essential to the modules' success (see also Cornejo et al., 2018). Of particular note is the evaluation process each module undergoes; at the end of each module, students have the opportunity to evaluate the course both in a Likert-style questionnaire as well as in long form responses. These comments are collated in an evaluative report and lecturers are required to respond to any complaints or suggestions for change. These responses are recorded, are expected to be acted upon, and are included for review in the following year's similar evaluative report.

DISCUSSION

Problem-based learning at the University of Twente takes different forms from module to module and from department to department. Certain modules are fully integrated single "atomic" units which cannot be usefully broken down into parts while others, such as the ones discussed in this article, have units that cohere with one another but are still recognisable as parts unto themselves. Other institutions structuring curricula in a similar way to the UT (Cornejo et al., 2018; Edström and Kolmos, 2014) also allow a variety of different modes of cohesion across the institution. Certainly at the first-year level the TEM projects are what Edström and Kolmos would term "discipline projects", ones where the students apply theoretical knowledge to relatively strongly framed practical problems, such as modeling the flight of an arrow in advanced technology or designing a solar inverter in electrical engineering. In the second and third years of the bachelor's programme the projects become "problem projects" where the problems to be addressed become increasingly ill-structured and complex. In this context of increasing participation in the engineering discursive community, TEM could be seen as encouraging the development of a discursive or core identity as an engineer through active engagement with engineering practice and discourse (Allie, Armien, Burgoyne, Case, Collier-Reed, Craig, Deacon, Fraser, Geyer, Jacobs and Jawitz, 2009; Craig, 2011; Craig, 2013).

Throughout the TEM modules, transferable non-technical skills are foregrounded as important. In each module the students need to write a report and present their work in front of an audience. In certain cases, such as in module 1 of Advanced Technology, writing skills and presentation skills are explicitly covered in dedicated workshops. Throughout the entire bachelor's programme the students have to work in groups where the workload necessarily needs to be shared between the group members. Research has shown that teamwork develops graduate skills that are valued by employers (King et al., 2017) and that active learning pedagogies, of which PBL is one, are aligned with teaching for equity (Tang, El Turkey, Cilli-Turner, Savic, Karakok and Plaxco, 2017), although concern has been raised (Beddoes and Panther, 2018) that teamwork practices could exacerbate gender inequalities in engineering. An investigation of gender inclusive teamwork practices in TEM could be an avenue for further research.

In addition to a group grade for the project report each member of the group is assessed individually on the same content, for instance through a short interview or a poster presentation. If one member of a group has failed a module and the rest of the group has passed, it will not be on the basis of the group work component but on another unit (such as calculus), which can then be reassessed through a second written test or an oral exam. A challenge is how to assess group project reports when some groups may have taken on additional strain due to members dropping out. In such cases the assessment has to take such difficulties into account and, again, interviews can help the teachers determine what is fair.

In some ways, mathematics is the unit that fits least well into the modules. Certainly, the topics covered in the mathematics courses are needed in the modules, but there is tension between

internal coherence within the sequence of interrelated mathematics courses (Calculus 1, Calculus 2, Vector Calculus) and coherence within the modules of which the mathematics courses are disciplinary units. The two modules discussed here are examples of that dilemma. In Advanced Technology module 1 (and indeed Electrical Engineering module 1, not discussed here) differential equations are covered first because of the demands of the Advanced Technology and Electrical Engineering units, however the rest of Calculus 1 is a selection of topics required to lay good groundwork for the two courses of calculus that follow and are not necessarily included for the Advanced Technology and Electrical Engineering modules themselves, such as limits, continuity and finding extrema. In Electrical Engineering module 2, the module requires very little of the calculus covered in Calculus 2 (mostly only needing integration techniques), primarily using differential equations skills taught in Calculus 1. Pedagogically speaking, it is not sufficient that the students encounter mathematics topics that their teachers know they will need, but that they can see the relevance of that mathematics (Dunn, Loch and Scott, 2018). Each first-year mathematics course is taught with one traditional two hour lecture per week, followed by small interactive classes called “guided self study” within which the teacher has the opportunity to choose exercises that are contextualized with the students’ study programme in mind if she chooses, and relatively traditional tutorials. Each programme has the opportunity to request that a “case study” also be included. The two modules discussed in this paper do not have case studies, but an example would be module 3 of Civil Engineering where a case study on traffic flow is included to make explicit connections between the (otherwise rather general and abstract) linear algebra unit included in that module and the civil engineering context. Ensuring that the sequence of first-year mathematics courses maintains an internal coherence while still cohering with the needs of the modules requires constant communication amongst the module team of teaching staff and is a matter of ongoing development.

Assessment remains a troublesome part of TEM. At time of writing, each module is passed or failed as a unit, which is worth 15 European Credits (ECs). The various parts of the module such as the project and its supporting disciplinary units all contribute to that module grade, often through a weighted average involving certain conditions or subminima. In modules such as the ones discussed here it is possible to pass all but one unit (for example the mathematics unit) and therefore have to repeat the entire module. This baseline requirement of TEM is not accepted in all departments; some modules structure their assessment requirements such that only parts of a module need be repeated. At the institution, the possibility of formally breaking up the “all or nothing” structure to make it possible to repeat only one part of a module is under discussion and is likely to be carried out. Another assessment challenge is how to introduce more formative and less summative assessment and thereby perhaps make assessment more student-centred (Visscher-Voerman and Muller, 2017; van den Berg et al., 2015). Graham (2012) observes that even very successful and institution-wide engineering education reform is vulnerable to a “drift” back to a traditional curriculum. On the other hand, Graham also observes that curriculum reform proves resilient if there is “an on-going focus on educational innovation and reinvention” (Graham, 2012, p. 3). Time will tell if the breaking up of the module grade is a sign of TEM drift or of reinvention.

CONCLUSIONS

The Twente Educational Model (TEM) has two primary foci. The first is the incorporation of interdisciplinarity into the curriculum to reflect the interdisciplinarity present in the institution’s research and that present in the industries and fields into which UT graduates may go. The second focus is to prepare its graduates for a rapidly changing world by teaching them current and valuable technical skills as well as transferable non-technical skills related to communication, presentation and effective teamwork. Six years of implementation of the

model have resulted in a mature system of project-based modules which are continually being evaluated for strengths and weaknesses.

Edström and Kolmos (2014) suggest three learning principles to guide the practice of project-based learning; I argue that TEM adheres to all three. First, the cognitive learning components of the modules are addressed through contextualised problems at the heart of the projects. The students need to analyse and define the problems which in the first year are generally well-defined. Secondly, the curriculum content in any module is interdisciplinary with skills from different disciplinary units needed to support the projects, such as calculus and circuit analysis. Thirdly, the social approach which is crucial to effective PBL is fulfilled by working in teams where communication occurs within and between teams; knowledge is created collaboratively.

After an in-depth study of engineering education reform across multiple institutions, Graham (2012) concludes that successful systemic change is often driven by “significant threats to the market position of the department/school” (p. 2). Certainly market forces did play a role in the university’s decision to bring about institution-wide change (Visscher-Voerman and Muller, 2017; van den Berg et al., 2015), but it is not enough for those market or other external forces to be present; throughout the institution the need for radical and curriculum-wide reform needs to be acknowledged and acted on with support from university leaders and management. Allowing plans for change to be influenced by current educational theory and innovative practice (in this case PBL) can result in successful and enduring reform.

This paper aims to contribute to the literature on curriculum renewal in higher education by presenting the Twente Educational Model and two successful implementations of the model in advanced technology and electrical engineering. While challenges still exist and details continue to change, the Twente Educational Model is an example of mature curriculum reform at a technical university that offers students an interdisciplinary education which can prepare them for a rapidly changing world.

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SUPPORTING LECTURERS TO MONITOR THEIR STUDENTS' LEARNING AND DEVELOP THEIR PRACTICE THROUGH ENGAGING WITH MATHEMATICS SUPPORT CENTRE FEEDBACK

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KEYWORDS: professional development, mathematics support, reflective practice

ABSTRACT

This paper analyses the engagement of lecturers of mathematics or mathematics-related subjects, with the feedback from their students' visits to a university mathematics support centre, as interpreted by the attending support tutor(s). This engagement is analysed in two ways. Firstly, via 43 lecturer responses to two end-of-semester electronic surveys conducted one year apart and secondly via analysis of reflective journals kept by lecturers the following year. This latter study, conducted with nine lecturers, involved them writing down their thoughts immediately upon reading the weekly email of mathematics support centre feedback relating to their module. We analyse these reflections and categorise them into three themes namely, reflection for action, knowledge of content and students, and student engagement.

The results suggest that the feedback received from the centre influenced lecturers' teaching practice in terms of content delivery and on placing further emphasis and time on important concepts. This study offers a route to leveraging the feedback collected at a mathematics support centre on student learning in order to close the feedback loop.

INTRODUCTION

Feedback on the mathematical content of student queries collected electronically at a mathematics support centre (MSC) has been available to lecturers at University College Dublin (UCD) since 2009 (Cronin & Meehan, 2015). The challenges associated with collecting such qualitative feedback on student learning and arguments for determining whether such feedback is worth collecting in the first place are important aspects to consider when attempting to evaluate the effectiveness of mathematics support (Curley & Meehan, 2015; Croft, 2009). How lecturers value and utilize such feedback on their students' learning is a largely under investigated phenomenon despite the benefits it affords (Cronin & Meehan, 2019). Indeed Croft, Grove and Lawson (2016) argue that both mathematics support and the mainstream teaching of mathematically rich modules can benefit by appropriate and effective channels of communication to facilitate exchange of information across the institution.

In this paper we explore the extent to which lecturers engage with mathematics support feedback and how it has helped shape their teaching going forward. In terms of examining their own pedagogy a lecturer's *reflective practice* provides a popular means for teacher professional development. Teacher reflection refers to teachers subjecting their beliefs and practices of teaching and learning to a critical analysis. One way to promote reflective practice

for both novice and experienced lecturers is through habitual journal writing. This case study sought to investigate in what ways, if any, regular journal writing based on engaging and reflecting on weekly mathematics support centre data promoted reflective and critical thinking on student learning among nine university mathematics instructors in Ireland over a 13-week period.

We examine how these lecturers of large first year university mathematics and statistics classes thought about and understood the feedback they received from MSC feedback, and the extent to which they engaged with it to monitor and further support their students' learning. To these ends we pose the following three research questions:

1. In what ways, if any, do lecturers find the electronic feedback provided by the MSC on students' visits useful?
2. What are the recurring themes, if any, among the written weekly submissions upon reading and reflecting on MSC feedback?
3. How can lecturers be supported in reflecting critically on their students' learning as interpreted via MSC feedback?

LITERATURE REVIEW

We give a brief overview of the literature on reflection as it pertains to the higher educational sphere before summarizing the mathematics learning support literature to date.

Reflection

Mathematics instructors at UCD have had the opportunity to engage with, and reflect upon, the feedback from their students' queries, collected electronically at the MSC since 2009 (Cronin & Meehan, 2015). Reflective practice is a key skill for instructors which affords them opportunities to learn from their experiences in a sustained and effective manner (Schön, 1983). Schön (1983) describes two types of reflection. *Reflection-in-action* occurs when the instructor's self-awareness of their pedagogy and skillset are utilised to handle contingent moments as they arise during teaching. This in-the-moment thinking on one's feet meta-cognition informs the instructor's next steps. *Reflection-on-action* refers to reflection occurring either before or after the teaching event has occurred. Reid (2004) argues that this type of reflection is more systematic; where reflection-in-action is immediate, and often implies split second decision-making, reflection-on-action takes more time, and involves looking at evidence, thinking about theories and alternatives. To these two reflective practices, Reid (2004) adds *reflection-for-action*, the forward planning, based on preceding reflection. This form of reflection, he contends, can and should be collaborative both with teaching peers and students. Boud, Keough and Walker (1985) define reflection as "those intellectual and effective activities in which individuals engage to explore their experiences in order to lead to a new understanding and appreciation".

Argyris and Schön (1974), and Brookfield (1995) argue that for reflection to lead to a change in practice it must consistently and critically evaluate and challenge personal assumptions and beliefs by confronting such values in the face of unsettling teaching experiences while exploring and imagining alternatives.

For feedback on student learning to be used effectively to change teacher practices requires staff to engage critically with the data to elicit positive change. Fuchs and Fuchs (1986) reported that whilst simply collecting data had some impact on student achievement, when instructors were required to interact with this data through reflection, the impact on student achievement was enhanced further.

Mathematics Support Literature

Mathematics support can be briefly summarized as that support which occurs outside of the regular timetabled teaching and learning activities associated with a module or programme and is usually delivered to non-specialist mathematics students. The most common form of mathematics support is the drop in model, while bookable appointments are also common. The growth of mathematics support in higher education in Ireland, England, Wales, Scotland and Australia has been well established (Cronin et al., 2016; Perkin et al., 2012; Ahmed et al., 2018; MacGillivray, 2009). There is evidence that such support has become a more embedded and sustainable activity within higher education (Grove et al., 2018) that shows no sign of abating as the needs for mathematics and statistics predominates such a diversity of higher education programmes today. Much of the scholarship in the area has centred on the evaluation, effectiveness and impact of the mathematics support initiative (Matthews et al., 2013). These studies focus on the users, and non-users, of mathematics support in terms of who they are, what their academic background is and how they use the service e.g. to merely pass their course or to improve their chances of excellence. Many local studies suggest that users have benefited in terms of retention, confidence, and success from such support and engaging those most at-risk and the non-engagers (Hillock et al., 2013; Rylands and Shearman, 2018)]. Other studies focus on who delivers mathematics support and include the preparation and training of tutors (Fitzmaurice et al., 2016; Grove and Croft, 2019). However there is little in the way of scholarship showcasing how the activities engaged with within mathematics support centres particularly can impact on mainstream lecturing and curriculum design at the modular or programme level. The 2015 all-Ireland audit of mathematics support provision recommends that:

> Mathematics Learning Support staff should collaborate and make use of institutional connections with module and programme coordinators to assist lecturers who may wish to reflect on their teaching practice to enhance further the learning experience of mathematics for their students (Cronin et al., 2016, p. xi)

A study of Cronin and Meehan (2019) suggests that feedback on student learning collected at a mathematics support centre is one of the most useful forms of feedback lectures receive, in that it is unique, useful, detailed and impacts positively on practice and resource development.

The UCD MSC Feedback Process

Feedback from each tutor-student interaction at the MSC is recorded electronically by the attending tutor(s), based on the nature of the content discussed by both parties. The tutor will typically record the interaction in two ways. Firstly the high level mathematical category of the visiting student's query is recorded, via a drop down menu, and then at a more granular level e.g. Mechanics -> bending moment diagrams – see Figure 1.

The tutor then adds a qualitative free-form response usually consisting of 2-3 sentences using the following structure: (1) what did the student state their issue/query was, (2) what did the tutor diagnose the issue to be (if different from (1)), and (3), what did both parties do to remedy the query – see Figure 2. This data is then anonymously (neither tutor nor student are identified) available to the relevant course coordinator electronically as soon as the session finishes. Each Friday, lecturers also receive a weekly email detailing all the feedback data from their students' visits to the MSC for that week in addition to a cumulative student visit count for the semester thus far.

Figure 1: Example of MSC tutor's interface when entering feedback

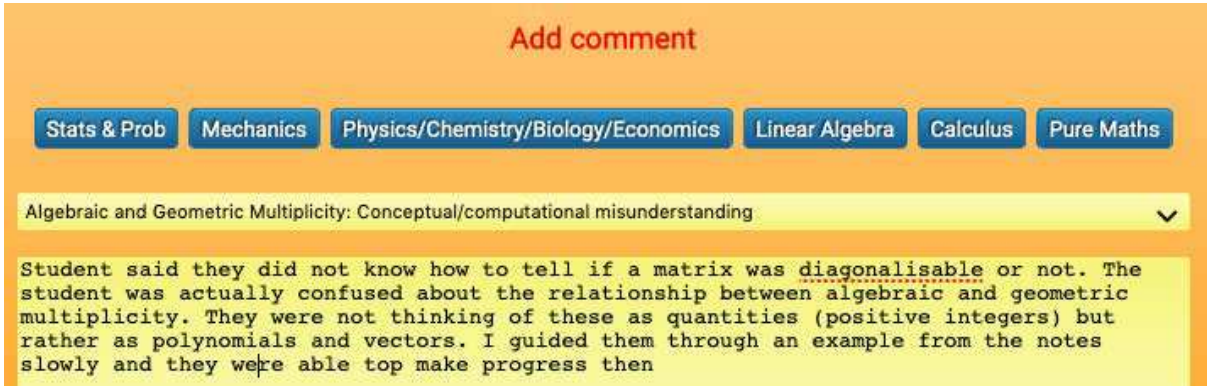

Figure 2: Example of tutor's free-form feedback on a student query in linear algebra

METHODS

This study was conducted at a large research-intensive university in Ireland between 2016-2019. An online survey was administered at the end of the first teaching semester in both of the academic years 2016/17 and 2017/18. The surveys were conducted with lecturers of large (>100 students) mathematics related subjects both from within the School of Mathematics and Statistics (SMS) and other schools e.g. School of Physics, School of Business, School of Economics.

The survey questions were identical each year and asked lecturers: if they read the electronic mathematics support centre feedback digests, and if so were they useful, and if so in what ways. Respondents were also asked if they were ever prompted to do anything different (e.g. with content delivery, assessment practice, lecture content) as a result of reading the feedback and finally, how they rated the MSC feedback in relation to other forms of feedback they receive on their students' learning.

In a follow-up study during the first semester of 2018/19 lecturers of mathematics classes within SMS were asked to volunteer in a reflective exercise whereby they would keep a diary of their reactions and responses upon reading weekly feedback from the MSC on their module. The reflective journal was kept by 9 lecturer participants teaching 10 different modules, 8 first year and 2 second year modules (see Table 1 for further details on the module class sizes, module stages and MSC engagement data). Six of the participants were male and 3 were female. Six of these participated in both the online surveys from the previous two years. The lecturers' teaching experience varied from 1 to 33 years with the majority (7 lecturers) having at least 10 years of lecturing experience.

These journals were written into once every Friday upon receiving the weekly electronic MSC email of their students' queries at the MSC for that preceding week.

The diaries were collected once by the researcher at the end of week 7 of semester and returned to the lecturers before the entries for week 8 were due and were collected again in week 15 of semester when formal teaching had ended. The journal entries were manually transcribed and then imported to the qualitative analysis software package *Nvivo* for analysis. Following the 6-stage thematic analysis approach of Braun and Clarke [22] the three themes of *Reflection for Action, Knowledge of Content and Students,* and Student *Engagement* were identified as most prevalent in the journals. This analysis consisted firstly of coding all lecturers' reflections before identifying patterns from these codes to establish the semantic themes relevant to the research questions.

RESULTS

We address research question one by analyzing the responses to the two electronic surveys conducted one year apart. Research questions two and three are addressed via results of the analysis from the reflective journals and follow-up conversations with participants.

There were 19 complete responses received from 37 lecturers surveyed (51%) in 2016/2017 where 14 respondents taught within SMS and 5 taught externally. In 2017/2018, there were 24 complete responses from 44 surveyed (55%), 12 respondents from SMS and 12 externally. We now present analysis of the lecturers' responses to the three survey questions most relevant to research question 1.

Usefulness of the feedback

All 43 (non-distinct) respondents over both years stated that the MSC feedback was useful to them. When asked in what ways the MSC feedback is useful and what lecturers do with it qualitative responses fell in to categories of 'Knowledge of students' weaknesses', 'Impact on practice' and 'Effects of the feedback on teaching'.

Knowledge of students' weaknesses and impact on practice

Lecturers were consistent in their responses in that the MSC feedback offers information on where students are weak in terms of the module content and it also reminds them of weaknesses in their students' prior knowledge. This data is then used to further emphasise such content in the opening minutes of lecturers or to devote extra time to the concepts causing most difficulty. Lecturers also use the MSC feedback as a proxy to devise revision lectures for the entire class.

> I find the information very useful. It gives me a good idea of the weaknesses of the students and I normally gather the problems and emphasize them in class by explaining better or giving more examples/exercises. I also use the reports for

> my revision week where amongst my standard revision topics I emphasize on the topics raised by students in MSC.
>
> If there are more than a couple of entries related to one topic/concept and relating to more than one or two distinct students I will address this issue in class. I might start a lecture with it stating that “I notice a few of you were asking about x, y and z in the mathematics help centre this week so I just want to clarify...” - this encourages more people to engage with the support the mathematics help centre offers then also
>
> It helps me assess which aspects of the module cause most difficulty. I then give them more time in class

Effects of the feedback on teaching

The survey question on whether the feedback prompted lecturers to do anything differently elicited a variety of responses. They ranged from changing lecture delivery style, lecture pace, and lecture notes to incorporating comments from the MSC feedback in to aspects of face-to-face components of instruction with the class. A common response was that extra time was given in lectures to address misconceptions or areas of difficulty arising from discussions in the MSC

> Yes, based on this feedback I have changed the lecture delivery this year. When I was spotting a good number of students seeking help on the exact same thing, I was picking it up and was addressing it in class explaining in a different way or giving more examples.
>
> Yes. I often revisit material that has posed problems. I also introduce revision material before each lecture and I have changed content and delivery in one first-year module as a result of the feedback.
>
> Yes, it might guide my opening minutes of the next week’s lectures or I might add a revision question to a homework sheet that came up frequently in the mathematics help centre.
>
> It helps to get a sense of what some of the students are finding difficult, and has informed the way I give explanations during the lectures. It guides me as to what I need to spend more time on. I incorporate comments on the MSC reports into my lectures, tutorials, revision.

Value of the feedback

The survey options on rating “MSC feedback compared to other forms of feedback on student learning within your module” were *excellent, very good, neutral, poor* and *very poor.* In 2017 (n=19), 9 responded *excellent* with a further 9 responding *very good*. One lecturer was not aware of any other forms of student feedback on the content of their students’ learning. In 2018 (n=24), there were 7 lecturers who chose *excellent*, 10 chose *very good*, 5 *neutral* with 2 respondents stating that they were unaware of any other such feedback to compare MSC student feedback to.

We conclude based on this evidence from the two electronic surveys, conducted one year apart, that lecturers of mathematics classes find MSC feedback on their students’ learning useful in the ways described above. This utility is reflected in the impact it has on instructors’ lecturing practice.

Reflective Journals

Analysis of the weekly journal entries indicates that the instructors wrote mostly about the actual content of the mathematics with which the students presented with at the MSC, e.g. limits, chain rule, integration by parts etc. They also commented on whether this was prerequisite or module-specific knowledge and on the duration of student(s) visits. As the classification of the mathematical content and areas of difficulty arising in mathematics support centres is not the focus of this paper (see Cronin et al., 2019 for such a discussion) these largely descriptive entries are not included in the following analysis which addresses research question two. Once again following the thematic analysis approach of Braun and Clarke (2006), three themes were identified as most prevalent in the journals; *reflection for action, knowledge of content and students*, and *student engagement*.

Reflection for action

Lecturers use MSC feedback to prompt them to implement various behavioral practices in their lectures, including; changes to content delivery in terms of pace and clarity, student questioning, and further emphasis on certain topics. They also used the feedback to modify their lecture notes, problem sets and assessments. Predominantly these actions involved revisiting content in lectures that the lecturers perceived as important, and to "improve", "rectify" and "clarify" lecture notes and/or problem sheets in a timely fashion. The MSC feedback also prompted others to add extra examples and worked-out solutions to existing lecture notes and e-learning tools. It also reminded lecturers to move more slowly and clearly in class. The journals also reflected the inclusion of: extra topics for revision lecturers, extra resources created by lecturers and added to the Virtual Learning Environment (Blackboard and Moodle), the dedication of lecture time to revise already introduced material before moving on with the module, and defining terms that were present in e-assessment software that were not defined in class.

> Visits about the graph of the exponential function which I will go over again in class since it is so important.

> There are problems with lecture notes as some concepts are not explained clearly. Will have to improve lecture notes next year.

Knowledge of content and students

Ball, Thames, and Phelps (2008) delineate the special forms of knowledge a mathematics instructor must possess in order to be an effective educator. In particular, they classify the domain of Pedagogical Content Knowledge, and its subdomains Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching as important factors in improving mathematics pedagogy. KCS allows mathematics instructors to develop their appreciation and understanding of common pitfalls students encounter and can provide them with valuable information on how to effectively support student learning.

A predominant theme throughout the journal entries was the ability of the MSC feedback to inform and enlighten lecturers as to the difficulties their students have both with module specific content and with assumed prerequisite material. It was stated in several accounts that this type of feedback is not discernible from lectures, regular tutorials or from grading assessments throughout term but is only revealed via MSC feedback. Lecturers reflected that is was important to be reminded how weak some students are and that some students do not possess the prerequisite knowledge to succeed in the module.

> It reminds me of how weak some students are and I guess that's very important for me

> Again a lot of visits and a lot of the topics seem to be the ones that cause trouble every year. However I am seeing more trouble with easier topics that don't usually cause trouble. I hope this is not a sign that the weaker students are really struggling

There was frequent concern among lecturers that many students were struggling with basic material e.g. algebraic manipulation, elementary errors and procedural difficulties, and that these issues were persisting well in to the term (weeks 9-13).

> So many questions about "the slope" at this stage [week 13] is worrying

> Their problems with signs and completing the square is very useful to know

Student Engagement

The final theme arising from the journal analysis was the issue of student engagement. This arose both in a reassuring manner and in a more negative aspect. Many participants were reassured that the engagement of students at the MSC was providing evidence that the learning objectives of the module were met and discussed in the MSC setting.

> My key reflection this week: This is exactly what I want the students to do, so I am confident that the problem sheet is meeting the learning objective in asking students to make or build their own examples and I view this feedback as evidence of this

It was also positively remarked that student engagement with the module content was occurring from an early stage in the semester also (from week 1 onwards) and that supplementary problems were also been discussed at the MSC.

> Good to see students engaging in revision/non-compulsory homework so early on in the semester

Many lecturers assume that because of the nature of the student query or because of the topic queried in the MSC that certain students must not have attended lecturers and/or tutorials. While we have no such attendance data to support these beliefs, anecdotally at least we can say that those who regularly attend MSC also regularly attend lectures and tutorials in the main and so that this may be an unfounded conclusion of lecturers. Also the issue of continuous assessment and midterm examinations driving attendance and engagement at the MSC was noted and welcomed by most lecturers. This was followed up by a concern among lecturers that the numbers doing poorly in summative assessments were not engaging with the MSC in the numbers they should be. This sentiment was echoed by lecturers who felt that first year students in particular are not yet in a self-monitoring or independent frame of mind and that the MSC is an underused resource.

> Knowing as I do that 30% of the class are likely to fail the final based on the midterm results, I am astonished that more students are not seeking help from the MSC

> The students have finally woken up – too late for me to address any problems in lectures though

> It is depressing that there have been no visits about this, though since looking at the CA marks, and the feedback from MSC tutors, there are a lot of students who need help

LIMITATIONS

In answering research question three it can be argued that there was little further insight to be gained from the journal analysis than was already present in the electronic survey analysis. This is perhaps good news in that such a study may be replicated easily by other mathematics support centres interested in linking their activities with the teaching and learning activities of the module. It also suggests that to involve lecturers in genuine critical and ongoing reflection of their practice may require extra support and collegiate collaboration in terms of participants being included as co-researchers rather than study subjects. Indeed in follow-up conversations with the participants of the reflective journal study many mentioned that a regular group meeting of lecturers of large first year classes would assist them to reflect more deeply on their students' learning.

The lecturer participants were not given any formal training in reflective practice or asked to read any scholarly articles on the topic and hence journal entries were *reflective* in varying degrees with some more descriptive than critically reflective in nature. Thus it would be interesting to replicate this study with a sample of lecturers who had engaged with the concepts of reflective practice. This may form part of an accredited professional development course for new lecturers and more experienced mathematics instructors. Based on analysis of the reflective journals and conversations held with lecturers after the journal study was completed it is conjectured that lecturers need further support in their response to student feedback.

FUTURE WORK

The next stage of the research is to devise a resource for lecturers to assist them when reading the weekly MSC feedback. This could include a regular meeting of lectures or a software feature that denotes whether the particular student query in question relates to module specific content or to assumed prior knowledge as perceived by the lecturer. There will also be a place to include comments from other sources of feedback received on their students' learning e.g. from timetabled tutorials, so as to assist lecturers triangulate feedback on their students' learning. This study describes an evolving and organic experience of engaging with student feedback as collected at a mathematics support centre. This form of immediate, formative and on-going feedback via the mathematics support centre was highlighted by instructors as useful and valuable and thus we recommend further research in this area, allied with the development of a similar feedback instrument for timetabled tutorial/laboratory sessions. In follow up conversations with the lecturer participants, many suggested that a focus group or group meeting of instructors could aid in assisting lecturers to reflect more deeply on their students' learning.

CONCLUSIONS

This study was concerned with the impact of MSC student feedback on lecturer practice, and the wider impact that engaging in such feedback practices can have on the learning-teaching environment. In addressing the research questions, and considering the evidence presented, it can be concluded that when provided with such a feedback framework (i.e. the MSC student feedback format) to support the teaching process, this feedback has the potential to have a positive impact on lecturer practice. Consistently engaging with MSC feedback can further enhance reflective capacities in lecturers; inform lecturers about the individual needs of their students; and, open up a dialogue of teaching and learning both in lecture theatres and tutorial settings.

In addition to responding to the research questions, the study also provided further insight into the use of student feedback as collected at a mathematics support centre to improve teacher practices that may be applicable in wider educational settings. Participants viewed the student feedback as useful as it allowed them to identify misconceptions held by students in the teaching and learning process. This information gained from students via the attending MSC tutor(s) enlightened lecturers to a range of factors influencing the teaching and learning process, for example: reflection-for action; knowledge of content and students; and, student engagement. As lecturers frequently spoke about the MSC feedback at their next lecture, this process also supported positive relationships between teachers and students, and enabled them to form a trustworthy and authentic partnership in teaching and learning. This suggests that this form of feedback may have potential to effectively inform wider curricula design and change in higher education.

A crucial factor to understanding the results of this study is that this was not an end-of-semester exercise in gathering student feedback on their satisfaction levels with the module's teaching, learning or assessment. Rather that this feedback offered an integrated, and on-going process designed to support lecturers' professional growth and reflective mindsets. The feedback assisted lecturers to develop a deeper understanding of their learners and how to respond to their needs. This study into the use of mathematics support centre feedback provides evidence that the dialogue between lecturers, tutors and students needs to be open, ongoing and should firmly establish the student voice as a key element in working to improve teaching and learning of mathematics at third level.

To support the learning improvement, students need to be given opportunities to communicate their learning needs and perceptions of the learning process. This does not mean that the voice of the student become the only driver of change. This form of formative feedback offered by MSC feedback, via a supporting mathematics tutor, gives just one way for lecturers to gain a perspective on their students' learning. Coupled with collective lecturer expertise, lecturers can develop an informed approach to their lecturing practice. Also of importance, is developing department leaders' capacities to support staff using feedback, without infringing on the process. Regardless of the research focus, if the conversation between lecturers, tutors and students is occurring in a safe and supportive way – then the benefits can only be positive, with all parties working in partnership to improve mathematical learning outcomes for students.

Table 1: MSC visitor statistics for the 10 modules in the reflective study

Module	No. of MSC visits	% of class visiting MSC	Class size	Year
Access Mathematics	174	64%	28	1
Economics Mathematics	21	19%	67	1
Engineering Calculus	177	19%	316	1
Foundation Mathematics	19	17%	84	1
Linear Algebra	361	35%	294	1
Mathematics Modeling	78	25%	130	1
Mechanics 1	38	19%	75	1
Mechanics 2	34	18%	74	2
Multivariable Calculus	168	51%	69	2
One-variable Calculus	80	42%	135	1
Totals	1,150	31% (Average)	1,272	

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WORSKHOP SUPPORT FOR FIRST-YEAR MATHEMATICS AND STATISTICS

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KEYWORDS: mathematics and statistics support, first-year, mathematics background

ABSTRACT

The Mathematics Education Support Hub ran mathematics and statistics support workshops for 17 different subjects from 2016 to 2018. The mathematics and statistics in these subjects was at first-year level. Data on workshop attendance was collected. For each student in a subject for which workshops were run, final marks, secondary school mathematics background and degree enrolled in were obtained from university records. This enabled an investigation into attendance at support workshops in relation to mathematics background and discipline of the degree, and into the effectiveness of support workshops and relationships to mathematics background.

Greater workshop attendance was linked to higher final marks, with noticeable and significant differences for different mathematics backgrounds. Unsurprisingly, students with stronger backgrounds performed better than those with weaker backgrounds at every level of workshop attendance. An interesting result is that students with no mathematics in their final year of secondary school showed a much greater rate of improvement in final mark for increasing workshop attendance than any of the groups with secondary school mathematics, surpassing those with an Elementary mathematics school background at a very low level of workshop attendance. Significant differences in workshop attendance were found for different level of mathematics background and different discipline of degree of study.

INTRODUCTION

Much has been written recently about the importance of STEM skills, for example, ‘STEM skills are essential to 75% of Australia’s growth employment areas’ (AMSI 2018; p.8). The report goes on to say that ‘It is essential to ensure Australia has the mathematical and statistical skills to remain internationally competitive and protect national security, population health and climate stability into the future’. The importance of mathematics and statistics is illustrated by the inclusion of these subjects in bachelor’s degrees in STEM disciplines including science, engineering and computing, as well as in non-STEM disciplines such as business.

Despite the importance of mathematics and statistics, many Australian universities do not have mathematics prerequisites for degrees with a mathematics and statistics component. For example, Belward, Matthews, Rylands, Coady, Adams and Simbag (2011) found that of the 17 Australian universities with more than 1800 students enrolled in the natural and physical sciences in 2005, only seven advertised secondary school mathematics as a prerequisite, with another five listing it as assumed knowledge. The Australian Mathematical Science Institute (AMSI) (2018; p.10) wrote ‘Universities must address the issue of pre-requisites’. Universities

without mathematics prerequisites for degrees containing mathematics and statistics must support mathematically underprepared students; such students are at risk of repeated failure.

The mathematics subjects offered in the final year of secondary school have been classified as Elementary, Intermediate and Higher, with Higher mathematics subjects being what Barrington and Brown (2014) called Advanced mathematics. For example, in New South Wales mathematics subjects classified as Elementary contain no calculus, unlike the Intermediate mathematics subject. The Higher mathematics subjects (Mathematics Extension 1 and Mathematics Extension 2) devote more class time to mathematics and include opportunities to understand proofs. Barrington and Brown (2014) report on an alarming move from 1995 to 2013 of students from Higher and Intermediate mathematics to Elementary mathematics. James (2019) reports that the proportion of students studying Higher mathematics in Australia in 2017 was the lowest level recorded in more than twenty years. The Choose Maths Gender Report (2017) contains estimates that in 2016, 20.9% of girls and 8.5% of boys did not study mathematics in their final year of high school. The problem of mathematically underprepared students studying mathematics and statistics at university has worsened as increasing proportions of secondary school students choose lower levels of mathematics and no mathematics in the last two years of school.

The problem of mathematically underprepared students is further exacerbated by increasing the proportion of the population who are studying bachelor's degrees. A result of the Australian government's decision to increase this proportion is that 24% of youths and adults in Australia in 2016 completed a bachelor degree or above, up from 18% a decade earlier (Australian Bureau of Statistics 2017).

Poorly prepared students are at risk of leaving university before graduating, thereby increasing the attrition rate. At Western Sydney University (WSU) the current strategic plan includes retention as a measure of success for two of its six strategic objectives. Indeed, a government report on retention states that universities are increasingly including retention targets in their strategic plans (Higher Education Standards Panel (HESP) 2017). The report in several places stresses the importance of the provision of support services for students. A report for the Australian Minister for Education on performance based funding (Wellings, Black, Craven, Freshwater & Harding 2019) proposed four measures for the funding scheme, one of which is attrition, meaning that attrition and retention will become even more important for universities.

Increasing numbers of mathematically underprepared students, together with the importance to universities of retaining students once at university, increases the importance of effective academic support for students.

Like many universities in Australia, WSU offers mathematics support for students. At WSU this support is provided by the Mathematics Education Support Hub (MESH). The HESP report (2017) assumes that academic support benefits students and there is much in the literature reporting improved performance by students who use mathematics and statistics support (for example, the studies cited in Rylands and Shearman (2018)). However, providing academic support is expensive and there are many different ways in which support can be provided. It is therefore important to evaluate the effectiveness of any support services provided, and for each student cohort that uses each service. Lawson, Grove and Croft (2019) note that evaluation can provide evidence for continued funding of mathematics support.

It has been reported that lack of engagement with mathematics support is a problem (Mac an Bhaird, Fitzmaurice, Ní Fhloinn & O'Sullivan 2013) and that there are different levels of use of support by students from different disciplines, and in some cases more use by stronger rather than weaker students (Mac an Bhaird, Morgan & O'Shea 2009; Pell & Croft 2008).

Lawson et al. (2019), in Section 6 of their extensive literature review of mathematics support, note that the majority of published work reports on who uses support and how often, and topics discussed. Some studies report on student feedback. These studies do not report on student achievement and retention. This paper studies the less reported connections between mathematics support and achievement. Lawson et al. (2019) state that the drop-in model of support is dominant; here the less common workshop model is studied. There is little or no research which considers the level of use of workshop support and mathematics background to predict performance gains.

The aim of the analyses presented here was to explore the use and effectiveness of MESH support workshops, especially in relation to mathematics background, and to also give some consideration to the discipline of the degree in which students were enrolled. The results could enable MESH to better serve all students and provide evidence to those who fund mathematics and statistics support at WSU, and more broadly, that workshop support appears to be effective.

Background of the university, students and workshop support

In 2018 WSU had a little over 48000 students. The university is a large multi-campus university in Sydney, Australia. There are no prerequisites for entry into degrees at WSU, so students in first-year subjects have mathematically diverse backgrounds. In the first-year subjects supported by MESH workshops, roughly two-thirds of students are underprepared for those subjects. By underprepared we mean no mathematics or Elementary mathematics in the last two years of secondary school.

MESH offers a variety of support services. In this paper we study the use and effectiveness of only the MESH workshops.

Almost all MESH workshops are run just before tests, assignments and examinations, as this is when students are most interested in support. The workshops run for one, two or three hours. The majority of in-semester workshops run for one hour, with some two hour workshops on *R* or *Microsoft Excel* for particular statistics subjects. Examination preparation workshops run at the end of semester for two or three hours.

The number of workshops run for a subject varies from three to seven different workshops, with some workshops run several times (at different locations or the same location if there is enough interest from students). A few workshops run in ‘lecture’ time. This can only occur when teaching staff do not always use all the time allocated, for example, if a three-hour lecture slot is booked for a two-hour lecture because sometimes the third hour is needed for a test.

Workshops are facilitated by MESH staff; students are encouraged to work in groups on problems prepared by MESH. For many workshops, sample tests and examinations are used; these tests and examinations are approved by the relevant academic to ensure that the content and level is appropriate. After each workshop, worked solutions are sent via email to students who attended so that students can continue to work on the problems and check their answers.

Apart from helping students to better understand the content which they are studying, the workshops aim to improve students’ collaborative learning skills as discussed by Laal and Ghodsi (2012) and their ability to relate potential assessment questions to the subject content. MESH staff are available to assist groups with this work as they require it. All workshops are designed to encourage students to develop their own abilities to solve problems by relating the problem to knowledge which they already have.

In Autumn and Spring semesters from 2016 to 2018, 17 different subjects were supported with MESH workshops. Some of these subjects ran in both semesters, some once a year. WSU offers a large number of mathematics and statistics subjects as many of these are designed specifically for particular cohorts of students (for example, science, engineering and computing). The subjects supported from 2016 to 2018 are first-year subjects, or the mathematical content in them is at first-year level. The subjects are

- 4 statistics subjects,
- 2 mathematics subjects for education students taught by School of Education staff,
- 3 mathematics subjects for engineering students,
- 2 sport and exercise subjects with mathematical content,
- 2 pre-calculus mathematics subjects,
- 3 calculus subjects (different to the engineering subjects),
- 1 discrete mathematics subject.

A computer programming subject was very briefly supported with workshops, then the support moved online. This subject is not included in the analysis presented here.

METHODS

Data

The data collected for MESH mathematics and statistics support workshops are

- student identifiers for attendees at each workshop,
- subject the workshop was run for.

Workshop attendance is counted as the number of workshops attended. Students who attended the same workshop twice have been counted as attending two workshops.

The university holds information on all students and from this MESH was given access to the following information about the subjects supported by workshops:

- student identifiers for all students in the supported subjects,
- secondary school mathematics background where this is recorded,
- the degree in which students were enrolled,
- students' final marks and grades in the subjects supported by workshops.

Students' secondary school mathematics background was classified as Unknown if there was no information in the university records, otherwise as None, Elementary, Intermediate or Higher. The Unknown group includes mature age students who have studied very little mathematics many years ago and international students with very strong mathematics backgrounds, among others. While we have no information about the mathematics backgrounds of students in this group, we expect the group to be far from homogeneous.

The final mark for each subject is given as a percentage. The grades awarded are based on the final mark: F (fail, 0-49), CF (compulsory fail, 50 or more but failed because a threshold requirement was not met), P (pass, 50-64), C (credit, 65-74), D (distinction, 75-85), H (high distinction, 85-100).

Student identifiers were used to link MESH workshop data with university data, enabling us to determine the proportion of students who took advantage of support workshops, their final result for the subject and how this compared to all students enrolled in the subject and in the discipline that was the focus of their study (via the degree in which they were enrolled).

Each student had a mark and grade recorded for each subject. Only students who had completed all mandatory components of the subject were included in the data set (students who dropped out officially or unofficially were removed, this was 15% of all students). Students were counted once for each supported subject they completed from 2016 to 2018. The totals of the number of students in the supported subjects is given in Table 1.

Table 1: The number of students in supported subjects by year and workshop attendance (a student is counted as attending if they attended at least one workshop).

	Year			
	2016	2017	2018	Total
Number of students in subjects	4125	3751	4405	12281
Number of students attending workshops	1169	928	1071	3168

Data analysis

The data analysis and diagrams were completed using the statistical software package *R* (R Core Team, 2018).

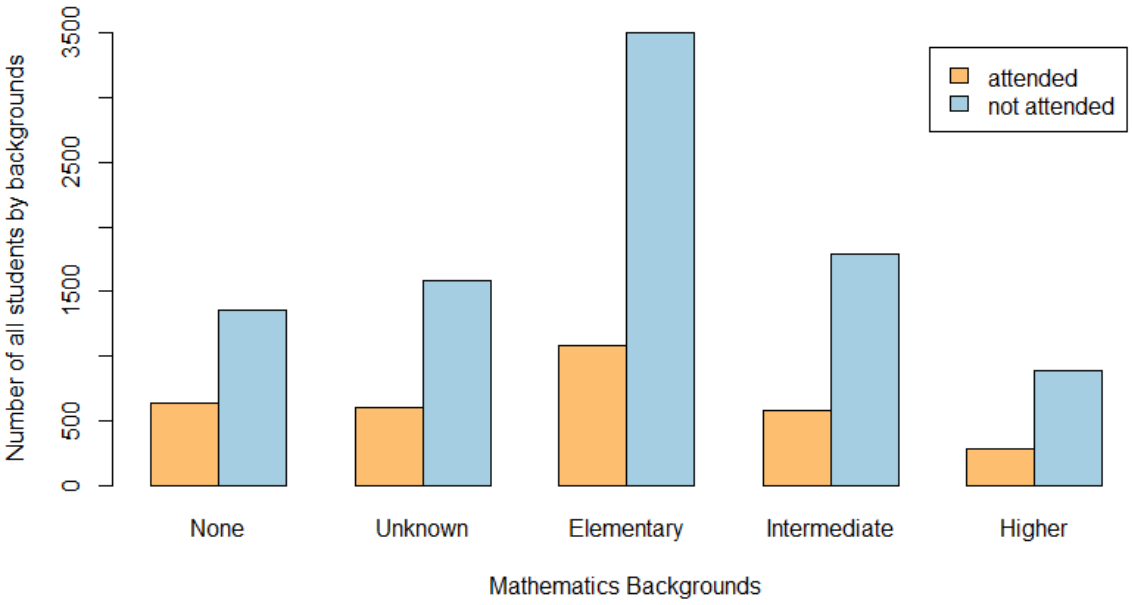

Figure 1: The numbers of students by background and whether or not they attended any MESH workshops. It is evident that workshop use was low for each background group and that the greatest use by proportion was by the None group.

The data set contains 12281 records, including 3168 records for students who attended at least one workshop for a subject. From this data it was found that 16% of the students did not complete any mathematics in their final two years at secondary school (the None group), 37% of the students studied elementary mathematics (the Elementary group), 19% studied intermediate mathematics (the Intermediate group), 10% studied higher mathematics (the Higher group) and in 18% of cases background was unknown (the Unknown group). Of the records indicating attendance at one or more workshop, 20% were in the None group, 34% in Elementary, 18% in Intermediate, 9% in Higher and 19% in the Unknown group. Figure 1 shows the mathematics backgrounds of students by attendance at workshops.

Students at WSU who are studying mathematics and statistics subjects are from various disciplines such as Science (including the health sciences), Engineering, Computing and IT, and Business and Commerce. Figure 2 shows the proportions of all students who attended at least one workshop (orange) and no workshops (blue) for these disciplines.

Students' secondary school mathematics backgrounds have been shown to have a significant effect on final marks in first-year tertiary mathematics subjects (Rylands & Coady, 2009), so students' gains from the workshops may be different for different mathematics backgrounds.

ANOVA was used to explore possible interactions between the variables of interest (Final Mark, Workshop Attendance, Mathematics Background and Discipline). Post hoc analyses were then done on variables for which significant ANOVA results were found.

To explore the effects and interactions of workshops and students' backgrounds on students' marks we used a multiple linear regression model with interactions. The explanatory variables used are mathematics background and workshop attendance. Mathematics background is a categorical variable with categories None, Unknown, Elementary, Intermediate and Higher. As MESH does not run the same number of workshops for all subjects, the proportion of workshops attended was used for each student. Hence Workshop Attendance is a numerical variable which takes values from zero to one. The resulting model estimates students' marks by workshop attendance for different levels of mathematics backgrounds. Discipline was then included in the model as a main effect to evaluate the difference between marks for each discipline.

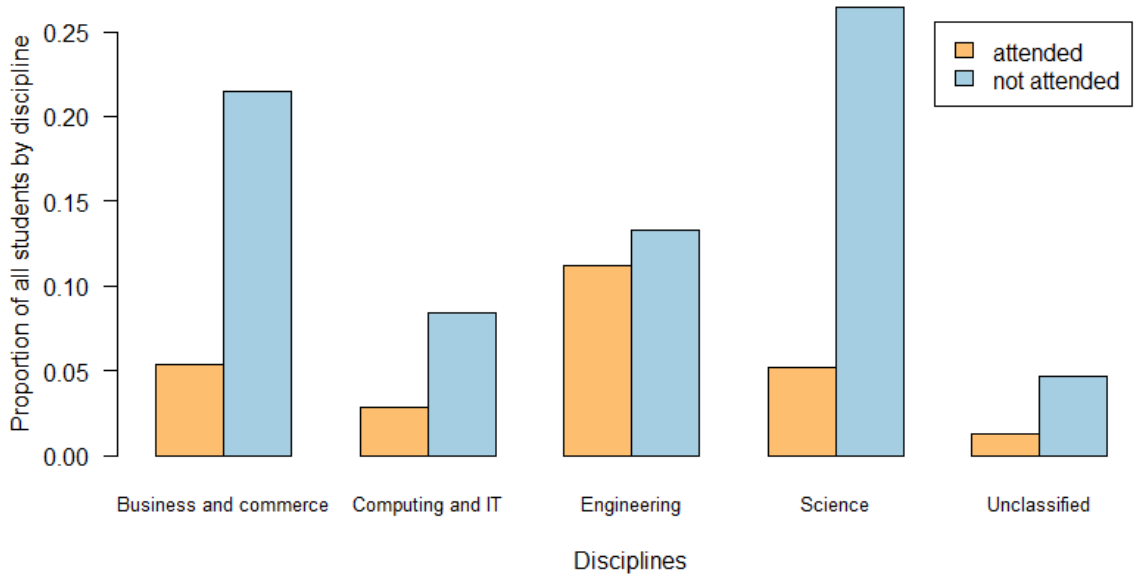

Figure 2: The proportions of students by discipline and whether they attended any MESH workshops or not. The proportionally high use by engineering students and the low use by science students can be clearly seen.

RESULTS

Table 2 shows the group comparisons of students who attended at least one workshop and those who attended none. Students were then grouped by final grades and mathematics backgrounds. Pearson's Chi-squared test showed a significant relationship between students' grades and workshop attendance ($\chi^2 = 82.43, p < 0.001$). A two-way ANOVA showed that the workshop attendance is associated with significantly different final marks for the subject studied ($p < 0.001$); mathematics background is associated with significantly different marks ($p < 0.001$); and the interaction between workshop attendance and mathematics backgrounds is significant ($p < 0.001$), which indicates that the relationship between workshop attendance and marks depends on the mathematics backgrounds.

Post hoc analyses with Tukey Honestly Significant Difference (HSD) of pairwise comparisons were performed on significant ANOVA results to identify which groups performed better. Students who attended workshops performed significantly better than students who attended no workshops ($95\%\ CI\ (0.91, 2.39), p < 0.001$). For the students who did not attend any workshops, the Higher group performed significantly better than the Intermediate group, and the Intermediate group performed significantly better than the Elementary, None and Unknown groups ($p < 0.001$). No significant differences were found between the Elementary, None and Unknown groups. For students who attended workshops, the Higher group performed significantly better than all other groups and the Elementary group performed significantly worse than all other groups ($p < 0.001$). No significant differences were found between the Intermediate, None and Unknown groups. Results of pairwise comparisons, including interactions, showed that None group students who attended at least one workshop performed significantly better than Unknown and Elementary group students who attended none.

For students who attended at least one workshop, there is a significant difference between the mean proportion of workshop attendance at each mathematics background level based on an ANOVA ($F = 6.48, p < 0.001$). A follow-up analysis was done by multiple pairwise comparisons with Tukey HSD corrections. This analysis showed that the mean proportion of workshop attendance for the None group is significantly greater than the mean proportion of for the Elementary, Intermediate and Higher groups (all $p < 0.01$).

Table 2: Mean marks, mean marks for background groups and percentages of grades obtained by workshop attendance (at least one workshop vs no workshops).

Characteristics	Students who attended no workshops (n=9113)	Students who attended workshops (n=3168)	Significance
Marks (μ) for None, Unknown, Elementary, Intermediate, Higher	55.6, 57.5, 56.6, 61.1, 65.7	60.4, 61.3, 56.4, 61.3, 68.2	***[a]
Grades (%) F, CF, P, C, D, H	27, 3, 31, 18, 14, 7	26, 1, 30, 18, 13, 12	***[b]

[a] Two-way ANOVA
[b] χ^2 test for independence
Significance codes: *** <0.001; ** 0.001–0.01; * 0.01–0.05

A two-way ANOVA was applied to marks (as a response variable) taking disciplines and mathematics backgrounds as factors. The results reveal that the mean marks for each

discipline are significantly different ($p < 0.001$); the mean marks for each mathematics background group are significantly different ($p < 0.001$) and that the interaction between discipline and mathematics background is significant ($p < 0.001$), meaning that the relationship between discipline and marks depends on the mathematics background. A post hoc analysis was not done as this was not the focus of the research.

The differences found by the two-way ANOVA comparing backgrounds and workshop attendance were further explored using multiple linear regression with interactions and including the proportions of workshops attended. The regression was used to give a model predicting the final mark, with two explanatory variables: mathematics backgrounds of students and the proportion of workshops attended. Results showed that not only do these two variables have a significant influence on the final mark, but also significant interactions exist between background and workshop attendance on marks. Significance levels of the main and interaction effects are given in Table 3.

Table 3: The coefficients in the multiple linear regression model for predicting final mark based on mathematics background and workshop attendance.

Explanatory Variables	Coefficients	Std. Error	Pr(>|t|)	Significance
(Intercept)	55.642	0.475	< 2e-16	***
BackgroundUnknown	1.732	0.649	0.008	**
BackgroundElementary	0.642	0.561	0.253	
BackgroundIntermediate	5.239	0.632	< 2e-16	***
BackgroundHigher	10.271	0.761	< 2e-16	***
Workshop.proportion	9.023	1.409	1.6e-10	***
BackgroundUnknown:Workshop.proportion	-0.597	2.051	0.771	
BackgroundElementary:Workshop.proportion	-6.478	1.818	3.7e-4	***
BackgroundIntermediate:Workshop.proportion	-7.135	2.075	5.9e-4	***
BackgroundHigher:Workshop.proportion	-5.399	2.653	0.042	*

Significance codes: *** <0.001; ** 0.001–0.01; * 0.01–0.05

From Table 3, the model obtained for the final mark *M* is

$$M = 55.6 + 1.7\mathrm{U} + 0.6\mathrm{E} + 5.2\mathrm{I} + 10.3\mathrm{H} + 9.0\mathrm{W} - 0.6\mathrm{UW} - 6.5\mathrm{EW} - 7.1\mathrm{IW} - 5.4\mathrm{HW} \qquad (1)$$

where W is the proportion of the workshops attended and the variables recording the mathematics background are U for Unknown, E for Elementary, I for Intermediate and H for Higher. Note that the background variables take the values zero and one only. Figure 3 shows the plot of the multiple linear regression model given in Equation (1).

Results suggest that there is a significant interaction effect based on mathematics backgrounds, that is, the effect of the workshop attendance is different for each background group.

Figure 3 shows that the None group benefit the most from workshops. There were no significant differences between student progress for those in the Unknown and None groups. The Higher group shows slightly better progress with the workshop support compared to the Elementary and Intermediate groups. Without workshop support, the Elementary and None groups do as well as each other, but interestingly the None group performs better than the Elementary group when they use workshop support. Not surprisingly, the Higher group

performed better than the other groups regardless of workshop support use. Overall, students' marks improved with workshop attendance at much the same rate for the Intermediate and Elementary groups.

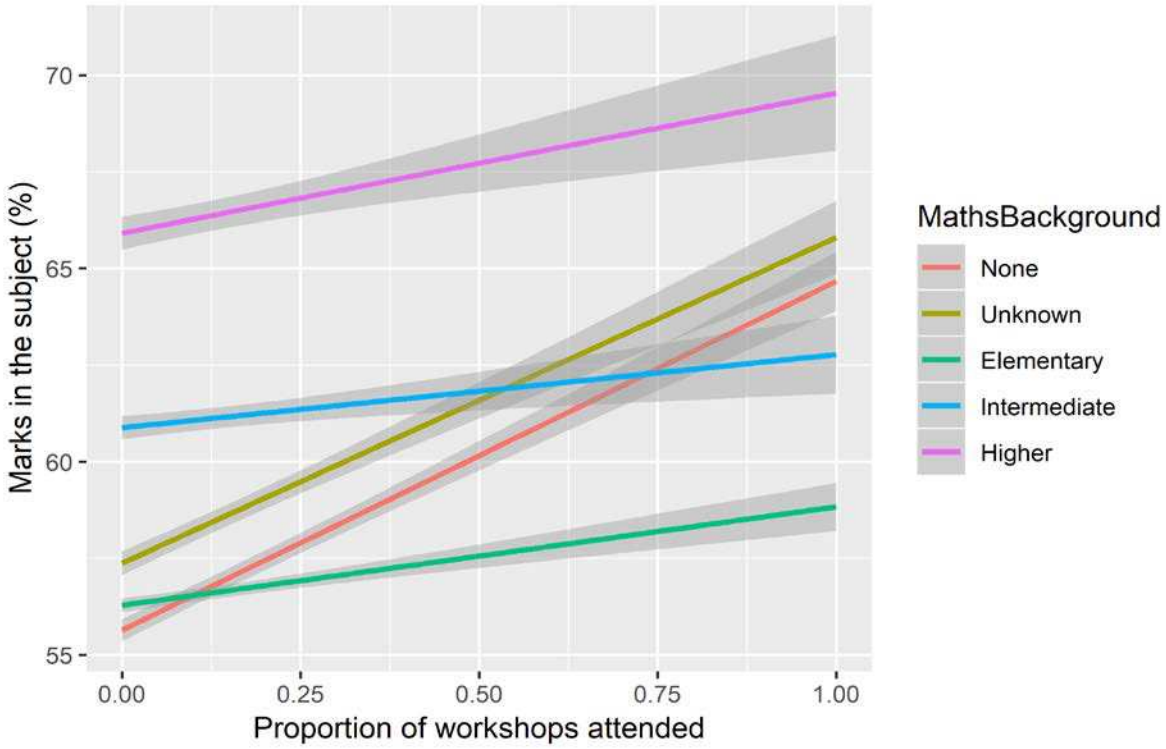

Figure 3: The multiple linear regression model (Equation (1)) showing effects on final marks based on workshop attendance by background. Confidence intervals (95%) for each regression line are shown. The greater improvement by in marks by attendance for the None and Unknown groups can be seen.

Multiple linear regression was used to give a model predicting the final mark, this time including variables for the discipline categories.

From Table 4, the model obtained for the final mark *M* is

$$M = 43.4 + 5.7\mathrm{C} + 16\mathrm{B} + 12.6\mathrm{X} + 14.1\mathrm{S} + 4.8\mathrm{U} + 0.5\mathrm{E} + 8.2\mathrm{I} + 13.8\mathrm{H} + 13.8\mathrm{W} - 1.4\mathrm{UW} - 4\mathrm{EW} - 5.3\mathrm{IW} - 4\mathrm{HW} \quad (2)$$

with discipline variables C for Computing and IT, B for Business and commerce, X for Unclassified and S for Science. The base discipline category is Engineering and the base background category is None. As for the background variables, the discipline variables take the values zero and one only.

Results show that Computing and IT students achieve higher marks than Engineering students, the Unclassified group achieve higher marks than Computing and IT students, Science students achieve higher marks than the Unclassified group, and Business and Commerce students perform better than all other disciplines. Note that students in different disciplines usually take different mathematics and statistics subjects. This model in Equation (2) gives no information on the rate of increase by workshop attendance for particular disciplines, neither does it give variation by background for particular disciplines.

Table 4: The coefficients in the multiple linear regression model for predicting final mark based on mathematics background, workshop attendance and discipline.

Explanatory Variables	Coef.	Std. Error	Pr(>|t|)	Significance
(Intercept)	43.370	0.562	< 2e-16	***
DisciplineComputing and IT	5.677	0.568	< 2e-16	***
DisciplineBusiness and Commerce	16.006	0.459	< 2e-16	***
DisciplineUnclassified	12.584	0.720	< 2e-16	***
DisciplineScience	14.111	0.438	< 2e-16	***
BackgroundUnknown	4.790	0.617	8.9e-15	***
BackgroundElementary	0.479	0.529	0.365	
BackgroundIntermediate	8.226	0.602	< 2e-16	***
BackgroundHigher	13.797	0.726	< 2e-16	***
Workshop.proportion	13.764	1.335	< 2e-16	***
BackgroundUnknown:Workshop.proportion	-1.385	1.935	0.474	
BackgroundElementary:Workshop.proportion	-4.044	1.716	0.018	*
BackgroundIntermediate:Workshop.proportion	-5.325	1.958	0.007	**
BackgroundHigher:Workshop.proportion	-3.970	2.503	0.113	

Significance codes: *** <0.001; ** 0.001–0.01; * 0.01–0.05

DISCUSSION AND CONCLUSION

Overall, the results suggested by the data analysis are positive; increasing workshop attendance, regardless of mathematics background, is associated with statistically significant increased final marks.

The large proportion of students in the None or Elementary groups (see Figure 1) is concerning, as it means that the majority of students studying mathematics and statistics at first-year level are poorly prepared for that aspect of their studies. The fact that many students are poorly prepared could put a downward pressure on academic standards, which could be exacerbated by rewards for universities to increase retention rates. This possibility has been acknowledged in an Australian Government Productivity Commission report (2019; p.18), along with the positive effects of support for increasing retention.

Proportionally, the group that made the most use of support workshops was the None group. This differs to the findings of Pell and Croft (2008) who reported that support was used more by better students.

The first model (Equation (1), Table 3 and Figure 3) suggests that the average mark across all mathematics and statistics subjects is over 50 without workshops. The intercept term, representing the average mark for the None category without workshops, is 55.6. This is only slightly lower (and not significantly different from) the Elementary category whose average mark without workshops is 56.2 (55.6 + 0.6). As expected, students in the Higher group perform significantly better than all others with an average mark without workshops of 65.9 (55.6 + 10.3). Workshop attendance is associated with increased marks for all levels of background, however, it is surprising is that the None group's improvement is much greater than any of the others with an overall increase of 9.0 marks for students who attended all workshops. By comparison, the Elementary group show an improvement of only 2.5 (9.0 –

6.5) marks for the same attendance level. The variations in marks for each background level is clearly illustrated by the different slopes of the lines in Figure 3.

That the None and Unknown groups improved more when attending workshops than the three groups who studied mathematics in their final year of secondary school was unexpected. It is concerning that the mean mark for the None group was greater than that for the Elementary group after only minimal workshop attendance and greater than the Intermediate group once workshop attendance surpassed 75%. The model suggests that it may be better for a student to take no senior mathematics and attend workshops at university than to study the subject at an elementary level. Whilst we have no explanation for this effect, we are led to wonder if there is something about the way that mathematics is taught in high school which leads to this discrepancy as the gains made by all students who studied at high school are similar.

We expect the None group to need the most academic support, so it is heartening that this group made the most use of workshop support and that the gains were higher than for other school background groups. However, it is disappointing to see that the Elementary group made the least gain for increasing workshop attendance. Further research to explain the differences between these two groups would be very valuable.

The second model, which includes effects of disciplines on final marks, is summarised in Equation (2) and Table 4. This model suggests that students from the None and Elementary groups who don't engage with workshops can expect to fail mathematics and statistics subjects in Engineering, the base discipline for the model, with average marks of 43.4 and 43.9 respectively. Computing and IT students with those backgrounds are also below 50 with average marks of 49.1 and 49.7 respectively. For the None group this amounts to 13.8 marks for those attending all workshops which means that these students will have a passing mark, on average, in all disciplines. This is also true for the Elementary group, however the improvement is not as great as the None group with an average improvement of only 9.8 (13.8 – 4.0) marks. We have not investigated the interactions between Discipline and the other variables so it is possible that different disciplines have different student background profiles. Students from different disciplines study different subjects at WSU, reflecting different discipline requirements; this is likely to be largely responsible for the differences in the discipline coefficients.

Of concern is the low proportion of students who attended workshops, regardless of background. Workshops were well advertised to students with all teaching staff emailing students and placing announcements on the university's learning management system. Mac an Bhaird et al. (2013), who also report low use of mathematics support services, found that the major reason is that students did not feel that they needed help, and that they would have sought help if they thought they needed it. Anecdotal evidence from MESH staff suggests that many students who have failed a subject and are repeating it still feel that they don't require help in the subject. An investigation of why students do not attend support workshops is a topic for further research.

The discrepancy in attendance between disciplines is interesting. Two of the mathematics subjects run for engineering students have two workshops scheduled in lecture time. This could partly account for the higher attendance for engineering. It is surprising to see that the proportion of business and commerce students who attended at least one workshop is higher than that for science. The majority of students studying science degrees at WSU are majoring in the life sciences; the mathematics and statistics requirements are a basic non-calculus mathematics subject or a first-year statistics subject. The lack of emphasis on mathematics and statistics and the low level of the minimum requirements in science might lead students

to feel that they don't need help in that area. Further study is needed to understand these differences.

In conclusion, this study agrees with other research in that mathematics and statistics support is associated with improved student results, however these improvements are not evenly distributed across all student backgrounds. It appears from this study that despite significant differences in results between mathematical backgrounds and discipline of study, it is worthwhile to provide mathematics and statistics workshops. Further study with a much more detailed model including interactions for all variables is needed to fully understand the effects of workshop attendance on different disciplines with different mathematics backgrounds. More data might be needed for such a study.

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MAKING THE GRADE: DO MATHEMATICS MARKS MATTER TO WOMEN IN STEM?

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KEYWORDS: gender gap, mathematics, STEM, retention, perceived academic performance

ABSTRACT

The disparity between female and male participation in fields that are underpinned by mathematics, for example, science, engineering and technology, is well known and has been researched for decades. Although significant attention has been focused on redressing this imbalance and programs have been purposefully designed to encourage and support girls in Science, Technology, Engineering and Mathematics (STEM), the gender gap persists at all levels of education and in the workplace.

We discuss preliminary results of a study conducted at a high-ranking Australian university. Participants of the study were undergraduate students pursuing a major that required tertiary mathematics. The aim of the study was to determine if students' perceptions of their mathematics results impacted on their decision to continue with their current major.

While students' satisfaction with grades was linked to their commitment to their current major for both male and female students, differences between genders were found with female students significantly more sensitive to their perception of performance in the critical first-semester of university than their male peers, impacting negatively on their intention to persist with their major.

INTRODUCTION

Female underrepresentation in mathematics has been in the spotlight nationally and internationally for decades. This abiding phenomenon is a complex, multi-faceted problem that many argue starts in primary school, with the so-called "leaky pipeline" describing the loss of female students from mathematics through all levels of education. As a consequence, the number of women continuing to careers in Science, Technology, Engineering, and Mathematics (STEM) fields is approximately one third the number of men (OECD, 2016).

Data from the last 40 years suggests that whilst the gender gap in completed STEM-related bachelor degrees closed during the 1990's, female representation in STEM at both the bachelor and PhD level has started to decline again (Miller & Wai, 2015). Women are significantly more likely to switch out of an engineering degree, for example, than men (Dickson, 2010), and one and a half times less likely than men to continue taking calculus-based mathematics subjects (Ellis, Fosdick, & Rasmussen, 2016), exacerbating the problem of female underrepresentation at levels beyond graduation, such as postdoctoral positions or in STEM professions (Hill, Corbett, & St Rose, 2010).

According to van den Hurk, Meelissen, & van Langen (2018), historically, the underrepresentation of women in higher education STEM programmes has been explained by arguing that girls did not perform as well as boys in school and as a consequence were not as academically prepared for the technical aspects of STEM degrees as boys were, but that this explanation has changed in recent years. Academically, female students are no longer

lagging behind male students in secondary school mathematics and science performance (Hanna, 2003; OECD, 2015; Thomson, Wernert, Underwood, & Nicholas, 2009) and the difference in STEM related ability is not large enough to explain why female students leave the field (Blickenstaff, 2005; Ceci & Williams, 2010; Eddy & Brownell, 2016; van den Hurk et al., 2018; Wang, Eccles, & Kenny, 2013). In fact, studies have found little difference between male and female performance at the undergraduate level in STEM disciplines (Eddy & Brownell, 2016), with some even finding that female students are entering engineering degrees better prepared than male students (Li, Swaminathan, & Tang, 2009; Watt, Eccles, & Durik, 2006).

One common perception is that women are leaving STEM degrees *because* of mathematics (Li et al., 2009; Steenkamp, Nel, & Carroll, 2017). Introductory calculus-based mathematics subjects have been shown to be associated with retention rates (Hutcheson, Pampaka, & Williams, 2011; Wake, 2011) and with students' decisions to leave STEM degrees, for both men and women (Chen & Soldner, 2013; Ellis et al., 2016). This has led to the perception that undergraduate mathematics subjects "weed out" low-performing students.

In this study we make no assumptions about students' actual performance in mathematics as demonstrated by their grades or the influence this may have on students' subsequent subject selection. Instead, we investigate the relationship between students' grade satisfaction and their commitment to persist with their major and whether this relationship is more salient for female or male students.

GRADE SATISFACTION AND RETENTION

Female students studying mathematics are, in general, less confident in their mathematics abilities than male students (Fennema & Sherman, 1978; Good, Rattan, & Dweck, 2012) and generally enter their degrees with lower self-competence beliefs than male students (Jagacinski, 2013). This is true even when female students' academic performance is on par with male students' performance (Frenzel & Goetz, 2007). Furthermore, this difference in perceived self-competence can actually widen as students progress through their STEM degrees (Dalgety & Coll, 2006; Hartman & Hartman, 2009).

This phenomenon seems to be related to STEM subjects in particular. Jagacinski (2013) compared first-year engineering students' perceptions of their competence with those of first-year psychology students. Female engineering students were found to report lower competence perceptions than male engineering students and both these groups reported lower competence perceptions than both female and male students in psychology. No significant differences were found between students' actual grades and Jagacinski argues that perception in one's abilities in STEM related subjects is more influential for female than male students, irrespective of actual performance.

Low competence beliefs have been correlated with reduced retention rates in STEM related majors, especially for female students (Burtner, 2005; Eddy & Brownell, 2016; Frenzel & Goetz, 2007; Good et al., 2012; Hall et al., 2015; Hartman & Hartman, 2009). Students with lower competence beliefs might focus their academic attention on avoiding low performance, for example, by withdrawing from the degree. In order to explain why low self-competence beliefs might affect female students more than male students, Nelson et al. (2013), using a lens of fear-of-failure, found that female engineering students are more likely than males to think that other students are aware of their own personal failures. As a result they are more likely than male students to experience feelings of distress when they feel others are aware of their failure. These concerns may then be reinforced when female students receive grades that they perceive to be poor. Dissatisfaction with grades can then cause feelings of

discouragement, which can reduce one's beliefs in their own ability to succeed (Hall et al., 2015; Marra, Rodgers, Shen, & Bogue, 2012; Nelson et al., 2013; Seymour & Hewitt, 1997). This is not to say that such students achieved low results, only that they *perceived* their results to be low. Poorer than expected results may subsequently discourage a student from continuing with their major (Hall et al., 2015; Nelson et al., 2013; Zarb et al., 2018). This relationship forms the focus of our study where we investigate students' *perceived* grade satisfaction, rather than their *actual* grades, and its influence on retention in their major.

METHOD

The purpose of this study was to explore the impact that students' perceptions of their university mathematics results had on their commitment to continue with their STEM major or degree and whether differences exist between genders.

We used a pre- and post-survey design. The results reported here form part of a larger study and we report only on survey questions relevant to grade satisfaction and retention.

Setting

The university in which this study took place is an Australian Group of Eight university that attracts high-achieving students. The Bachelor of Science at this university is comprised of a diverse array of discipline areas and is the primary pathway to engineering and health science majors. As a consequence, students can easily change between majors that demand high levels of quantitative competence and others that include very little additional mathematics study, without changing degrees.

Participants

Participants in the study were undergraduate students studying selected mathematics subjects. These included a range of subjects from first-year to third-year level subjects with varying enrolment sizes. Students in the subjects were studying in a range of undergraduate majors including commerce, engineering, science and medical pathways as well as mathematics and statistics.

Table 1: Number of participants by gender and year-level for Survey One and Survey Two

	Number of Participants		
Survey Number & Year-Level	Female	Male	Total
Survey One			
1st Year	46	61	107
2nd Year	22	28	50
3rd Year or above	12	30	42
Survey Two			
1st Year	46	65	111
2nd Year	36	63	99
3rd Year or above	11	26	37
Total Participants	173	263	436

Two surveys were administered, one in the middle of semester and the other after students had received their end of semester results. The first survey attracted 199 student-responses (80 female; 119 male) with 247 students answering the second survey (93 female; 154 male). Surveys were emailed directly to students who could choose whether to respond or not. Students were encouraged to respond to both surveys so that comparisons between students'

attitudes over time could be made, but only 47 students (12 female; 35 male) completed both surveys. A summary of participants by year-level is shown in Table 1.

Note that *year-level* refers to the number of years the student has been enrolled at university, not the subject level. For example, a student in their second year of study could be studying a first-year mathematics subject. This student would be grouped as a second-year student. Because the number of students in their third year of study or above was low, these students were grouped together as third-year students. We have not included an analysis of the 47 students who completed both surveys due to the small sample size and overrepresentation of male students in this group.

Design

Students completed two questionnaires both containing the same questions. The first (Survey One) was administered mid-semester, well before the examination period. We chose the middle rather than beginning of semester to allow first-year students time to experience university life (Tinto, 2017). The second (Survey Two) was administered after students had received their final examination results. The results from the surveys were then compared to see if receiving examination results had any influence on students' level of commitment to their major.

Measures

Demographic. Demographic questions included gender and year-level.

Grade satisfaction in mathematics. We assessed grade satisfaction with the following question: "How happy were you with the results you obtained for your mathematics subject?" Students could respond with a slider that ranged between values from 0 (I was not happy with my results) to 100 (I was happy with my results). For Survey One, students were asked to consider results from the last university mathematics subject they studied. Students studying their first mathematics subject at university were asked to consider their secondary school (high school) results[2], since they had not yet received any university results. For Survey Two, all students were asked to consider their results from the subject they had just finished.

Student retention. We measured students' intent to continue with their major with the following: "Taking into account any other pathways you might be interested in, how committed do you feel to your current pathway?" Students could respond using a slider ranging from 0 (not committed) to 100 (very committed).

RESULTS

First, we report on how students perceived their mathematics results. Next, we explore whether a relationship exists between students' perception of their results and their intent to continue with their current STEM major.

Student perception of mathematics results

Gender: Students were grouped by gender and whether they had completed Survey One or Survey Two. ANOVA revealed significant differences between students' responses to their perception of their mathematics results $(F(3, 451) = 3.32, p = .020)$. Post hoc analysis showed female students in Survey One $(M = 65.46, SD = 29.06)$ were significantly happier with their results than male students in Survey One $(M = 58.41, SD = 18.48)$ and significantly happier with their results than both female students in Survey Two $(M = 52.37, SD = 31.32)$

[2] All students had studied a calculus-based mathematics subject in their final year of secondary school.

and male students in Survey Two $(M = 54.37, SD = 31.32)$. Hence, before receiving end of semester results, female students had the greatest grade satisfaction.

Gender and year-level: Students were then grouped by gender, year-level, and survey. An ANOVA was conducted separately for female and male students. Significant differences were found for female students $(F(5, 167) = 3.25, p = .008)$. Post hoc analysis showed significant differences between first-year female students in Survey One $(M = 71.52, SD = 23.65)$ and first-year female students in Survey Two $(M = 48.57, SD = 30.36)$, where students in Survey One had greater grade satisfaction than students responding to Survey Two. No statistically significant differences were found between any other groups. This suggests that female students are leaving secondary school and entering university with a positive outlook on their results which then drops once they receive their first set of university results.

A similar analysis was conducted for male students which again revealed significant differences between groups $(F(5{,}267) = 2.99, p = .012)$. Post hoc analysis showed that, as with female students, first-year male students in Survey One reported being the happiest with their mathematics results $(M = 66.26, SD = 23.19)$, which was significantly higher than second-year male students in both Survey One $(M = 45.93, SD = 35.93)$ and Survey Two $(M = 49.44, SD = 32.50)$. Unlike female students, the difference between first-year male students in Survey One and Two was not significant.

Figure 1 summarises students' mathematics grade satisfaction. Both genders reported entering university happy with their results. This falls when students receive their first set of university results, which was significant for female but not male students. For male students, grade satisfaction continues to fall into second-year which is a significant decline from when they entered and, in fact, female students remained happier with their mathematics results than male students.

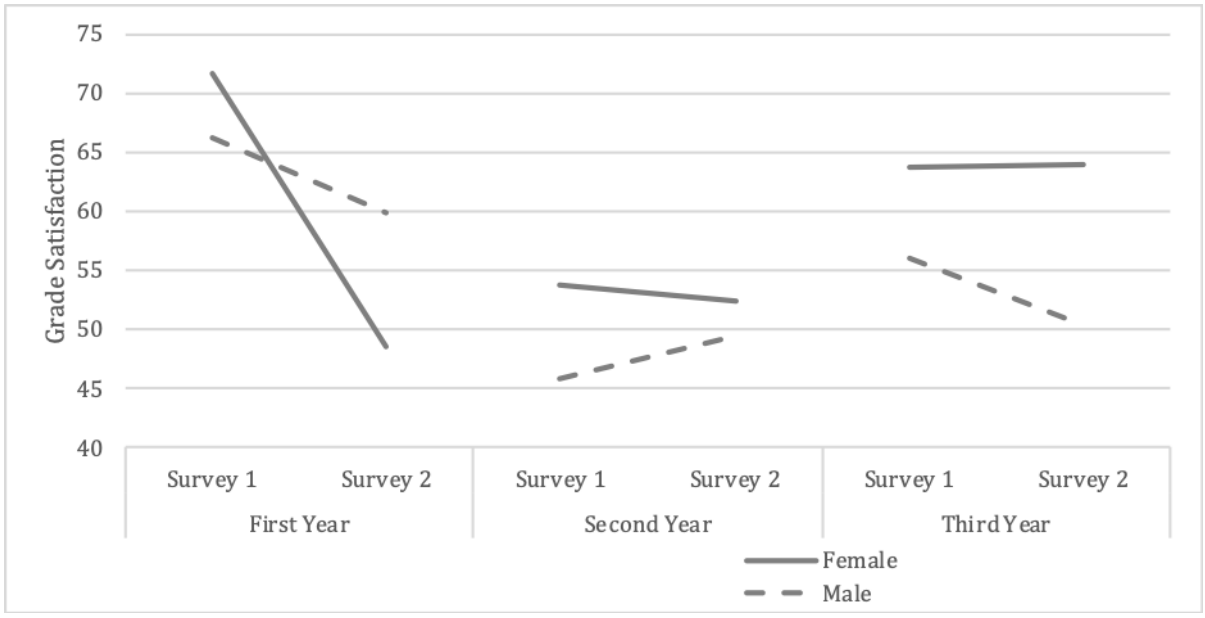

Figure 1: Mathematics grade satisfaction grouped by gender where 0 represents not happy with results and 100 represents very happy with results

Influence on commitment

A general linear regression was used to explore the relationship between how committed students feel to their current degree and grade satisfaction, while considering gender, year-

level and survey. First, we simultaneously tested for the effects of all variables as well as the interactions between them. Three two-way interaction effects were found to exist. Which survey students completed was found to interact with gender and with year-level and also with students' happiness with results. An interaction between gender and grade satisfaction did not exist. No three-way interactions were found. The model was run again, removing unnecessary interaction effects.

The main effect of students' year-level was that the longer students had been at university, the greater their level of commitment, which was significant $(F(2,446) = 10.866, p < .001, \eta^2 = .047)$. The main effect of which survey students completed was also significant, with students completing Survey One feeling more committed than those completing Survey Two $(F(1,446) = 14.410, p < .001, \eta^2 = .032)$. Grade satisfaction also had a significant main effect, where students that were happier with their results felt more committed to their major $(F(1,446) = 14.967, p < .001, \eta^2 = .033)$. The main effect of gender was not found to influence commitment to a student's current major $(F(1,446) = 1.856, p > .05, \eta^2 = .004)$. However, the interaction effect between gender and survey was significant $(F(1,446) = 5.453, p = .020, \eta^2 = .012)$. A fall in how committed students feel to their major occurs from Survey One to Survey Two for both genders, but the decline is more pronounced for female than male students (See Figure 2). The interaction effect between students' year-level and survey was also significant $(F(2,446) = 7.831, p < .001, \eta^2 = .035)$. First-year students, whether completing Survey One or Survey Two, were equally committed to their major. However, second- and third-year students completing Survey One were more committed to their major than those completing Survey Two, and this gap widened the longer students had been at university. Lastly, the two-way interaction between which survey students completed and how happy they were with their results was no longer significant $(F(1,446) = 3.642, p > .05, \eta^2 = .008)$.

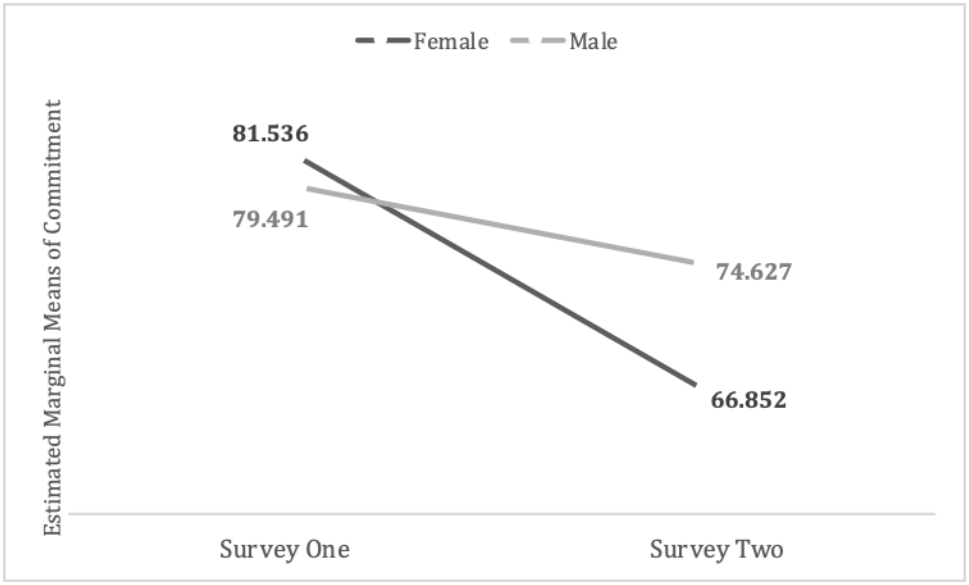

Figure 2: Interaction between gender and survey with respect to students' intent to continue studying their current major

DISCUSSION

The results of this study suggest that the way students feel about their university results in mathematics does influence their decision to continue studying their STEM major where

students who reported wanting to continue with their current major were more likely to report higher grade satisfaction.

Results suggest that students are entering university happy with their school mathematics results but then feel dissatisfied with their university results. Notably, female students were found to enter university with the greatest grade satisfaction but experienced a significant drop once they received their first-semester university marks. In contrast, male students experienced a similar, but not as pronounced, decline in grade satisfaction for first-year which continued to decline into second-year while female students remained happier with their mathematics results than male students in second and third-year.

One interpretation for the decline in grade satisfaction during first-year university is a perceived gap between school mathematics and university mathematics (Brandell, Hemmi, & Thunberg, 2008; Luk, 2005). For example, Solomon and Croft (2016) interviewed undergraduate students studying mathematics. Students recognised a difference between school and university mathematics, wanted to learn and understand their work, but felt unsupported in doing so. As a result, their confidence in their mathematics abilities dropped. Our results could suggest that the move from school to university mathematics resulted in a loss in self-confidence which was reflected in a decline in grade satisfaction. While grade satisfaction did not recover for male students, it did for female students. One interpretation is that female students who experienced poor grade satisfaction during first-year did in fact choose to opt out of mathematics and change their major leaving behind a greater proportion of second and third-year female students who had higher grade satisfaction. If this is the case, then first-year mathematics subjects might indeed act as a "weeding out" subject that has a greater influence on female students than male students. It may therefore be necessary to address students' expectations as they transition from school into university as to what mathematics is at university and what an acceptable result looks like.

Despite both female and male students in this study reporting that they were less satisfied with their university results than they were at school, the relationship between grade satisfaction and retention was stronger for female than male students. Our results support findings from the literature that suggest lower than expected results can discourage students from continuing with their major (Hall et al., 2015; Zarb et al., 2018).

We did not find evidence that male students enter university with greater grade satisfaction than female students, nor did we find evidence demonstrating that this gap widens as students continue through university (See Dalgety & Coll, 2006; Hartman & Hartman, 2009; Jagacinski, 2013). Instead we found that female students in this study consistently reported higher grade satisfaction than male students, except for first-year female students receiving their first set of university results. This inconsistency with the wider literature may be due to the way in which we operationalised perceived academic achievement. We only asked students about grade satisfaction rather than including a wider range of prompts typically used to address students' confidence in their ability to master content (e.g., prompts from the Motivated Strategies for Learning Questionnaire; Pintrich, Smith, Garcia, & McKeachie, 1991). If the way students feel about their results does not relate to perceived academic achievement, then this could explain why our results do not align with wider literature. Additional studies would be needed to explore the relationship between grade satisfaction and perceived academic ability.

RECOMMENDATIONS

First-year female students' response to their mathematics results is concerning. Female students' confidence as they enter university appears fragile. First-year educators should not

assume that students enter university with strong beliefs in their academic ability. This needs to be built up and reinforced as students transition to an environment that requires students to be more independent and resilient. Addressing women's low grade satisfaction at transition may be crucial for later retention.

LIMITATIONS

We had planned to have students complete both Survey One and Survey Two to assess changes in individual student' attitudes. Unfortunately, only a small number of students completed both surveys, limiting the number of comparisons we could make. Hence, we treated responses from each separate survey as individual groups of students. While we were unable to have the majority of students answer both surveys, our study does have the advantage of collecting students' perceptions across two points in time which gives a better measure of the impact an institution may have than measuring at one point in time only (Eddy & Brownell, 2016).

There are also issues involved in relying on self-reported data only. Students were invited via email to take part in the two surveys, and therefore self-selected to take part. This runs the risk of overrepresenting students who are either highly interested or dissatisfied with their mathematics subjects.

Our findings support the argument that female students' mathematics results are influential in their decision to continue with their major (Li et al., 2009; Steenkamp et al., 2017). However, we did not ask how students felt about their overall degree results to ascertain whether they were happy or unhappy with other university results alone or in comparison to mathematics results. Future research might include how students feel about both their mathematics results and their overall course results to assess the impact mathematics specifically has on students' choice to persist with STEM pathways. Future research might also consider whether similar findings exist in other discipline areas or whether they are confined to STEM disciplines only.

CONCLUDING REMARKS

In summary, a relationship was found to exist between students' satisfaction with their results and how likely they were to continue with their STEM major. Students with lower grade satisfaction were more likely to report wanting to leave their current STEM major. This relationship was more salient for female than male students. Results also suggested that students transitioning from school to university experience a decline in grade satisfaction which was again, more prominent for female students. Consequently, supporting transitioning students' academic self-efficacy, especially female students, by addressing students' expectations of mathematics at university could begin to address the leaks in the pipeline for female students in STEM.

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MAKING MATHEMATICS TEACHERS: THE BELIEFS ABOUT MATHEMATICS AND MATHEMATICS TEACHING AND LEARNING HELD BY MATHEMATICS TEACHER EDUCATORS AND PRE-SERVICE TEACHERS

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ABSTRACT

How pre-service teachers experience the mathematics they are taught at university potentially influences their beliefs about mathematics and hence also how they will teach mathematics when they graduate. This paper explores similarities and differences between the beliefs about mathematics and mathematics teaching and learning held by three groups: university-based mathematics teacher educators who teach either mathematics or mathematics pedagogy and secondary mathematics pre-service teachers. Eighty-two academics and twenty-five pre-service teachers were surveyed and the beliefs and differences between the groups were characterised using descriptive statistics and a χ^2 (chi square) test of homogeneity. Generally, the respondents had a Problem-solving view of mathematics and mathematics teaching, but there were some differences between the beliefs of the three groups. Where these differences occurred, pre-service teachers, more so than mathematics teacher educators, tended to lean towards Instrumentalist views and traditional teaching practices. These initial findings suggest a need for further exploration of the beliefs and teaching and learning practices experienced by pre-service teachers.

INTRODUCTION

In Australian Initial Teacher Education (ITE) programs, secondary mathematics pre-service teachers (PSTs) are taught by two categories of mathematics teacher educators (MTEs), those who teach mathematics content (MTEC) and those who teach mathematics pedagogy (MTEP). The research literature in mathematics teacher education is replete with accounts of mathematics pedagogy courses and their impact on pre-service teachers, but less attention has been given to the role of university mathematicians who teach content courses (Leikin, Zazkis, Meller, 2018). This research aims eventually to understand how pre-service teachers make sense of the possibly different perspectives on mathematics and its teaching and learning that might be communicated to them during their university studies by these two types of MTEs, and how they make choices about their own beliefs as beginning teachers. The study reported here contributes to this overall aim by addressing the following research questions:

(1) What are the beliefs about mathematics and mathematics teaching and learning espoused by MTEs who teach mathematics content courses, MTEs who teach mathematics pedagogy courses, and the pre-service teachers they teach?

(2) What similarities and differences can be observed between the beliefs of these three groups?

BELIEFS

Many researchers claim that teachers' classroom practices are strongly influenced by their conceptions or beliefs about mathematics and how it is best taught and learnt (McLeod, 1992; Mosvold & Fauskanger, 2014). Teachers' classroom practices, in turn, influence their students' beliefs about both mathematics and their ability to learn mathematics. For this reason, it is assumed that teachers' beliefs need to change before they can embrace new classroom practices. However, others argue that the connection between teacher beliefs and practices is more complex and contextually constrained; for example, Guskey (1986) suggests that teacher beliefs can change as a consequence of changes in their behaviours. Lerman (2001) maintains that beliefs espoused by teachers are related to the contexts in which they are elicited, and that specific situations are "*productive* of beliefs, practices, purposes, and goals, not reflective of them" (p. 44, emphasis added).
Philipp (2007) defined beliefs as:

> … psychologically held understandings, premises, or propositions about the world that are thought to be true. … Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions towards action (p. 259).

Here beliefs are considered as "anything that an individual regards as true." (Beswick, 2005, p. 39).

Ernest (1989) described three different conceptions of mathematics, which he labeled Instrumentalist, Platonist, and Problem-solving views. The Instrumentalist view considers mathematics as a collection of procedures, facts and skills. The teacher's role is as an instructor to assist students to master skills and procedures. The Platonist view considers mathematics as a structured, unchanging body of knowledge where the teacher as an explainer helps students develop conceptual understanding. The Problem-solving view considers mathematics to be created by human endeavour where the teacher is a facilitator of students' learning so that they become confident in problem posing and solving. Generally a person's beliefs do not align with only one of Ernest's views, but tend to be a mixture, and this is reflected in their teaching.

Although the school mathematics curriculum in Australia (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2018a) does not mandate any particular approach to teaching or set of epistemic or pedagogical beliefs, it does promote the notion of mathematical proficiency as intertwined strands of understanding, fluency, problem-solving and reasoning. These strands define the "mathematical actions" that can be used for learning mathematics. When solving problems, students are to "develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively" (Australian Curriculum, Assessment and Reporting Authority (ACARA), n.d.b). Hence it is important for PSTs to be exposed to a problem-solving view of learning mathematics as well as the other notions of proficiency articulated in the Australian Curriculum: Mathematics.

APPROACHES TO TEACHING AND LEARNING

It is often thought that in classrooms there are two broad types of teaching – traditional teaching methods, on the one hand, and those which support constructivist learning on the other hand. In a traditional learning environment the teacher is the authority in the classroom

who delivers the information to the students. This approach might be said to reflect the traditional lecture format of teaching mathematics commonly found in universities. Often the primary learning goal in traditional teaching is procedural fluency. Alternatively, constructivism is built on the tenets that knowledge is not passively received but needs to be constructed by learners through connecting and revising current knowledge. Students construct knowledge when they have opportunities to actively engage with tasks and discuss and justify their thinking. These discussions lead to learners revising and reorganising their knowledge systems.

Constructivist learning is based on the assumption that each learner "constructs" their own knowledge based on their personal experiences (Fosnot, 2013; Steffe & Gale, 1995). Constructivism is built on the four following tenets: (1) Knowledge is not passively received by the learner but actively created. (2) Learners generate new ideas by reflecting on what they are doing and integrating the new ideas into their knowledge structure. (3) Each learner has their own interpretation of the world based on their present and past experiences. (4) Learning is a social process where discussion allow learners to build "taken-as-shared understandings" (Clements & Battista, 1990; Cobb, Wood & Yackel, 1993; Fosnot, 2013).

A teacher's beliefs about mathematics and how it is best learnt, whether by individuals constructing their knowledge through engaging with activities and discussions or by memorising facts and procedures, are related to their approach to teaching mathematics. When school curriculum documents emphasise mathematical problem solving it is important to understand whether the beliefs of PSTs align with this view and also to probe the beliefs of the MTEs who teach them.

RESEARCH DESIGN AND DATA COLLECTION

The survey used in this study was developed by Beswick (2005) as part of her study of teachers' beliefs, in which factor analysis revealed two factors corresponding to Ernest's (1989) Problem-solving and Instrumentalist conceptions of mathematics and the related beliefs about constructivist and traditional learning and teaching methods respectively. The survey was administered online and had 26, five-point Likert scale items to elicit responses (strongly disagree, coded as 1, to strongly agree, coded as 5). A link to the survey with an invitation to participate was emailed to Australian mathematicians, statisticians, and mathematics educators involved in ITE programs via the Australian Mathematics Society, the Mathematics Education Research Group of Australasia, and the Heads of School/Faculty of Mathematics or Science and Education at all Australian universities with a request to forward to the relevant staff.

All items in the survey were completed by 82 (from 120) respondents who represented 35 different Australian universities and five international universities. Thirty-three respondents (40%) were female and forty-nine (60%) were male, with a median age of 46 years. Sixty respondents (73%) taught mathematics or statistics content courses, and 22 (27%) taught mathematics pedagogy. The respondents had a wide range of qualifications which included: 44 with a PhD in mathematics (54%); 12 with a PhD in education (15%); 11 with a PhD in mathematics and a Graduate Diploma in Education (GDE) (13%); and 15 with masters, honours or bachelor's degrees as their highest qualification (18%).

The same survey was sent to pre-service secondary mathematics teachers at three universities in South East Queensland. Twenty-five (from 39 respondents) completed all items. Nineteen of the pre-service teachers were studying both mathematics and education courses in an undergraduate ITE program and six were studying education courses only in a

postgraduate ITE program as they had completed their mathematics study in a previous qualification.

The data analysis was conducted using *SPSS* and included calculation of descriptive statistics and a χ^2 (chi square) test of homogeneity (or independence) to examine differences between respondent groups (MTEs who teach mathematics content, MTEs who teach mathematics pedagogy, and PSTs). For a large proportion of items the χ^2 test for independence violated the assumption that there was a minimum of five elements in each cell when all three categories of respondents were included, even when the five response levels were collapsed to three (disagree, undecided, disagree) or two categories (disagree and undecided, agreed). Therefore the χ^2 tests were completed pair-wise for the respondents, using two levels of agreement. First, the respondents were grouped into three pairs for comparison: (1) MTEs who teach content and MTEs who teach pedagogy; (2) MTEs who teach content and PSTs; and (3) MTEs who teach pedagogy and PSTs. Levels of agreement were grouped into 'disagree and undecided' and 'agree'. When the assumption that there was a minimum of five elements in each cell was violated a Fisher's exact test was used to determine whether there was any difference in the proportion of MTEs who taught mathematics content, MTEs who taught mathematics pedagogy, and PSTs who agreed or strongly agreed with each item (Lund Research Ltd, 2018).

ANALYSIS AND DISCUSSION OF RESPONSES

The results are presented in two sections, discussing the participants' beliefs about mathematics and beliefs about teaching and learning mathematics. Throughout, "agreed" has been used to indicate "agreed or strongly agreed" and "disagreed" for "disagreed or strongly disagreed" for ease of reading. Rather than report on the whole contingency table for the χ^2 test of homogeneity for each of the combinations of respondents and levels of agreement, only the percentages for "agreed" have been included in Table 3 or those with a statistically significant difference. This addresses brevity and allows for ease of identifying trends.

Beliefs about mathematics

Two items in the survey referred to beliefs about mathematics. The numbers of participants who agreed or strongly agreed with each item are given in Table 1. Generally participants agreed (>90%) that mathematics was "beautiful, creative and useful" and "both a way of knowing and a way of thinking" but not (≤ 20%) that mathematics was "computation". The χ^2 test for homogeneity indicated no statistically significant differences between the MTEs and PSTs. Hence most respondents held a Problem-solving view of mathematics.

Table 1: Survey responses (agreed and strongly agreed) about Beliefs Mathematics

		Number and % of Educators expressing agreement			
No.	Item	MTEs who teach Mathematics	MTEs who teach Pedagogy	Pre-service teachers	View
9	Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking.	59 98%	20 91%	24 96%	PS
20	Mathematics is computation.	7 12%	1 5%	5 20%	I

PS – Problem-solving conception; I – instrumental conception

Beliefs about Teaching and Learning Mathematics

Table 2 shows the number of respondents who agreed with each item about teaching and learning mathematics. Only the numbers and percentage of respondents who have agreed have been included. The items have been listed in descending order of agreement by MTEs teaching mathematics content. Shading has been used to identify responses representing agreement of ≥ 90% or ≤10% of the respective groups. Looking down the table of items makes it easy to identify those that attracted the highest and lowest levels of agreement, while looking across the table at individual items allows for visual comparison of the extent of similarity and difference between the three groups of respondents. For the items marked with a * there were statistically significant differences between the proportions of respondents as determined by multiple pairwise χ^2 tests of homogeneity. These differences are discussed later.

The first observation that can be made about the responses is that there was strongest agreement amongst all three groups for items aligned with a Problem-solving conception of mathematics and constructivist approaches to mathematics teaching and learning, and strongest disagreement for items aligned with Instrumentalist conceptions and traditional teaching and learning approaches.

Table 2: Survey Responses for Beliefs about Teaching and Learning Mathematics (agreed and strongly agreed) (Beswick, 2005)

Item	Number	Number and % of Educators expressing agreement			View
		MTEs who teach mathematics (MTEC)	MTEs who teach pedagogy (MTEP)	Pre-service teachers (PST)	
6	Knowing how to solve a mathematics problem is as important as getting the correct solution.	58 97%	22 100%	22 88%	PS Cons
13	Justifying the mathematical statements that a person makes is an extremely important part of mathematics.	58 97%	21 95%	21 84%	PS Cons
3	It is important for students to be given opportunities to reflect on and evaluate their own mathematical understanding.	57 95%	22 100%	24 96%	PS Cons
4	It is important for teachers to understand the structured way in which mathematics concepts and skills relate to each other.	57 95%	22 100%	25 100%	P Cons
*5	Effective mathematics teachers enjoy learning and “doing” mathematics themselves.	57 95%	20 91%	20 80%	PS Cons
*10	Allowing a student to struggle with a mathematical problem, even a little tension, can be necessary for learning to occur.	57 95%	21 95%	19 76%	PS Cons
1	A vital task for the teacher is motivating students to solve their own mathematical problems.	55 92%	22 100%	24 96%	PS Cons
2	Ignoring the mathematical ideas that students generate themselves can seriously limit their learning.	53 88%	21 95%	23 92%	PS Cons
*15	Teachers can create, for all students, a non-threatening environment for learning mathematics.	46 77%	18 82%	24 96%	PS Cons
*7	Teachers of mathematics should be fascinated with how students think and intrigued by alternative ideas.	45 72%	22 100%	21 84%	PS Cons

11	Students always benefit by discussing their solutions to mathematical problems with each other.	39 65%	18 82%	19 76%	PS Cons
12	Persistent questioning has a significant effect on students' mathematical learning.	39 65%	16 73%	17 68%	PS Cons
14	As a result of my experience in mathematics classes, I have developed an attitude of inquiry.	39 65%	15 68%	14 56%	PS Cons
*8	Providing students with interesting problems to investigate in small groups is an effective way to teach mathematics.	35 58%	20 91%	19 76%	PS Cons
*17	There is an established amount of mathematical content that should be covered at each grade level.	34 57%	6 27%	18 72%	I Trad
*16	It is the teacher's responsibility to provide students with clear and concise solution methods for mathematical problems.	33 55%	6 27%	17 68%	I Trad
*18	It is important that mathematics content be presented to students in the correct sequence.	31 52%	7 32%	22 88%	I Trad
*19	Mathematical material is best presented in an expository style: demonstrating, explaining and describing concepts and skills.	29 48%	5 23%	14 56%	I Trad
25	It is important to cover all the topics in the mathematics curriculum in the textbook sequence.	7 12%	0 0%	2 8%	I Trad
24	Listening carefully to the teacher explain a mathematics lesson is the most effective way to learn mathematics.	6 10%	1 5%	4 16%	I Trad
22	I would feel uncomfortable if a student suggested a solution to a mathematical problem that I hadn't thought of previously.	4 7%	2 9%	2 8%	I Trad
23	It is not necessary for teachers to understand the source of students' errors; follow-up instruction will correct their difficulties.	4 7%	0 0%	1 4%	I Trad
*21	Telling the students the answer is an efficient way of facilitating their mathematics learning.	1 2%	1 5%	5 20%	I Trad
26	If a students' explanation of a mathematical solution doesn't make sense to the teacher it is best to ignore it.	1 2%	0 0%	0 0%	I Trad

PS = Problem-solving conception; I – Instrumentalist conception; P = Platonist conception; Cons = constructivist teaching methods; Trad = traditional teaching methods

MTEs who teach mathematics pedagogy appeared more likely to endorse items aligned with a Problem-solving conception and constructivist teaching approaches (ten items with ≥ 90% agreeing) than MTEs who teach mathematics content (seven items with at least 90% agreeing) or PSTs (five items with ≥ 90% agreeing). For example, 100% of MTEs teaching pedagogy agreed on the importance of knowing how to solve a problem rather than just getting the right answer (item 8), students reflecting on their understanding (item 3), teachers motivating students to solve their own problems (item 1), and teachers being fascinated by how students think (item 7). All members of this MTE group also agreed with item 4, on the importance of teachers "understanding the structured way in which mathematics concepts and

skills relate to each other", which perhaps resonates more strongly with Platonist than Problem-solving conceptions.
All three groups were almost equally likely to withhold endorsement from items aligned with an Instrumentalist conception and traditional teaching approaches (between four and six items with ≤10% of each group agreeing). For example, only one respondent agreed that it is best to ignore a students' explanation of a mathematical solution if this does not make sense to the teacher (item 26) and only MTE who taught content and one MTE who taught pedagogy agreed that telling students the answer is an efficient way of facilitating their mathematics learning (item 21). There was similarly low level of agreement amongst all groups with statements about the lack of necessity for teachers to understand the source of students' errors (item 23) and feeling uncomfortable if a student suggested a solution to a problem that the teacher had not thought of previously (item 22).

Looking across the blocks of shaded items describing Problem-solving conceptions (i.e., those listed in the top part of Table 2), there appears to be more similarity between the beliefs profiles of the two groups of MTEs than between PSTs and either group of MTEs. For example, at least 90% of MTEs who teach content (MTEC) and MTEs who teach pedagogy (MTEP) were in agreement with items, 6, 13, 3, 4, 5, 10, and 1, whereas for only three of these items – items, 3, 4, and 1, concerning student reflection, teacher understanding of mathematical structure, and teacher motivation of students – were there at least 90% of PSTs in agreement.

The pairwise χ^2 test of homogeneity found statistically significant differences between pairs of respondent groups for ten questionnaire items, five that aligned with a Problem-solving conception of mathematics and five aligned with an Instrumentalist view. Table 3 summarises these statistically significant results: it was generated from the individual contingency tables following the multiple pairwise χ^2 tests of homogeneity. Each line in Table 3 represents a separate running of the χ^2 test that compared the two groups for which the percentage of Educators expressing agreement is shown. The items have been ordered so that the highest agreement for MTEs who teach mathematics are listed first.

Table 3: Differences in Beliefs about Teaching and Learning Mathematics – percentage of respondents who agreed (χ^2)

Item	Abbreviated item	MTEC	MTEP	PST	$X^2(1)$	p	view
5	Effective teachers enjoy learning and "doing" mathematics	95%	_	80%	4.657	0.045	PS Cons
10	Struggle can be necessary for learning.	95%	_	76%	6.729	0.017	PS Cons
15	Teachers can create a non-threatening learning environment.	77%	_	96%	4.539	0.057	PS Cons
7	Teachers should be fascinated by students' thinking and alternative ideas	75%	100%	_	6.731	0.008	PS Cons
8	Students should investigate interesting problems in small groups	58%	91%	_	7.735	0.005	PS Cons
17	There is an amount of content to be covered at each level	57% -	27% 27%	_ 72%	5.567 9.368	0.018 0.002	I Trad
16	Teachers should provide clear, concise solution methods	55% -	27% 27%	_ 68%	4.962 7.768	0.026 0.005	I Trad
18	Mathematics should be presented in the correct sequence	52% -	- 32%	88% 88%	9.924 15.63	0.002 0.000	I Trad
19	Mathematics is best presented in an expository style.	48% -	23% 23%	_ 56%	4.349 5.379	0.037 0.020	I Trad
21	Telling students the answer is an efficient way of facilitating learning	2%	_	20%	9.041	0.008	I Trad

PS = Problem-solving; I = Instrumentalist; Cons = constructivist teaching methods; Trad = traditional teaching methods

Patterns of similarity and difference between the groups of MTEs and PSTs are discussed separately for the set of items aligned with Problem-solving and Instrumentalist views.

With regard to Problem-solving beliefs (items 5, 10, 15, 7, and 8), two different patterns were observed. First, for items 5, 10, and 15 there was a statistically significant difference between MTEs who teach mathematics content and PSTs. While both groups were highly likely to agree with statements about effective mathematics teachers enjoying learning and "doing" mathematics themselves (item 5) and the necessity of allowing students to struggle and experience a little tension while solving a mathematical problem (item 10), a higher proportion of MTEs teaching mathematics content than PSTs expressed agreement with these items. On the other hand, a higher proportion of PSTs than MTEs teaching mathematics content agreed with the statement that teachers can create, for all students, a non-threatening learning environment (item 15). Taken together, these differences might be interpreted as PSTs feeling less certain about the benefits of struggle while doing mathematics, and instead valuing a learning environment in which student struggle is supported or even reduced.

The second response pattern in the set of items aligned with Problem-solving conceptions revealed statistically significant differences between MTEs who teach mathematics content and those who teach mathematics pedagogy. MTEs who teach pedagogy were more likely than MTEs teaching mathematics content to agree that teachers should be fascinated with how students think (item 7) and that providing students with interesting problems to investigate in small groups is an effective way to each mathematics (item 8). Both of these items refer to practices that might be more commonly aligned with the goals of a mathematics pedagogy course – focusing on learning how to teach mathematics – than those of a mathematics content course.

With regard to Instrumentalist beliefs (items, 17, 16, 18, 19, and 21), another two patterns of statistically significant difference were observed. First, for items 16, 17 and 19, PSTs more closely resembled MTEs who teach mathematical content than those who teach pedagogy, as both the former groups endorsed Instrumentalist conceptions and traditional teaching approaches. These approaches involved teachers providing students with clear, concise solution methods; the existence of an established amount of mathematical content to be covered at each grade level; and presenting mathematical material in expository style. Also, for item 18, PSTs differed from both groups of MTEs in agreeing more strongly with the importance of presenting students with mathematics content in the correct sequence.

DISCUSSION AND CONCLUSIONS

The overarching goal of this study is to develop insight into how pre-service teachers of secondary school mathematics make sense of the perspectives on mathematics, and mathematics teaching and learning, that may be explicitly or implicitly communicated to them through the university courses they study in mathematics content and mathematics pedagogy. Typically, in Australian universities, these courses are respectively taught by mathematicians and education academics who have few opportunities to develop a shared understanding of each other's goals and teaching approaches in preparing future school teachers. Because the *Australian Curriculum: Mathematics* promotes a view of mathematical proficiency that encompasses understanding, reasoning, and problem solving alongside more traditional goals concerning procedural fluency, it is important that pre-service teachers are exposed to what has been described as a Problem-solving conception of mathematics, and the constructivist teaching and learning practices aligned with this conception (Ernest, 1989).

This paper addressed two research questions. The first sought to characterise the beliefs of mathematics teacher educators who teach either mathematics content or mathematics pedagogy and of the secondary mathematics pre-service teachers whom they teach. Analysis of questionnaire responses indicated that all groups of respondents supported a Problem-solving conception of mathematics, and mathematics teaching and learning, rather than an Instrumentalist conception and associated teaching and learning approaches. The second research question probed differences in the beliefs of the three respondent groups, and here some interesting patterns emerged. On a small number of items, the only statistically significant differences were between the MTEs who teach mathematics content and those who teach mathematics pedagogy, with the latter group more likely to agree with a Problem-solving conception and practices that reveal and support students' mathematical thinking. However, on a larger number of items, the statistically significant differences distinguished the pre-service teachers from one or both groups of MTEs, almost always with the PSTs expressing more agreement for Instrumentalist views, or less agreement for Problem-solving views, than the other group(s).

The relationship between beliefs and teaching practices is complex and influenced by contextual variables (Beswick, 2005; Guskey, 1986; Lerman, 2001). While it is not possible to claim a direct relationship of influence between the beliefs of mathematics teacher educators and the beliefs of the pre-service teachers whom they teach, the findings of this study suggest that all these participants in the initial teacher education enterprise express beliefs somewhere along the spectrum from Problem-solving to Instrumentalist, and in patterns of similarity and difference that resist simple classification. Nevertheless, the propensity for pre-service teachers in this study to express Instrumentalist views points to the importance of encouraging them to discuss their own past and continuing experiences of learning mathematics as part of their mathematics pedagogy courses, including their emotional experiences. As many PSTs will be learning mathematics content at university concurrently with mathematics pedagogy as part of their initial teacher education program, this could provide opportunities to explore beliefs about mathematics and how mathematics is learned and taught in schools and universities. Discussions exploring beliefs about mathematics and how mathematics is learned and taught should improve the understanding of future teachers, as well as those who teach them, about the possibilities and constraints inherent in different educational settings.

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MATHEMATICAL TEXT COMPREHENSION: THE CASE OF A COHORT OF PRE-SERVICE MATHEMATICS TEACHERS

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KEYWORDS: mathematical text comprehension, pre-service teachers, text comprehension theory

ABSTRACT

In this study ability of a cohort of pre-service mathematics education teachers to comprehend and learn from a novel mathematical text was investigated. In South Africa from grade 10 to 12 learners have to choose between mathematics and mathematical literacy. The subject mathematical literacy is described in the Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011, pg 8) of South Africa as *not* having a focus on abstract mathematical concepts and being based on elementary mathematical content. The investigation also set out to determine whether there are differences between the mathematical text comprehension abilities of participants with mathematics and mathematical literacy backgrounds. Study participants were presented with a text based on the concepts of floor and ceiling and were required to study the theory and worked examples and then attempt exercises provided in the text. No teaching or assistance was offered for the entire duration of the exercise. Findings indicate that a majority of participants struggled to comprehend and learn from the texts containing ordinary English and mathematical English (with and without symbols). No statistically significant difference was found between text comprehension abilities of mathematics and mathematical literacy participants.

INTRODUCTION

It is generally accepted that every academic discipline has a distinctive ‘language’ that sets it apart from one another. Gaining competence in a discipline includes learning the ‘language’ of the discipline. Schleppegrell (2007) contend that learning the ‘language’ of a discipline cannot be separated from learning in the discipline. The ‘language’ of mathematics is considered to be complex by many (Österholm, 2006). Thus, if the ability to read mathematical texts (the ‘language’ of mathematics) is not fairly well-developed learning will be hampered.

Mathematics teachers need to be fairly competent in the use and understanding of mathematical texts in order to develop their learners’ understanding (van Jaarsveld, 2018). In the South African school mathematics curriculum this competency is alluded to where teachers are called upon to use correct notation and mathematical language in the classroom to enhance learners’ understanding (DBE, 2016). A prerequisite for the use of correct mathematical language and notation is the ability to read, comprehend and learn from mathematical texts. Another South African report of the DBE (DBE, 2013) states that many grade 12 learners experience problems when conceptual understanding is required, which is premised on understanding of verbal and text-based information.

It is thus obvious that pre-service mathematics teachers should be exposed to the independent reading of mathematical texts to develop the mentioned competency. Our knowledge of pre-service mathematics teacher education programmes in South Africa is that there are not any that focus specifically on the development of competency in the reading and comprehension of mathematical texts. This study investigated the mathematical text reading

and comprehension abilities of a cohort of pre-service mathematics education students at a university in South Africa.

Pre-service mathematics teachers enter university mathematics courses with either mathematics or mathematical literacy as exiting mathematics courses during the final school year. Another objective of the study therefore was to determine how students with different mathematical backgrounds (mathematics or mathematical literacy) would compare in terms of mathematical text reading ability.

LITERATURE REVIEW

Watkins (1979) investigated text comprehension of two groups of college students. One group had high prior knowledge while the other group had low prior knowledge of the content used in the study. The study found that mathematical texts with symbols did not appear to have either a negative or positive effect on problem solving abilities of low prior knowledge students whereas it appeared to help high prior knowledge students. Mathematical texts written in ordinary English benefitted both low and high prior knowledge students.

A study by Kintsch (1986) focused on the type of mental representations (*textbase* or *situation model*) formed when students read texts. The objective was to determine how these mental representations contribute to and interact during problem solving activities. A finding of the study was that participants recalled problems already solved (Arithmetic word problems) on the basis of the *situation model*—the reconstruction of the problems occurred in terms of solution performance. Hubbard (1990) provides a theoretical explanation on why people find it difficult to read mathematical texts and argues for the inclusion of reading and study skills in first year mathematics courses.

Österholm (2006) compared the reading comprehension of high school and university students of two mathematical texts and one historical text. The results of the study indicate that reading comprehension of the historical text and the mathematical text without symbols was similar. On the other hand there was a difference in reading comprehension between the mathematical text without symbols and the text with symbols. He concludes that the mathematical symbols in the text had a bigger influence on reading comprehension for both groups of students. Furthermore, there were no differences between the university and the high school students in terms of mathematical text reading ability.

Research by van Jaarsveld (2018) focused on pre-service teachers' written descriptions of procedures they were employing to solve presented problems. Students' use of mathematical vocabulary was the focus of the study in order to identify student misconceptions. Consideration of the findings led to a recommendation that mathematical vocabulary should be a focus of initial mathematics teacher education.

Berger (2019a) investigated two pre-service teachers' reading of a section of a prescribed mathematics textbook and found that they used productive and less productive ways for reading to learn mathematics compared to a hypothetical expert reader. She also researched reading styles of practising teachers (Berger, 2019b) and identified five different kinds of reading styles.

This brief literature review presented some studies and their findings related to mathematical text comprehension abilities by both school and students in tertiary institutions including pre-service mathematics teachers. A few studies explored the relationships between mathematical background and text comprehension, part of the focus of the research reported in this article.

THEORETICAL FRAMEWORK

Kintsch (1994) suggests a distinction between a memorizing of a text and learning from it during reading. He contends that in order to learn from a text the reader of a text should be able to do any or all of the following in conjunction with prior knowledge: make inferences from the text, can use the text to solve novel problems or to elaborate on the text. He terms this deep understanding of a text. This is contrasted with shallow understanding needed to memorize the content of a text in order to more or less completely reproduce the content. For mathematical problem solving this means that solution procedures of worked examples will be memorized and hence be imitated only if presented problems are very similar.

Memory formation of idea units when reading a text depends on at least three factors: 'whether the information within that idea unit is encoded' (made part of the memory structures); 'the extent to which connections between that idea unit and other related ideas or topics are encoded' and 'whether retrieval cues for accessing that idea unit are available at time of a test' (Rawson and Kintsch, 2004, p. 324).

Text comprehension theory explain how mental representations is formed when a text is read. From text comprehension theory different levels of comprehension of a text are possible (van Dijk & Kintsch, 1983; Kintsch, 1992). A mental representation is a *surface component* if the words, phrases and the linguistic relations between them are encoded in memory. In this case no mental processing or reasoning is done with the words and phrases except linking with the ordinary meaning of the words and phrases.

If in addition to the *surface component* the semantic structure of the text are encoded without adding anything that is not explicit in the text then the level of comprehension are known as the *textbase*. For a *textbase* representation no inferences are made from the text. To create a *textbase* some prior knowledge is required. In this instance the knowledge is of a general kind that is required to 'decode' the text. Elaboration of the information in the text by using prior knowledge and its subsequent integration with the prior knowledge is the level of comprehension referred to as the *situation model*. The prior knowledge required to create a *situation model* is more specific with respect to the content of the text.

According to Kintsch (1994) text comprehension theory clarifies the mental engagements of low and high prior knowledge readers. He claims the theory predicts that high and low prior knowledge readers will equally be able to reproduce the text but that high prior knowledge readers will in addition be better at reconstructing and elaborating on the text. This is due to readers with low prior knowledge not having a prior context for the content of the text and hence lacking retrieval cues to engage with the content. Readers with high prior knowledge, however, have several retrieval cues available to engage with the content of a text. The reader with high prior knowledge will therefore be able to use the content of the text to solve novel problems whereas this is not automatic for readers with low prior knowledge.

Accordingly, for this study text comprehension theory predicts that pre-service mathematics teachers entering university study with mathematical literacy, deemed to be a less demanding mathematical option, will display weaker text reading ability than those entering university mathematics teacher education programmes with mathematics.

Learning in a discipline presupposes becoming literate in it. Literacy however is hard to define (Kalman, 2008) and contentious. This article prefers the term content literacy since it is more relevant. Content literacy is defined as 'the ability to read, understand and learn from texts from a specific subject area' (Österholm, 2006, p. 329). It is deemed as consisting of three knowledge components: general literacy skills, content specific literacy skills and prior

knowledge of content (McKenna & Robinson, 1990). According to Österholm (2006) general and content-specific literacy skills are the general knowledge that is required to read and comprehend a text. These two literacies are not dependent on the detailed content of a specific text and is primarily utilized to 'decode' texts and to create a *textbase* in the mental representations of the reader. He further contends that prior knowledge of content is referent to knowledge that is connected to the content of a specific text and is used to create a *situation model* in the mental representation.

It was argued above that learning from a text implies that deep understanding of the text was developed. Since this implies inter alia that inferences are made in conjunction with prior knowledge it implies that new mental connections are made. New mental connections between prior knowledge and new knowledge imply that conceptual knowledge, knowledge rich in relationships (Hiebert and Lefevre (1986), was expanded and enhanced. This in turn suggests that conceptual understanding, the comprehension of mathematical concepts, operations and relations (National Research Council, 2001), was attained to some degree.

Previously, it was indicated that a reader learned from a text when she/he is able to use information from the text to solve novel problems based on the text. This implies that a measure of ability in mathematical task solving can be employed to indirectly measure text comprehension. Consequently, if a reader can solve a procedural problem, different from that presented in the text (delimiting algorithmic reasoning based on procedural knowledge), then it can be inferred that a certain degree of procedural understanding from the text was attained and that at least a *textbase* was created as a mental representation. Conversely, if the reader cannot supply a correct solution to such problems, but do provide correct solutions to nearly similar problems (familiar algorithmic reasoning based on procedural knowledge) then the inference is that procedural understanding has not been achieved at the correct level and that the mental representation created is *surface component*.

Similarly, if a reader provides a plausible argument for a conceptual problem based on the text (but not exactly like that presented in the text) then it can be concluded a certain degree of conceptual understanding was acquired and parts of a *situation model* as a mental representation was created. Correspondingly if a plausible argument to such problems is not provided, then the inference is that a *surface component* has been created as a mental representation. However, if such a reader employs conceptual knowledge from the text correctly in instances where presented problems are near-similar to those in the text, then it is taken that a *textbase* was formed as a mental representation.

In this article the analysis of mathematical text comprehension was used to determine, which mental representation was formed for which of the content literacies. The analysis scheme is presented in Table 1.

Table 1: Analysis of mathematical text comprehension

Content literacy skill requirement (knowledge components)	**Mental representation**
General literacy skills	*Surface component* (words and phrases encoded without its specific meaning in the mathematical context; only the operational meaning of mathematical symbols is discerned which allows for imitation of only very similar problems)
General and content specific literacy skills	*Textbase* (words and phrases together with semantic structure is encoded;

	relations that are explicitly specified in the text are encoded; no implicit connections are made i.e. no inferences are made from the text; general operational and semantic meaning of mathematical symbols are discerned but is not always specific to the provided text which allows for imitation, and low level of local creativity in problem solving activities)
General and content specific literacy skills and prior knowledge of content	*Situation model* (words and phrases and semantic structure of text is encoded; both explicit and implicit connections are made i.e. text is connected and integrated with prior knowledge; both specific and general operational and semantic meaning of mathematical symbols are discerned which allows for imitation and local creative reasoning.)

METHOD

Twenty pre-service mathematics teachers of a class of 84 students who were registered for the 2nd year of a mathematics course participated in the study. Participation was voluntary. The section reported on in this article was a once-off three hour session on a Saturday morning, not part of the normal contact sessions. The sample was thus a convenience one and the first author presented the course and the extra session.

Students enrolled for the course had either completed mathematical literacy or mathematics as their school-leaving mathematical subject. Mathematical literacy deals more with the applications of mathematical constructs and is done in the last three years of school. The applications are generally based on real-life situations. Most of the texts presented to learners in mathematical literacy are in ordinary English with and without mathematical symbols. To a lesser extent some texts include mathematical English with symbols. The pure symbolic form is not featured in mathematical literacy texts.

The mathematics curriculum students followed in their last three school years had a fair amount of engagement with texts that contain mathematical English with and without symbols as well as pure mathematical symbolism. Since a major part of the text presented to participants for this study contained mathematical English with symbols the expectation was that students who had mathematics at the school level would have more prior knowledge of such texts and therefore would be able to perform better in problem solving activities based on the text than those students entering university with mathematical literacy due to the differences in exposure to texts with mathematical English. Table 2 provides a summary of the demographics of the participants.

Table 2: Demographics of study participants

	Number of students	Average age	Mathematics	Mathematical Literacy
Female	11	23	7	4
Male	9	23	5	4
Total	20		12	8

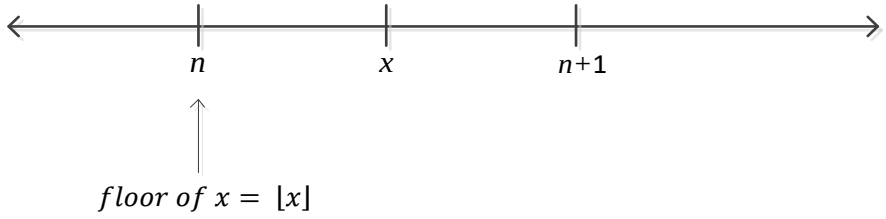
Participants were presented with a copied mathematical text from a discrete mathematics textbook (Epp, 2011). The text dealt with the concepts of floor and ceiling of number theory. These constructs were not part of the content of the mathematics courses the students had followed. The concepts were completely new to the students. The text contained definitions and worked examples based on the definitions. The text also contained an exposition of direct proofs of properties of floor and ceiling. Students had to study the theory and worked examples and then attempt the given exercises. No teaching or assistance was offered for the entire duration of this exercise.

Some of the exercise problems required imitative reasoning based on procedural knowledge for their solution. Problems that required students to compute the floor and ceiling of numbers (questions 1 through 4) were similar to the worked examples. Students were also presented with an additional question requiring them to compute floor and ceiling (referred to as question A) that were not contained in the textbook exercises and that were slightly different.

Some problems were of a higher cognitive level and required creative reasoning based on conceptual knowledge to solve them. Questions 6 and 7 were based on worked examples in the text and required conceptual understanding of the definitions of floor and ceiling. Questions 6 and 7 were slightly different to those presented in the text since it required a conceptual understanding of ceiling whereas worked examples were based on the concept of floor. The two questions were classified as local creative reasoning based on conceptual knowledge. Questions that required the construction of a direct proof also fell in this category but were classified to be on a higher cognitive level than questions 6 and 7. Students were allowed to discuss with one another while working through the text, but were required to attempt the exercises individually.

Definitions of floor and ceiling were given in four formats in the text. These format are Ordinary English (OE) without symbols; Mathematical English (ME) with symbols (Watkins, 1979); symbolic and graphical. Table 3 exemplifies these different formats. All questions presented to students were based on these definitions.

Table 3: Ways in which statements were presented

Ways in which statements were written	**Statements**
OE without symbols	The floor and ceiling of the number are the integers to the immediate left and to the immediate right of the number (unless the number is itself, an integer, in which case its floor and ceiling both equal the number itself).
ME with symbols	Given any real number x, the floor of x, denoted $\lfloor x \rfloor$, is defined as follows: $\lfloor x \rfloor = that\ unique\ integer\ n\ such\ that\ n \leq x < n+1$
Symbolically	$\lfloor x \rfloor = n \Leftrightarrow n \leq x < n+1$
Graphically	

Questions students worked on were divided into three categories: computational, conceptual and proof questions. The proof questions are not dealt with in this article because of space constraints. For the computational questions 1 mark was awarded for a correct solution and 0 for an incorrect solution. A total of 12 marks were possible for the computational questions since there were 6 questions where floor and ceiling had to be calculated. Two conceptual questions were posed. Two marks were awarded for each question and hence a total mark of 4 was possible. Based on scores and making inferences from student answers, students were placed into categories by using Table 1.

The analysis focused on mathematical text comprehension of all participants and the differences in performance of the students with mathematical literacy (*math lit*) and mathematics (*math*) as school mathematics background.

Student scores for the different categories of questions were the data and *IBM SPSS version 25* was used to analyse the data. As argued above mathematical text comprehension is operationalised through the test scores.

PRESENTATION OF RESULTS AND DISCUSSION

Table 4 shows the descriptive statistics for all participants.

Table 4: Descriptive Statistics for all participants

	N	Minimum	Maximum	Mean	Std. Deviation
Q A (a)	20	0	100	22.50	41.279
Q A (b)	20	0	100	42.50	46.665
Q1	20	0	100	82.50	33.541
Q2	20	0	100	90.00	30.779
Q3	20	0	100	72.50	44.352
Q4	20	0	100	70.00	47.016
COMPUTATIONAL MARK	20	0	100.0	63.333	27.0909
Q6	20	0	100	37.50	35.818
Q7	20	0	50	10.00	20.520
CONCEPTUAL MARK	20	0	50	25.00	19.868

The descriptive statistics shows that students performed satisfactorily in questions 1 to 4, but performed less well in question A and questions 6 and 7.

The internal consistency was determined by calculating Cronbach's alpha. The statistics is presented in Table 5 and 6. The overall Cronbach's alpha is greater than .7 hence the test has satisfactory internal consistency. Two items have values less than .3 (Q6 and Q7) this implies that these items might be measuring something different than the test as a whole. This will be discussed later.

Table 5: Reliability Statistics

Cronbach's Alpha	N of Items
.743	10

Table 6: Item Total Statistics

	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
Q A (a) ⌊3⌋; ⌈3⌉	493.333	29514.620	.557	.696
Q A (b) ⌊−7⌋; ⌈−7⌉	473.333	31839.181	.307	.744
Q1 ⌊37,999⌋; ⌈37,999⌉	433.333	32163.743	.485	.711
Q2 ⌊17/4⌋; ⌈17/4⌉	425.833	33244.883	.439	.718
Q3 ⌊−14,00001⌋; ⌈−14,00001⌉	443.333	28944.444	.544	.698
Q4 ⌊−32/5⌋; ⌈−32/5⌉	445.833	27613.304	.595	.687
COMPUTATIONAL MARK	452.500	29598.684	.943	.660
Q6	478.333	37839.181	.000	.778
Q7	505.833	37902.778	.100	.753
CONCEPTUAL MARK	490.833	37279.971	.189	.746

The independent-samples t-test was used to determine if there is a statistically significant difference between the two groups since the overall score were reasonably normally distributed. The statistics are shown in Tables 7, 8, 9 and 10.

The group statistics show that the mean (n = 8; mean = 76.042**)** of the *math lit* group is much higher than that of the *math* group (n = 12; mean = 54.861**).** The null hypothesis is that the means of computational mark are the same for the two. The null hypothesis is retained at the .05 significance level (t(18)= 1.813, sig (2-tailed) = .087). Hence no significant difference was observed between the means of *math* and *math lit* for computational mark. The magnitude of the differences in the means was large (eta squared = .154) (Cohen, 1988), implying that 15.44% of the variance in mathematical text comprehension is explained by mathematical background (*math* versus *math lit*).

Group statistics for conceptual mark show that there is a 5.2% difference between means of the *math* and *math lit* groups. Similarly, for the conceptual score the null hypothesis was that the means are the same across categories for *math* and *math lit* groups. The null hypothesis is retained at the .05 significance level (t(18) = .564, sig(2-tailed) = .580). Hence no significant difference was observed between the means of the two groups. The magnitude of the differences in the means was small (eta squared = .0173) (Cohen, 1988). This implies that 1.73% of the variance in mathematical text comprehension is explained by mathematical background for conceptual score.

Table 7: Group Statistics for computational mark

	MATH (M)/MATH LIT (L)	N	Mean	Std. Deviation	Std. Error Mean
COMPUTATIONAL MARK	MATH LIT	8	76.042	24.9752	8.8301
	MATH	12	54.861	25.9804	7.4999

Table 8: Independent Samples Test for computational mark

			COMPUTATIONAL MARK	
			Equal variances assumed	Equal variances not assumed
Levene's Test for Equality of Variances	F		.031	
	Sig.		.863	
t-test for Equality of Means	t		1.813	1.828
	df		18	15.582
	Sig. (2-tailed)		.087	.087
	Mean Difference		21.1806	21.1806
	Std. Error Difference		11.6821	11.5853
	95% Confidence Interval of the Difference	Lower	-3.3626	-3.4327
		Upper	45.7237	45.7938

Table 9: Group Statistics for conceptual mark

	MATH (M)/MATH LIT (L)	N	Mean	Std. Deviation	Std. Error Mean
CONCEPTUAL MARK	MATH LIT	8	28.13	16.022	5.665
	MATH	12	22.92	22.508	6.498

Table 10: Independent Samples Test for Conceptual mark

	CONCEPTUAL MARK	
	Equal variances assumed	Equal variances not assumed
F	2.825	

Levene's Test for Equality of Variances	Sig.	.110	
t-test for Equality of Means	t	.564	.604
	df	18	17.861
	Sig. (2-tailed)	.580	.553
	Mean Difference	5.208	5.208
	Std. Error Difference	9.236	8.620
	95% Confidence Interval of the Difference Lower	-14.195	-12.912
	95% Confidence Interval of the Difference Upper	24.612	23.329

The Levine test for equality of variances indicates that the two groups were of similar variance for both computational and conceptual scores with significant values of .863 and .110 computational score and conceptual score respectively at the 0.05 level of significance.

As mentioned previously for the computational type questions students were presented with the following additional question, indicated as question A) that were not contained in the textbook exercises and that were slightly different. This was done in order to determine how well the text and worked examples were read and understood: *Compute the floor and ceiling of each of the following: (a) 3 and (b) -7.*

The text presented to participants contained worked examples for the following values of x: $\frac{25}{4}$; 0.999; -2.01. The text therefore did not contain examples where the presented number was an integer. Three (3 of 8 (37.5%)) of the *math lit* students gave a correct solution to both questions and only one (1 of 12 (8.33%)) of the *math* students provided a correct solution. However only one of the *math lit* students used the correct symbol for floor and ceiling whereas the *math* student used the correct symbol (see Figures 1, 2 and 3 for examples of responses).

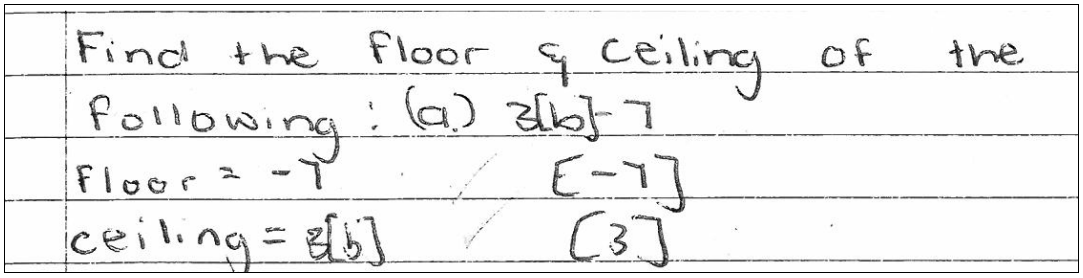


Figure 1: Math lit student 1 – response (incorrect symbolism) to question A

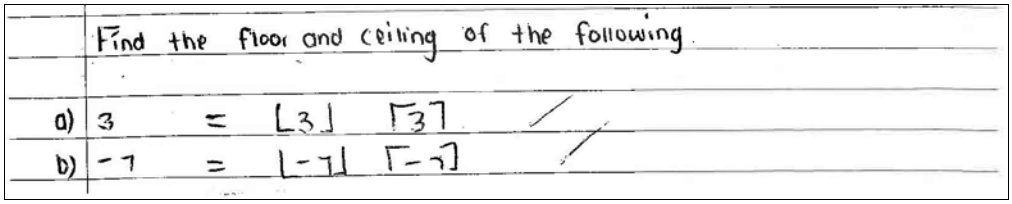


Figure 2: Math lit student 2 – response (correct symbolism) to question A

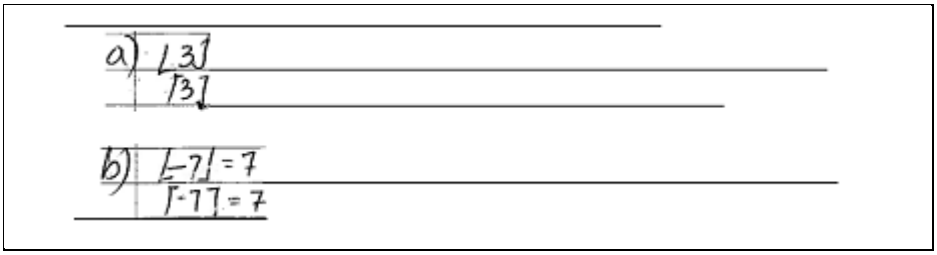


Figure 3: Math student 1 - response (correct symbolism) to question A

Questions 1 to 4 also required students to compute the floor and ceiling of given numbers. However these questions were different from question A in that it was similar to the worked examples (it contained fractions and decimal numbers). The majority of students performed well with these questions (mean =79%).

As mentioned previously the solutions to questions 6 and 7 required conceptual understanding of the concepts of floor and ceiling. The following are the questions:

Question 6: If k is an integer, what is $\lceil k \rceil$? Why?
Question 7: If k is an integer, what is $\left\lceil k + \frac{1}{2} \right\rceil$. Why?

The majority of students did not perform well in these questions (means of 38% and 10% for question 6 and 7 respectively). A comparison of the *math lit* and *math* groups shows that both groups had a mean score of 38% for question 6. However for question 7 the *math lit* group had a mean score of 19% compared to 4.2% for the *math* group. Six of (6 out of 8) the *math lit* group provided a correct response to question 6 but could not provide a correct reason for their response. See Figure 4 for an example of correct answer but incorrect symbolic reason of a *math lit* student. It is plausible to suggest that this is since they lack in ability to firstly read and subsequently to reason and present an argument with mathematical symbols.

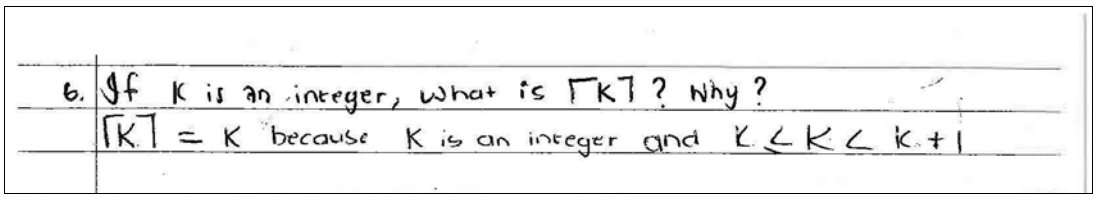


Figure 4: Math lit student 2 – response to question 6

Many of the *math lit* group could not provide a coherent argument for question 6 although it appeared as if they knew a reason but could not articulate it (see Figure 5 for an example). It is plausible to suggest that this is a result of their prior reading experiences within mathematical literacy where few instances occur where they are required to provide arguments in symbol form. Hence a majority of these students could comprehend the symbolic text to some degree, but did not learn from it. In the majority of cases therefore these participants' mental representation of the text was *surface component* with only a very slight degree of *textbase*.

In the case of the *math* group 3 participants provided a correct response and reason and 3 provided a correct response with an incorrect reason for question 6 (see Figure 6 for an example). The fact that some of the *math* group could provide correct reasons can be explained by their previous reading experiences. Their previous reading experiences included

instances where they had to read symbolic mathematical texts and then reason and provide arguments with it.

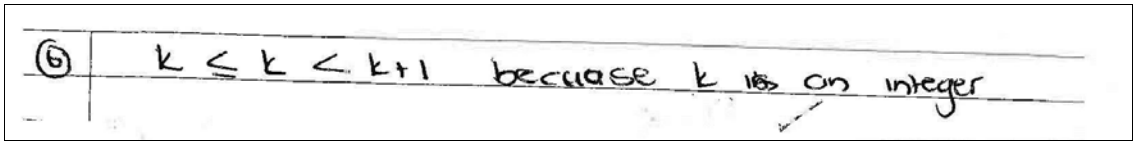


Figure 5: Math lit student 1 – response to question 6

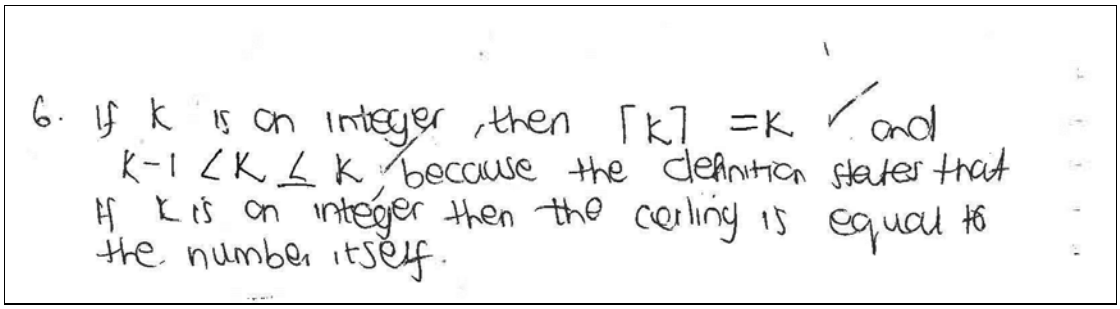


Figure 6: Math student 3 – response to question 6

In the case of question 7 only 4 of all participants provided partially correct responses. Two of these responses were not coherent giving the impression that students did not comprehend and learn from a similar example provided in the text.

Three *math lit* participants provided partially correct responses to question 7. These three participants provided a partially correct symbolic reason, but an incorrect answer (see Figure 7 for an example)

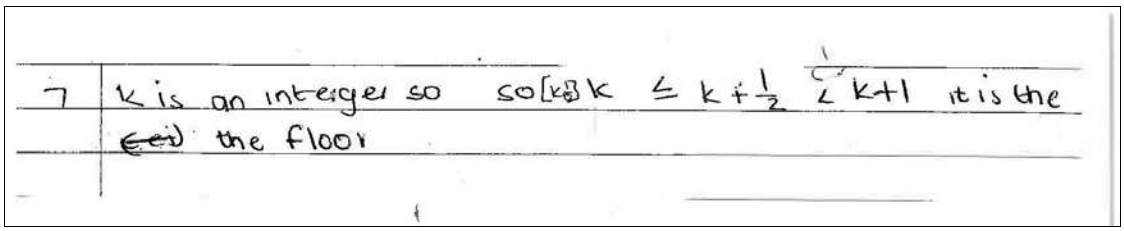


Figure 7: Math lit student 4 – response to question 7

Only one *math* participant provided a partially correct response for question 7. This is an indication that the *math* group struggled more than the *math lit* group with this question. Even student 3 who provided a correct answer and reason for question 6 could not master question 7 (see Figure 8). The majority of the *math* group therefore could not read, comprehend and learn from similar examples provided in the text. One can therefore conclude that the mental representation of these groups were also mostly *surface component* and a slight degree of *textbase*.

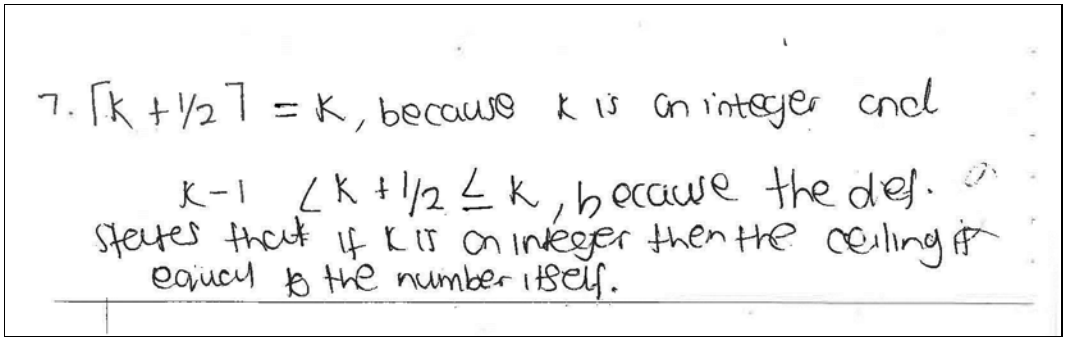


Figure 8: Math student 3 – response to question 6

CONCLUSION

Overall the results indicate that there were not substantial differences between the two groups on their ability to comprehend and learn from a novel mathematical text. Furthermore the findings of this study is similar to the finding of Österholm (2006) that participants who studied less mathematics than their counterparts have approximately the same text reading ability in terms of mathematical texts with symbols.

Both groups performed reasonably satisfactorily with familiar algorithmic reasoning based on procedural knowledge type questions (questions 1 to 4). The fact that the majority of participants could not use information from the text in a new way for these type of questions imply that their reading abilities did not allow them to make inferences from the text but they could imitate similar solutions to similar questions. Thus, the majority of participants formed a *surface component* mental representation of the text. This therefore suggests that for the text of computational type problems participants' content literacy skills was largely general literacy skills and only a low level of specific literacy skills.

All participants struggled with the conceptual type questions (local creative reasoning based on conceptual knowledge). The given text was primarily in mathematical English with symbols. Participants thus had to read and comprehend both mathematical English and mathematical symbols. This can possibly be explained by a low level of comprehension of the examples provided. Thus, the words and symbols were not encoded with its semantic structures. It is plausible to suggest that the mental representation for the majority was *surface component* with only low level of *textbase*.

The findings resonates with that of Österholm (2006) that indicates that mathematical symbols had a larger influence on reading comprehension of both high and low prior knowledge students.

The mathematical text reading abilities of the majority of participants in this study is not at the desired level. It is our contention that mathematical text comprehension abilities should be explicitly developed. This is particularly necessary since students will be called upon to engage in more self-study with the pervasive use of electronic media for distributing mathematical knowledge.

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CROSSING THE LIMINAL SPACE: STUDENTS' UNDERSTANDING OF CONFIDENCE INTERVALS

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ABSTRACT

This paper describes responses first-year university students gave to a survey asking them about their understanding of Confidence Intervals, after their first introduction to the topic in a statistics course at two tertiary institutions in Australia. Their responses indicate that whereas the participants could explain that Confidence Intervals were used to estimate the value of a population mean, in general they were confused about the theory that enabled probabilities to be assigned to such intervals. Some participants were also confused about the terminology used in inferential statistics. The results of this study suggest that instructors should not underestimate how difficult students find this topic. We must be careful to include questions that require explanations of understanding, not just numerical answers that can be learnt by rote. We conclude that more research is needed into student understanding of confidence intervals, including the suggestion from this study that Confidence Intervals might be considered as a Threshold Concept in advancing students' statistical thinking.

INTRODUCTION

It will be apparent to most, if not all, educators that there are some concepts with which their students have particular difficulty. These same concepts may have been difficult for the educators when they were themselves students. In the area of inferential statistics Confidence Intervals (CIs) form one of these difficult concepts and it is because of our own experience, both as students and educators, that the authors carried out the study described in this paper.

There is however the suggestion that some concepts are more than just 'difficult' in the traditional sense. In 2003 Meyer and Land (2003) introduced the idea of *Threshold Concepts,* concepts they postulate are more than just important for learning of the discipline, but concepts that lead to seeing the subject in new way. They describe a *Threshold Concept* as:

> [A concept that] can be considered as akin to a portal, opening up a new and previously inaccessible way of thinking about something. It represents a transformed way of understanding, or interpreting, or viewing something without which the learner cannot progress…there may thus be a transformed internal view of subject matter, subject landscape, or even world view…with the transition to understanding proving troublesome. (Meyer & Land, 2003, p.1)

Threshold concepts are often examples of *troublesome knowledge* (Perkins, 1999), that is, knowledge that is conceptually difficult, counter-intuitive or alien. Meyer and Land are not mathematicians and their work in general focuses on understanding, describing and

scaffolding student difficulties in a general cross-disciplinary sense. However, they do consider some examples from mathematics (limits) and inferential statistics (sampling distributions) which they postulate have the characteristics of threshold concepts. Limits were initially identified as potential threshold concepts based on the work of Tall (1992), and recent studies have provided further evidence to support this claim (Oates, Reaburn, Brideson, & Dharmasada, 2017; 2018). With respect to inferential statistics, Meyer and Land (2003) cite the work by Kennedy (1998,) who describes a situation familiar to many instructors of statistics: many students, at the completion of their first statistics course, will know how to do "mechanical things such as compute a sample variance, run a regression, and test a hypothesis, but they do not have a feel for the 'big picture'" (p. 142).

What makes some knowledge "troublesome"? Perkins (1999) suggests that it is because some knowledge is complex and may appear paradoxical and inconsistent to the student. The language used by the discipline may also add to a student's confusion. Meyer and Land (2003) state that students who are in the process of coming to understand troublesome knowledge may be left in a state of "liminality" (p. 13), where students "shift between previous and new understandings of a concept" (Nicola-Richmond, Pépin, Larkin & Taylor, 2018, p. 102). If students cannot cross this "liminal space" (Nicola-Richmond et al., p. 102) the students resort to using *mimicry* (Meyer & Land, p. 13) – using procedural knowledge to cover up their lack of understanding. Crossing this space involves a *transformation* in students' thinking as they *integrate* what they consider to be unrelated pieces of information. Meyer and Land then propose that the final product is *irreversible*, as the students will never see the concept in the old way again. One question this study raises is whether CIs might also be considered threshold concepts in progressing students' statistical thinking?

CONFIDENCE INTERVALS

Confidence intervals (CIs) estimate a population parameter based on the evidence provided by a sample. They give a range in which it is considered likely the value of the population parameter will lie with a measure of this likelihood. Although point estimates of population parameters are useful, they are unable to give an idea of how precise an estimate is. In contrast, CIs indicate both the direction of an effect and an idea of its size, and they also have the advantage of being in the same units as the data being measured (du Prel, Hommel, Röhrig, Blettner, 2009). It is reasons such as these that have led to calls for confidence intervals to be used routinely in addition to, or even instead of, *p*-values (e.g. Cumming & Fidler, 2005), and why some journals now require effect sizes in their papers (Cumming, 2012).

To understand the process of calculating a CI, students need to integrate several pieces of knowledge. They will need to know the characteristics of normal distributions. They also need to understand the link between a population and the samples taken from it. If it were possible to take a large number of randomly selected, fixed-sized samples from a population, then the means for these samples would form a normal distribution with a mean equal to that of the original population mean. If the sample size is large enough, this normal distribution will result even if the original population was not normally distributed. These properties form the basis for the *Central Limit Theorem (CLT)*. The standard deviation of this new population is narrower than that of the original population and is equal to the standard deviation of the original population divided by the square root of the sample size; this is called the *standard error of the mean*. Students then need to appreciate that because sample means belong to a normal distribution this distribution will have the same characteristics of any other normal distribution. This is the basis for the calculation of the 95% confidence interval for the mean. Approximately 95% of the possible sample means will be found within two standard errors of the population

mean. The consequence is that if we were to calculate an interval that starts at two standard errors below the sample mean and ends at two standard errors above the sample mean, this interval will include the value of the population mean 95% of the time we carry out this process. Unfortunately, we never know if our estimate is one that does include the value of the population mean.

From this brief description it should not be surprising that the concept of a confidence interval, which integrates several pieces of knowledge, should be difficult for students to understand. Whereas the idea that the value of a sample mean is likely to be near the value the population mean is intuitive, the theory that is introduced to assign probabilities complicates the process. This is not helped by the poor intuition that students may have about probability in general, for example the common misconception of the 'gamblers fallacy' and the difficulty of describing probability at all. This is made worse by students' previous experience in mathematics where much of their time would have been involved in trying to find the exact *correct* answer.

There is extensive research to suggest that confidence intervals are indeed difficult and this, with the complexity of the concept, suggests that CIs are a form of troublesome knowledge. Studies of students in undergraduate statistics courses (delMas, Garfield, Ooms and Chance (2007; Chance and McGaughey, 2014) have found that students are likely to believe that CIs give the percentage of data values that lie between the two limits, or that CIs contain the stated percentage of all possible sample means. This agrees with the findings of Reaburn (2014), who also found that there was confusion between the terms standard deviation and standard error. Errors in interpretation are not only confined to students. In study of researchers, Cumming (2006) found many held the misconception that "if the sampling were to be repeated, X% of the replicate sample would be within the original X% CI (Cumming, 2006, p.2).

Because of the complexity of the concept it should not be surprising that students use *mimicry* to cover up their lack of understanding; this can go undetected by the instructor unless questions are asked that require explanation of the students' thinking. For example, a student may be able to carry out the calculations and report a result such as "the 95% confidence interval for the mean weight of the students is between 45 kg and 46 kg", with little to no understanding of what this statement actually means.

In this study we asked two questions:

1. What are student understandings of confidence intervals after their first exposure to the concept?
2. To what extent have these students crossed a 'threshold' in their statistical thinking?

THE STUDY

The participants of this study were volunteers from tertiary introductory statistics units at two medium sized universities in Australia. Two weeks after confidence intervals were introduced into the units, emails were sent out to the students requesting them to take part in an anonymous survey with the additional option of undertaking an interview. Out of a possible 300 students, we received 14 completed surveys and to date no interviews have been carried out. The questions that were asked on this survey are described in Figure 1. All the questions were informed by the literature, but have not as yet been established as a validated scale, so we acknowledge any findings may be open to interpretation.

Analysis

Student answers were analysed using error analysis (Radatz, 1980). This involves a careful reading of student answers not only to identify them as correct or incorrect but to identify errors in student thinking. Originally error analysis was proposed as a tool for instructors to identify concepts that need to be retaught and to alert instructors to where teaching methods need to be altered (Borasi, 1987), but is also useful for researchers who also wish to examine student thinking.

Part A

Scenario

It has been claimed that the proportion of overweight school students in Australia is increasing. A research team is examining, among other variables, the weight of Year 8 students in your state.

Part of this process involved taking a random sample of Year 8 boys across your state and measuring the weights of these students. The mean weight of this sample was calculated.

Q1: Looking at Scenario A, what was the purpose of calculating the sample mean?

Q2: For Scenario A, the researchers then calculated the 95% Confidence Interval for their data. What is the purpose of this calculation?

Q3: For Scenario A, the result of the calculation of the Confidence Interval was reported as: The 95% Confidence Interval for the mean weight of boys in Year 8 in this state is between 41kg and 49kg. Write in a sentence or two how you would explain this statement to a friend who has not studied statistics.

Q4: In the previous question, what does the "95%" refer to?

Part B

Researchers in South Australia also took a sample of Year 8 girls and measured the sample mean for their heights. Coincidentally, their calculated sample mean is the same as that for the heights of the girls in your state. However, their sample size is larger than for the study in your state. The 95% confidence interval for the mean height in one of these two states (A) is between 155cm to 159cm. The 95% Confidence interval for the other of these two states (B) is between 156cm to 158cm.

Q6: Which one of these confidence intervals comes from South Australia? Explain your answer.

Part C

Q1: Write in your own words the meaning of the Central Limit Theorem. If you cannot do this, say "I don't know" and continue to the next question.

Q2: Explain the difference between the standard deviation and the standard error. If you cannot do this, say "I don't know" and continue to the next question.

Q3: The Central Limit Theorem states that if the sample size is large enough, the sample mean calculated from this sample belongs to a normal distribution regardless of the distribution of the original population. Tick next to all the boxes for the following statements if they are true:

a) If the sample size remains fixed and is large enough, approximately 95% of all the possible sample means will be within two standard deviations of the population mean.
b) If the sample size remains fixed and is large enough, approximately 95% of all the possible sample means will be within two standard errors of the population mean.
c) Sample means form a normal distribution only if the original population had a normal distribution.
d) The standard error of the mean refers to the mistakes we might make when calculating confidence intervals.
e) The standard error of the mean is the standard deviation of the distribution of the sample means.

Figure 1: The survey questions

RESULTS

Because the survey was anonymous, it is not possible to identify the participants. The participants were approximately equally distributed between males and females and between the two participating universities. In the presentation of the results, the participants have been identified as Participants A to M so that any consistencies and inconsistencies in their responses can be followed through the discussion

ANSWERS TO PART A – SCENARIO

Q: What is the purpose of calculating the sample mean?

All participants except one explained that the sample mean was to estimate the population mean with little to no further explanation. One participant, however, identified that an inferential process was being used, “So that an inference can be made in regards to the mean of the population.” (Participant K)

Q: For Scenario A, the researchers then calculated the 95% Confidence Interval for their data. What is the purpose of this calculation?

In general, the participants stated that the CI gave an interval estimate to varying degrees of detail. For example: “To get an interval estimate of the population mean of Year 8 boys” (Participant A). Other participants mentioned the level of confidence: “Provides us with an interval in which we can be 95% sure that the true population mean lies in” (Participant D). Participant I showed some understanding of the process, but the answer indicates a misunderstanding of the terminology: “This calculation tells us that the true mean of the whole population is likely to exist within a set of parameters (with a 95% confidence rate in its existence within the boundary)”.

In contrast, Participant K, demonstrated a possible more fundamental lack of understanding that was inconsistent with their answer to the previous question: “To determine a reasonable range for the population mean weights that can be expected based on the sample.” Here it would have been useful to be able to follow up the participants. Either the participant did not believe that population mean to be a single value, or the plural use of the word “weight” was an error.

Q: For Scenario A, the result of the calculation of the Confidence Interval was reported as: The 95% Confidence Interval for the mean weight of boys in Year 8 in this state is between 41kg and 49kg. Write in a sentence or two how you would explain this statement to a friend who has not studied statistics Meaning of CIs, to someone who has not studied statistics.

Most of the participants gave answers that accurately describe the situation but very few could give an answer without any of the technical language. For example: “I am 95% sure that the actual mean is between 41kg and 49kg, given the sample that I have taken”. (Participant H). An exception was Participant A who stated: “The mean weight of Year 8 boys is most likely between 41kg and 49kg.”

Again, participant K demonstrated a lack of understanding, which was consistent with the previous answer suggesting that CIs are about the ranges where individuals are likely to be: “In 95% of cases, the weight of the boys is expected to be between 41 and 49 kg”,

Q: What does the 95% refer to?

Four of the participants gave the answer “confidence” or “confidence intervals” (participants B, E, G, and N). From these answers alone it was not possible to determine what, if any, understanding is present. Participant L’s answer suggested a deeper level of understanding:

"How confident statisticians are that the mean weight is inside the specified interval (41 and 49kgs)".

This was in contrast to other answers where the level of understanding was unclear, for example Participant N: "The amount of certainty we have about our mean range, its 95%." Participant D, who, up to now had been notable for concise and accurate answers, gave an answer that not only indicated a lack of understanding of how confidence intervals work but was unclear in meaning: "Our level of confidence – i.e. the middle range of 95% of the weights."

ANSWERS TO PART B

Which one of these confidence intervals comes from South Australia? Explain your answer.

Nine of the participants stated that the CI from South Australia had the narrower interval, even though there was some misuse of terminology, replacing "narrower" with "smaller". For example: "Because South Australia had a larger sample size and so should have a smaller confidence interval". (Participant L)

Participant D gave a detailed explanation. Whereas this answer indicated understanding of the process, the answer also indicated the tendency of students to regard samples as being "accurate" or "inaccurate":

> The larger the sample size, the more data you can collect, thus resulting in a more accurate representation of the population. For these reasons you can be more confident that your data are accurate, and as a consequence, the interval in your CI becomes smaller.

The other five participants suggested that the wider confidence interval would be the one that came from South Australia, indicating that as there were more data the interval would be wider. Participant M is an example: "A, as there was a larger sample size there is likely to be more results included in a confidence interval."

It was interesting to note that no participant referred to the formula for the standard error, $s/\sqrt{n}$, to demonstrate that as n increases, the value of the expression decreases.

ANSWERS TO PART C

Write in your own words the meaning of the Central Limit Theorem. If you cannot do this, say "I don't know" and continue to the next question.

Six of the participants indicated that they did not know. Participant H indicated understanding of the principle but appeared to suggest that it is each single mean that follows a normal distribution: "If our sample size is large, then the mean of each possible sample, of the given size, should follow a normal distribution." Participant I was clear that it is the sample means that follow the normal distribution: "The CLT states that the sampling distribution of the sample means follows a normal distribution as the size of the sample increases (even if the population distribution is not normal)."

In contrast, Participant G was confused about what follows a normal distribution, but had appreciated that there was something about a sample size of 30 that was important: "The central limit theorum [sic] suggests that equal to or more than 30 pieces of data collected will follow an approximately normal distribution."

Explain the difference between the standard deviation and the standard error.
Five participants indicated that they did not know the answer; four of these had answered "I don't know" to the previous question.

Participant H described the standard error by its formula alone but made an error in identifying "*n*": "Standard error is equal to the standard deviation over the square root of the number of samples." Participant B gave a correct answer but did not give enough information for the reader to determine if his/her knowledge was accurate: "One is about the population and one is about the summary statistic." Participant A gave more information: "The first refers to the sd for the population (or sample) an [sic] the second refers to the sampling distribution of the mean."

Participant G focused much more on the idea of deviation: "The standard deviation is how much the deviation exists from the mean value versus the standard error measuring how far away from the population mean the sample mean is."

In contrast, Participant C suggested that the standard deviation of a population is a known value and whereas the participant knew that standard errors are connected to samples, the participant did not describe how: "Standard deviation is a known value of the entire population, standard error is from a sample".

Participant M picked up on the idea that a confidence interval may not include the value of the population mean but indicated a lack of understanding of the process. "Standard deviation is the incraments [sic] from the mean, whereas the standard error is similar to confidence interval - set of values for buffereing [sic] in case the mean is wrong."

CLT –WHAT STATEMENTS ARE TRUE?

The final question on the survey gave the participants a list of statements with which they were asked to agree. The number of participants who agreed with each statement are listed in Table 1.

Table 1: Number of participants who agreed with each statement about the Central Limit Theorem

Statement	Number of participants who agreed
a. If the sample size remains fixed and is large enough, approximately 95% of all the possible sample means will be within two standard deviations of the population mean.	9*
b. If the sample size remains fixed and is large enough, approximately 95% of all the possible sample means will be within two standard errors of the population mean.	7*
c. Sample means form a normal distribution only if the original population had a normal distribution.	4
d. The standard error of the mean refers to the mistakes we might make when calculating confidence intervals.	3
e. The standard error of the mean is the standard deviation of the distribution of the sample means.	10

* Two participants agreed to both statements

SUMMARY AND DISCUSSION

There are too few participants in this survey to be representative of their respective student groups as a whole. Nevertheless, their answers gave indications of how students may be confused by CIs and the theory behind their calculations.

The questions in the survey started with questions that were easy to answer and became more difficult as the survey progressed. As the survey progressed the students tended to become less precise in their use of the mathematical terminology, and the number of demonstrated errors in understanding increased. As such, this survey gives an indication of the liminal space that students need to traverse before full understanding of the principles behind CIs is achieved.

Judging from answers to the survey, two of the participants could be considered to have crossed the liminal space in that they made no errors. Three of the participants made minor errors only (for example agreeing with both statements (b) and (c) in Table 1). It is likely that these participants have crossed the liminal space but without further elucidation, it cannot be certain. The other nine participants made major errors in the answers to the questions and gave only minimal evidence that they understood the process of calculating a CI.

This survey also confirms two aspects of troublesome knowledge, the need to integrate several pieces of knowledge, and how the use of language can be a source of troublesomeness. Each community of practice has its own discourse which will be less familiar to those who are new to the community (Wenger, 2000). The answers to this survey indicate that some participants had problems with the term “standard error of the mean” and did not fully understand the meaning of the term “95% confident”. Further questions were needed in regard to the latter point; it was possible that participants were using mimicry in their answers, but this could not be determined by the survey as it now stands. It is planned that further surveys will ask for further elucidation about the consequences of the Central Limit Theorem. That is, that sample means have the same characteristics as any other normal distribution.

The results of this survey have implications for educators. They highlight the possibility of mimicry in student answers, that unless questions are asked that prompt students to explain their thinking, they may produce answers that are technically correct but with little underlying understanding. In addition, they indicate that educators should not underestimate how difficult students may find CIs to be. Both CIs and *p*-values are necessary if researchers and others are to make good judgements about the results of research. Finally, many students did show evidence of dwelling within a ‘liminal space”, which in turn suggests that confidence intervals may indeed be a threshold concept for students in progressing their statistical thinking, but more research is needed to fully establish this hypothesis.

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STUDENT-LECTURER PARTNERSHIPS IN UNDERGRADUATE MATHEMATICS QUESTION DESIGN

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KEYWORDS: undergraduate mathematics, student partnership, assessment, blended learning

ABSTRACT

Our exploratory study examines the benefits of student-lecturer partnerships in course design at a university level. This project is situated within a larger design research project investigating a blended learning intervention in a stage II service mathematics course. A mathematics *Lecturer* and a *Student* entering postgraduate mathematics study both independently composed questions for online pre-lecture quizzes related to the calculus section of the course. Utilising Schoenfeld's (2010) theory of goal-oriented decision making, we unpack the complexity of the design process by examining the three fundamental factors: Resources, Orientations and Goals (R/O/G). Using this theoretical lens, we interpret the results of the study by accounting for the differences between the *Lecturer*'s and the *Student*'s quiz questions through an analysis of their R/O/Gs. Our findings suggest that interpreting the differences in question construction provides insight into student learning of mathematics from both student and lecturer perspectives as well as how students engage with blended learning resources. The systematic approach that we describe, utilising the R/O/G framework for an analysis of the design process, can be used for developing and refining the assessment by other student-lecturer partnerships in other educational settings.

INTRODUCTION

'We know what we are, but know not what we may be.'
– William Shakespeare

Opportunities for teaching innovations and technological advancements are rapidly changing how university mathematics courses are taught. Handouts become slides. Assessments are submitted virtually. Calculations become code. All while new ways of assessing students' learning of mathematics are continuously being developed. In this new age, there are advantages to students and lecturers becoming both educators and learners.

Our project involved a second-year general mathematics course *Lecturer* inviting a *Student* to write questions for the course. The project aimed to explore the types of questions deemed conducive to the learning of mathematics by a teacher compared to a student. An important feature of the setting for this explorative study is a blended learning environment. Blended learning, the integration of face-to-face and online instruction, is now widely adopted as the 'new normal' in course delivery across tertiary institutions. In mathematics classes, this new modality of instruction is commonly seen at all levels, yet the extent to which it is effective raises important questions about its pedagogical merit and the responsibility of instructors with its evaluation (for reviews, see Borba, Askar, Engelbrecht, Gadanidis, Llinares, & Aguilar, 2016).

Background setting

In our study, the *Lecturer* comes from a pure mathematics background and has been lecturing both undergraduate and postgraduate university courses for twelve years. She was a part of a blended learning initiative for a second-year course at the University of Auckland and began including short online quizzes between lectures in 2016 (Evans, Kensington-Miller, & Novak, 2019).

The quizzes are worth 7% of the final grade and require the students to answer two multiple-choice questions online before the next lecture, assessing the content of the previous lecture. The students are allowed two attempts at completing each quiz and their highest score is recorded. Each question is randomly selected from a bank of questions containing 2-3 versions (for example, different numerical values). The time limit is set for 30 minutes to provide enough time for students to revise the material while taking each quiz.

The impact of the incorporation of quizzes into this course was previously researched and reported. The findings suggest that this relatively small change in course instruction can improve efficiency and effectiveness of educational exchange. Researchers analysed data from multiple sources and provided evidence that this intervention resulted in a sustained increase in frequency of students' engagement with mathematics, increased attendance of lectures and improved grades (Evans, Kensington-Miller, & Novak, 2019). Our study was designed in this setting, taking into account and building on the findings from the previous research.

The *Student* involved in the project was entering postgraduate study in Mathematics with a focus in Mathematics Education after completing an undergraduate degree majoring in Pure Mathematics and English. She had taken the course herself with a different lecturer, but prior to the incorporation of blended learning and quizzes.

Theoretical background

A theoretical concept relevant to our research is the notion of partnership between the *Student* and the *Lecturer*. In the UK Higher Education Academy's framework for partnership in learning and teaching in higher education, it is stated that in these partnerships, 'staff experience renewed engagement with and transformed thinking about their practice, and a deeper understanding of contributions to an academic community' (HEA, 2014, p. 2). Involving tertiary students in the instructional design is well established in higher education but often comes with challenges and concerns for both the academic staff and students (Money, Dinning, Nixon, Walsh, & Magill, 2016). Some recent research has been carried out and provided insights into successful practices in forming student partnerships in tertiary education (Healey, Flint, & Harrington, 2014), but it has not been specific to mathematics.

The Catalyst Project (Jaworski, Treffert-Thomas, & Hewitt, 2018) at Loughborough University is a recent research endeavour related to exploring the process and results of partnerships between mathematics students and educators. The university runs a one-year course for Foundation Students (FSs) who do not hold the correct qualifications to start the degree they are intending. Student Partners (SPs), who were former FSs, partnered with lecturers to design computer-based tasks for FSs. The team investigated how their SPs engaged with designing the tasks and how the FSs interacted with the task. The analysis of The Catalyst Project is still in its early stages but is providing valuable insight into how FSs learn and may prove beneficial to the SPs. Our project, like The Catalyst Project, explores the partnership between educators and learners but differs in its design, data collection, research questions, and overall aims.

We examine the differences in the quiz questions written by the *Lecturer* and the *Student* utilising the theory of decision-making developed by Alan Schoenfeld in 'How We Think' (Schoenfeld, 2010). According to this theory, an inspection of a teacher's decision-making process during a teaching-learning interaction can be conducted through the examination of three fundamental factors:

- Teacher *Resources*—primarily knowledge, but also including classroom resources such as technological gadgets (tablets, mobile phones, clickers, etc.);
- Teacher *Orientations* to the domain—in essence, what they consider important which is shaped by their beliefs and attitudes towards mathematics;
- Teacher *Goals* for the teaching interaction—in essence, what they are trying to achieve in a particular teaching-learning event (Schoenfeld, Thomas, & Barton, 2016).

There is previous research done at the University of Auckland in using Schoenfeld's (2010) theory of decision-making as a tool for lecturers' professional development (Oates & Evans, 2017; Paterson & Evans, 2013; Schoenfeld et al., 2016). One of the research questions from Schoenfeld et al. (2016) was, 'How can Schoenfeld's resources, orientation and goals (R/O/G) framework be adapted to support lecturer professional development?' Participating lecturers were asked to engage with their R/O/Gs. These were then reflected on in relation to a short video excerpt from a lecture and used to catalyse discussion around lecturer in-the-moment teaching decisions. They concluded that the adapted R/O/G framework was effective in stimulating and centering their discussions.

We extend on this application of the R/O/G framework by having the *Student* and the *Lecturer* maintain an active awareness of their R/O/Gs during the construction of their questions. Using this theoretical lens, we interpret the results of the study by accounting for these differences through an analysis of their R/O/Gs. We hypothesised that gaining insight into student R/O/Gs (student perspectives) can be beneficial to the lecturer in course development.

Our main research questions were:

- How can analysing questions devised by a student for assessing learning in a course compared with questions devised by a lecturer support the development of courses featuring blended learning?
- How can Schoenfeld's R/O/G framework be used to account for perceived differences between what a student finds valuable to student learning and what a lecturer finds valuable to student learning in a course featuring blended learning?

METHOD

Methodological framework

This research was conducted as part of a larger design research project investigating the impact of online quizzes between lectures in a university mathematics course. Design research differs from traditional experimental research designs in that initial concepts for learning are constructed but may be adjusted during the testing process. In education, conducting purely experimental research often results in an inability to generalize, as natural learning environments contain numerous variables that are impossible to replicate exactly. Design research aims to advise, 'namely to give theoretical insights into how particular ways of teaching and learning can be promoted' (Bakker, 2018, p. 8) through interactive and iterative cycles of development and research, as characterized in Figure 1 (adapted from Goodchild, 2014).

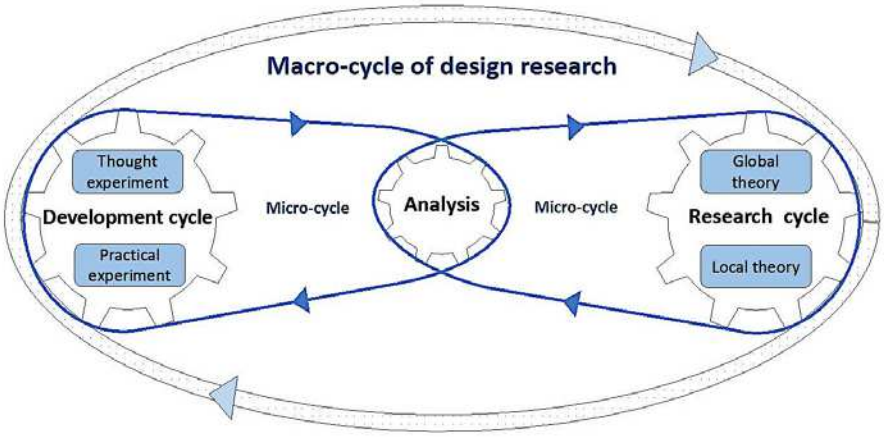


Figure 1: Design research: Cogwheels in motion, chain-driven by design principles

As mentioned in the introduction, the first macro-cycle of the design research project was completed during 2016-2018 with results reported in Evans et al. (2019). This project represents a *Research micro-cycle* (see Fig. 1 on the right) of this larger design research project, building on the findings from the first macro-cycle. The findings from this *Research micro-cycle* are used to inform future *Development* and *Research* cycles of the project. The knowledge yielded by design research is commonly summarized as design principles, which change and develop through the cycles. Design principles are typically summarized in the following form.

- If you want to design intervention X [for purpose/function Y in context Z]
- then you are best advised to give that intervention the characteristics C_1, C_2,…, C_m [substantive emphasis]
- and to do that via procedures P_1, P_2, …, P_n [methodological emphasis]
- because of theoretical arguments T_1, T_2,…, T_p
- and empirical arguments E_1, E_2,…, E_q (Van den Akker, 2013, p. 67)

We will offer the design principle that resulted from this study in our findings.

Study setting

As part of a summer research project, the *Lecturer* assigned the *Student* to research literature that discusses the use of Schoenfeld's R/O/G framework in mathematics education research. The *Student* then wrote two multiple-choice questions for each of the ten lecture topics in the Calculus section of the course. If the questions were found to be valuable, then they would be included in future online quizzes for the course. An important difference of method between our research and that of The Catalyst Project (Jaworski et al., 2018) is the blinded process we engaged in developing the questions in contrast to the gradual collaborative process where SPs were given feedback as they progressed. The only instruction given to the *Student* was to consider the R/O/G framework when writing the questions in order to analyse the decision-making process later. The *Lecturer* and the *Student* did not share their R/O/Gs with each other and did not discuss the content of the course, with the intention of avoiding an influence on the question design process. The *Student* was provided with access to all the course materials with the exception of previous quizzes.

The *Lecturer*'s questions in this analysis were used in the second semester of 2017 in Mathematics XXXX course – a large service stage II course with 450 students enrolled. She described her method of writing questions to be quick and direct. She comments, 'I was true to my R/O/G the whole way – just two main learning outcomes from the previous lecture only.'

The *Student* wrote the questions over a period of five weeks and was in a position to spend significant time thinking about the types of questions she wanted to ask, as well as research, draft, and review them. She kept detailed notes on how each question related to her R/O/G. Once completed, the *Lecturer* and the *Student* came together to compare their quizzes and consider the resulting implications, with their R/O/Gs as a foundation for discussion.

QUESTIONS OVERVIEW

The questions written by the *Lecturer* were similar to the questions in the coursebook, with a primary focus on students successfully reproducing the method taught in class. In practice, these questions had a very high success rate for students, with the large majority answering correctly and well within the 30-minute time limit. The three fundamental types of questions asked by the *Lecturer* aimed for students to:

- practice the method;
- recall definitions with correct mathematical notation;
- recall theorems/claims.

Three types of questions emerged for the *Student*. These can be distinguished through the goals for students in the course to:

- practice the method;
- build intuition and understanding through the use of visualisation;
- build intuition and understanding through the use of non-examples.

Below we compare questions on the same topic and share noteworthy examples.

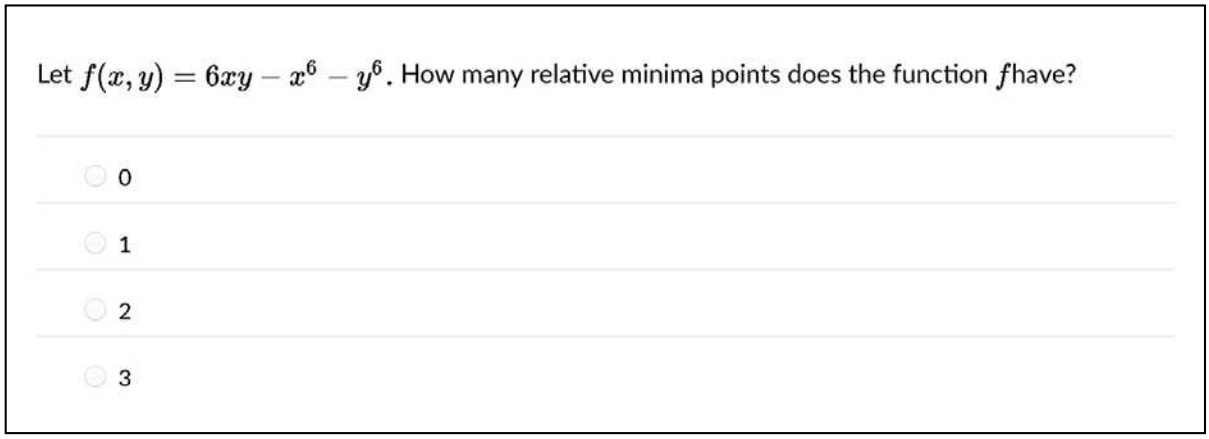


Figure 2: Lecturer-written question on optimisation

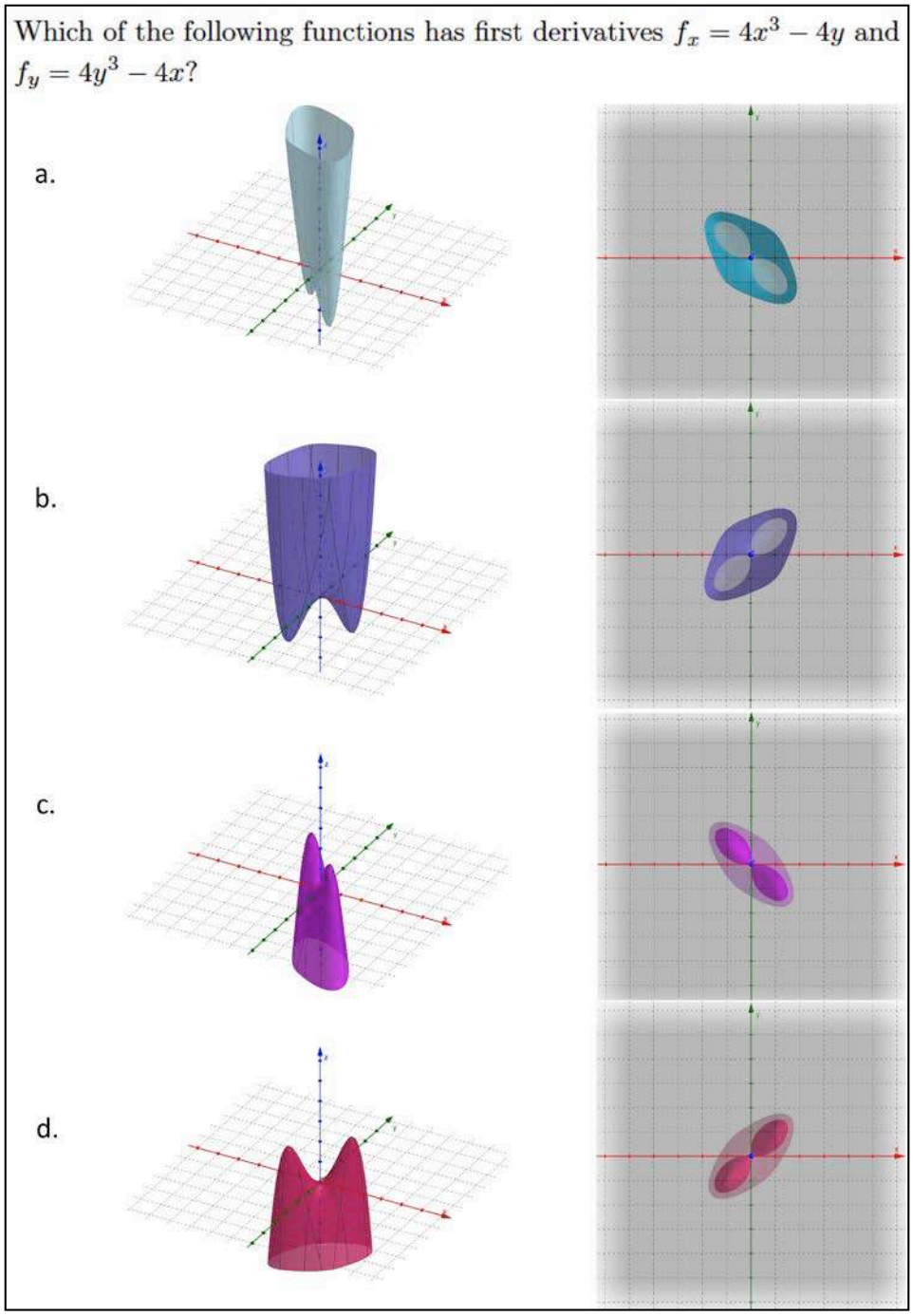


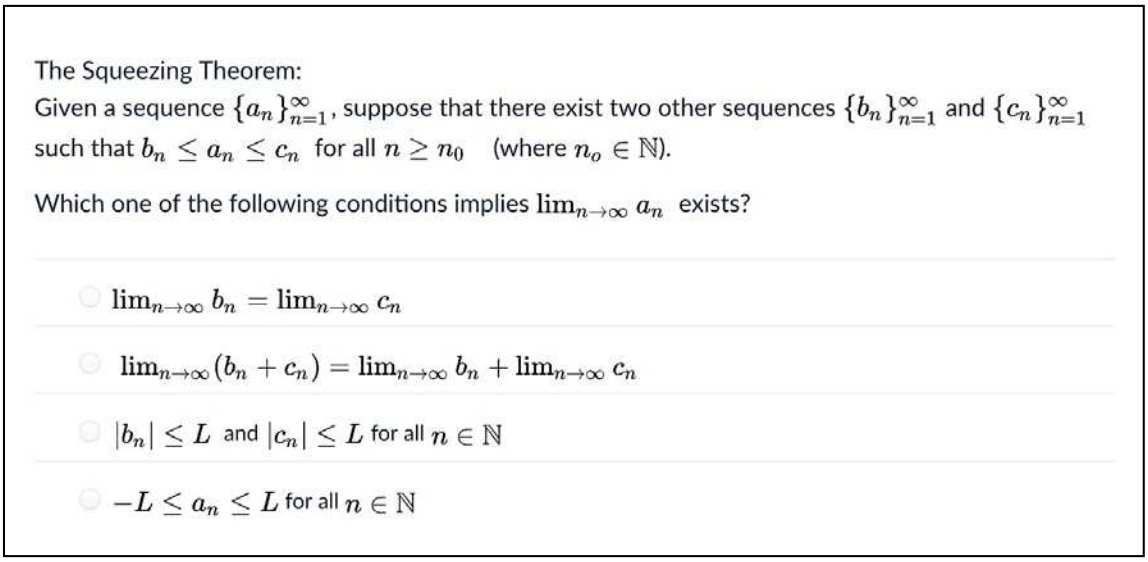
Figure 3: Student-written question on optimisation

Both questions on optimisation (Figures 2 and 3) have the intention of getting students to identify critical points, though the *Student*'s question is not as transparent. The *Student* comments, 'This question extends the student thinking from simply reproducing a method. Visualising assists in their understanding of the concept and helps create connections in the mathematics.' The *Student* explores using visual representations in several questions, while the *Lecturer* does not use any visuals. The *Lecturer* comments:

> The reason I did not use any visuals is because, coming from pure maths background, neither have I possessed sufficient technological capability, nor had previous

experience in the use of technological resources that could be easily incorporated into our new Learning Management System (Canvas), which was rolled out at the University in early 2016, when I first wrote the questions. After I wrote them, other requirements of my busy academic life took over, so it was never a priority to revisit the quizzes or upskill myself and find out about new resources available for integration with Canvas.

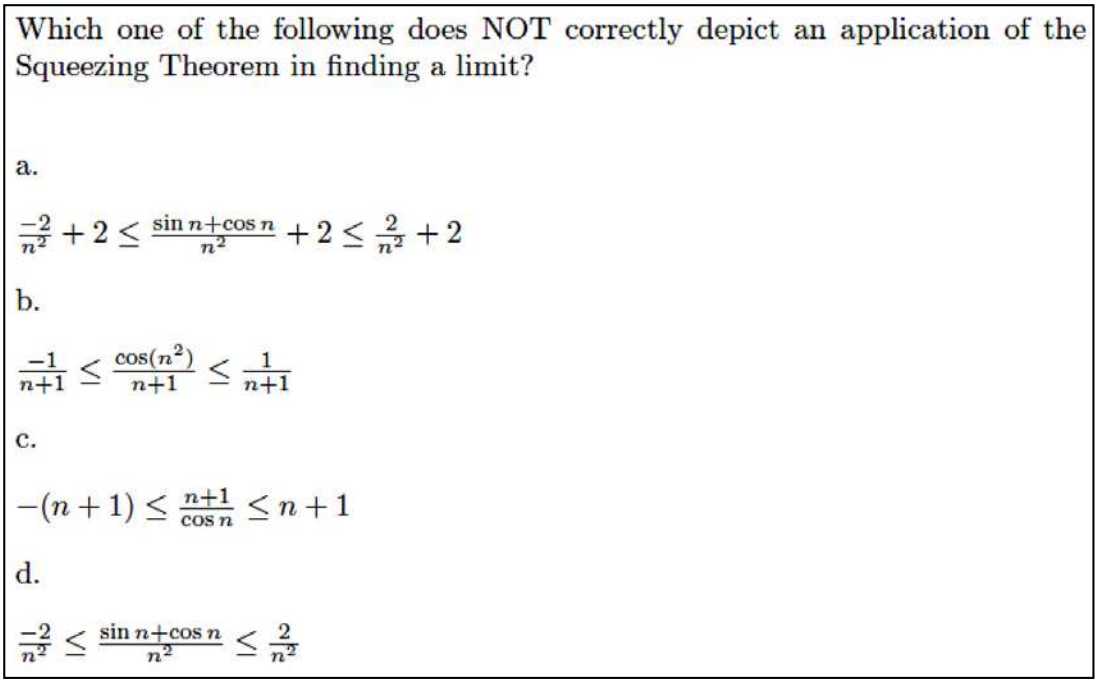
The lack of time and incentives to develop familiarity with technologs of teaching and learning. Lecturers' background and their academic environment shape their R/O/Gs in a profound way. We present the detailed analysis of the data through the R/O/G lens in the next section.


The Squeezing Theorem:
Given a sequence $\{a_n\}_{n=1}^{\infty}$, suppose that there exist two other sequences $\{b_n\}_{n=1}^{\infty}$ and $\{c_n\}_{n=1}^{\infty}$ such that $b_n \leq a_n \leq c_n$ for all $n \geq n_0$ (where $n_o \in \mathbb{N}$).

Which one of the following conditions implies $\lim_{n\to\infty} a_n$ exists?

- $\lim_{n\to\infty} b_n = \lim_{n\to\infty} c_n$
- $\lim_{n\to\infty}(b_n + c_n) = \lim_{n\to\infty} b_n + \lim_{n\to\infty} c_n$
- $|b_n| \leq L$ and $|c_n| \leq L$ for all $n \in \mathbb{N}$
- $-L \leq a_n \leq L$ for all $n \in \mathbb{N}$

Figure 4: Lecturer-written question on the Squeezing Theorem


Which one of the following does NOT correctly depict an application of the Squeezing Theorem in finding a limit?

a.

$\frac{-2}{n^2} + 2 \leq \frac{\sin n + \cos n}{n^2} + 2 \leq \frac{2}{n^2} + 2$

b.

$\frac{-1}{n+1} \leq \frac{\cos(n^2)}{n+1} \leq \frac{1}{n+1}$

c.

$-(n+1) \leq \frac{n+1}{\cos n} \leq n+1$

d.

$\frac{-2}{n^2} \leq \frac{\sin n + \cos n}{n^2} \leq \frac{2}{n^2}$

Figure 5: Student-written question on the Squeezing Theorem

Figures 4 and 5 comprise essentially the same question. The *Lecturer*'s question demands a recognition of the mathematical notation and a recall of the statement of the Squeezing Theorem, while the *Student*'s question checks if they understand what the notation means and focuses their attention on an incorrect application of the Squeezing Theorem – a non-example of a sort. While both are crucial to student progression in the course, the formal definition can be found easily both in the coursebook and online. The *Student* included no questions asking the class to reproduce the statement of the theorem. We can see a similar pattern in Figures 6 and 7. The *Student* again takes into account what information is immediately available to the students taking the quizzes.

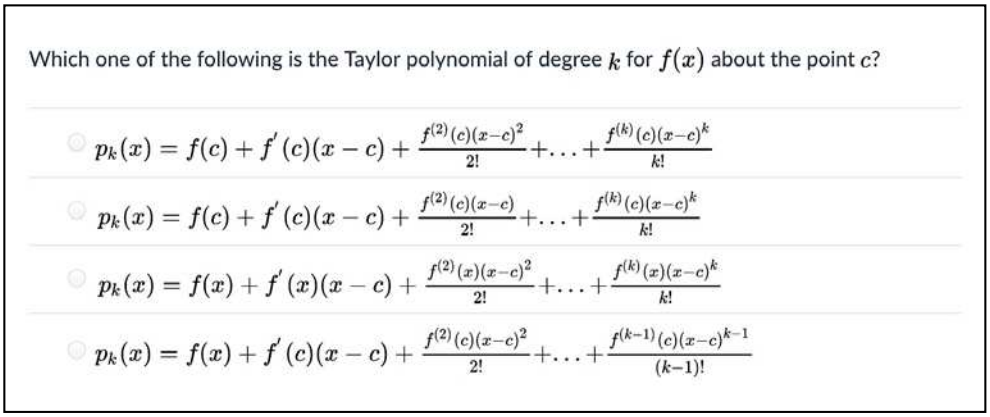


Figure 6: Lecturer-written question on Taylor polynomials/Taylor series

Which one of the following combinations of statements is true?

a. A Taylor series is a representation of a function as an infinite sum of terms, which is used to approximate the value of the function.

   Lower degree Taylor polynomials provide better approximation about a centre.

   Inside the domain of the Interval of Convergence, the Taylor series is an unsuitable approximation to the function.

b. A Taylor series is a represenation of a function as an infnite sum of terms, which is used to approximate the value of other functions.

   Lower degree Taylor polynomials provide better approximation about a centre.

   Inside the domain of the Interval of Convergence, the Taylor series is an unsuitable approximation to the function.

c. A Taylor series is a polynomial used to approximate only other polynomials.

   Higher degree Taylor polynomials provide better approximation about a centre.

   Outside the domain of the Interval of Convergence, the Taylor series is an unsuitable approximation to the function.

d. A Taylor series is a polynomial used to approximate only other functions.

   Higher degree Taylor polynomials provide better approximation about a centre.

   Outside the domain of the Interval of Convergence, the Taylor series is an unsuitable approximation to the function.

Figure 7: Student-written question on Taylor polynomials/Taylor series

The *Student* frequently employed ‘combination of statements’ questions (as seen in Figure 7), which focused on either interpreting the mathematics in more colloquial terms outside the standard definition or extending student thinking by drawing attention to non-examples. The *Student* felt this style of question prevented easily looking up solutions online and required thinking about the language of mathematics.

Similar intent – to prevent students from looking up solutions online – is observed in the questions that demanded an application of a method. In her notes for the question in Figure 8, the *Student* states, ‘avoiding just googling the solution through splitting the question into parts without final solution.’ In contrast, the *Lecturer*’s questions on integration all had the form with solutions for the final integral.

Correctly select the *parts* and the integration by parts equation to $\int \frac{\ln(3x)}{x^8}dx$.

a.

$u = \ln(3x)$ $\quad$ $du = \frac{1}{x}dx$

$v = -\frac{1}{7x^7}$ $\quad$ $dv = \frac{1}{x^8}dx$

$\int \frac{\ln(3x)}{x^8}dx = -\ln(3x)\frac{1}{7x^7} + \int \frac{1}{7x^7}\frac{1}{x}dx$

b.

$u = \ln(3x)$ $\quad$ $du = \frac{1}{x}dx$

$v = \frac{1}{7x^7}$ $\quad$ $dv = \frac{1}{x^8}dx$

$\int \frac{\ln(3x)}{x^8}dx = -\ln(3x)\frac{1}{7x^7} - \int \frac{1}{7x^7}\frac{1}{x}dx$

c.

$u = \ln(3x)$ $\quad$ $du = \frac{1}{3x}dx$

$v = \frac{1}{x^8}$ $\quad$ $dv = \frac{1}{7x^7}dx$

$\int \frac{\ln(3x)}{x^8}dx = \ln(3x)\frac{1}{x^8} - \int \frac{1}{x^8}\frac{1}{3x}dx$

d.

$u = \frac{1}{x^8}$ $\quad$ $du = -\frac{8}{x^7}dx$

$v = \ln(3x)$ $\quad$ $dv = \frac{1}{x}dx$

$\int \frac{\ln(3x)}{x^8}dx = \ln(3x)\frac{1}{x^8} - \int \ln(3x)\left(-\frac{8}{x^7}\right)dx$

Figure 8: Student-written question on the integration by parts

Another distinction to be noted is that the *Lecturer* included questions with real-world contexts (e.g. Figure 9) where the student included none. The *Student* revealed she felt real-world questions served little purpose in understanding the mathematics itself and were unnecessarily time-consuming for students in the context of these quizzes.

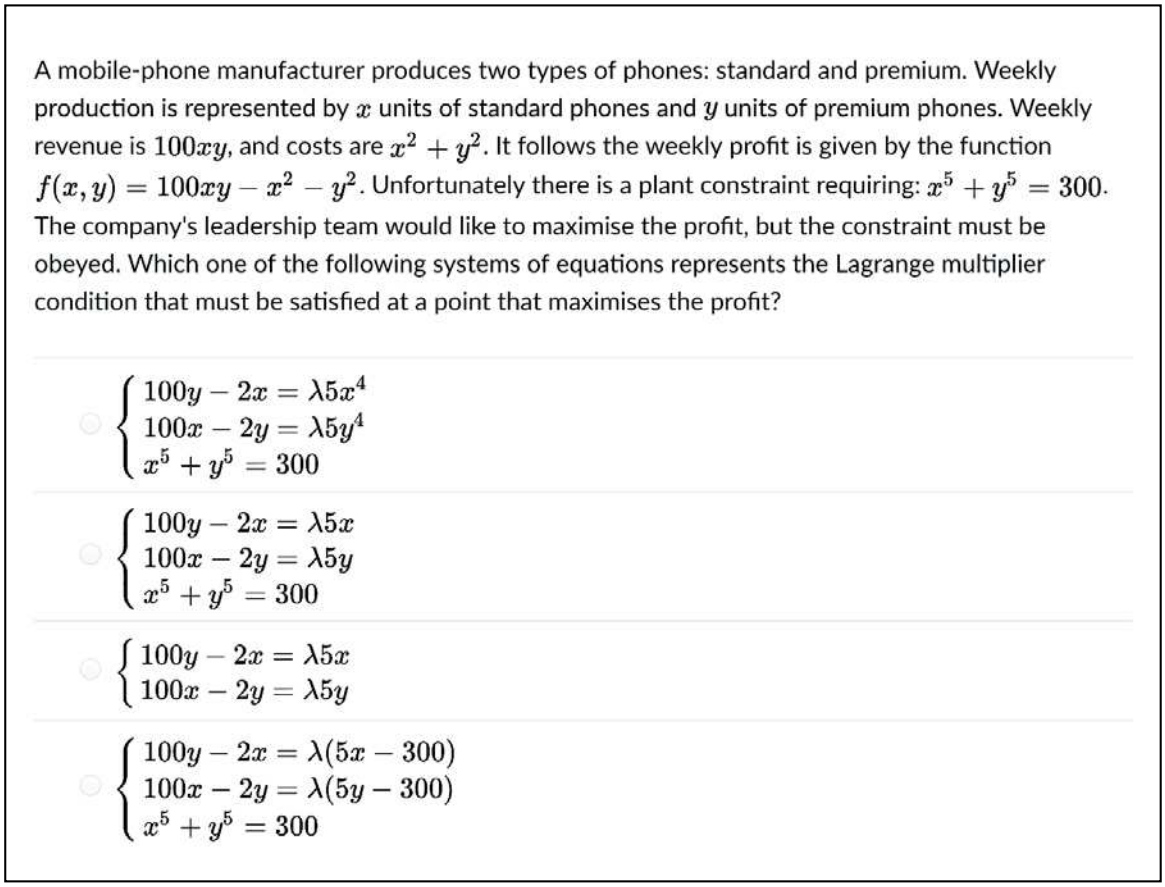


Figure 9: Lecturer-written question on Lagrange multipliers

A comparison of the key points that were chosen by the *Student* and the *Lecturer* for each lecture was made. Outlined in Table 1 are examples of the key points identified by the *Lecturer* and the *Student* as targets for assessment by the quizzes for the three lectures in which the distinction was observed.

Table 1: Examples of key points from lectures to be assessed in quizzes as determined by the *Lecturer* and the *Student* – three lectures with the most distinction

Lecture topic	Both	Lecturer	Student
Constrained Optimization	- optimising function with constraints	- interpreting real-world questions	- interpreting through graphical visualisation
Sequences: Introduction		- finding the n^{th} term formula for a sequence - recalling the Squeezing Theorem	- using the Squeezing Theorem correctly (via non-example) - finding limits of sequences/

		establishing convergence
Taylor Series	- recalling Taylor and Maclaurin polynomials formula - finding Taylor and Maclaurin polynomials	- interpreting definitions through the use of non-examples and colloquial terms - establishing convergence of power series

Table 2: Summary of the key points identified by the *Lecturer* and the *Student* from all lectures in the study

	Same key points	Shared one key point	Different key points
Number of quizzes	6	2	2

RESULTS

We can see in Table 2 that the *Lecturer* and the *Student* frequently valued the same main learning outcomes from each lecture. What was more varied was how they addressed assessment of those learning outcomes.

It is plausible that a key difference that emerged lay in the core, not necessarily conscious, belief of what will create a successful student in that course – this core belief determines the Orientations of the *Lecturer* and the *Student*. This, according to Schoenfeld's (2010) theory, in turn, orientate the formation of their Goals. It is important to note that most of the students in the course are not mathematics majors. The course content is skills driven to serve the needs of other majors like finance, economics, physics, computer science, and chemistry. There are also almost no proofs in the course. The *Lecturer*'s questions were in line with the idea that practice and repetition will create a student who can fulfil all of the requirements of the course, and thus, provide the tools needed to satisfy their major. The *Student* took the approach that a depth of intuitive understanding will develop better recall and confidence in the subject.

Explaining the differences through the ROG lens

The primary Resource of both the *Student* and the *Lecturer* was their knowledge of the material. The *Lecturer* had more depth and experience in her mathematical knowledge and knowledge of the course, and the student cohort, while the *Student* had been through the process of learning the content more recently. Essentially, we have the *Lecturer*'s insider knowledge of how a large population of students learn mathematics, and the *Student*'s insider knowledge of how an individual student learns mathematics.

Interestingly, while both the *Student* and the *Lecturer* included course materials under their Resources, the *Student* also explicitly included the Internet. The consequence of this is a key finding of our study. This awareness suggests a greater familiarity with working online and an understanding of the advantages and disadvantages of this in taking mathematics courses. From secondary school to postgraduate study, it is common to find solutions to very similar examples, if not identical assessment questions, on numerous websites. This plethora of mathematical resources can be hugely beneficial to the aspiring mathematician. However, it has the possibility to be detrimental to those students who seek to get through assessments quickly, likely restricting their quality of engagement with the content and not reinforcing their

understanding. As the blended learning intervention that was the setting for this study reported a significant improvement in course performance across all students (Evans et al., 2019), it seems probable that most students are not abusing the online and independent nature of the quizzes. However, we can still seek to further improve their design. The *Student* in describing her design process comments, 'I attempted to write questions (where possible) that are not easily "google-able"'. Concluding the project, she states:

> There are many calculators online that solve everything from series to integration by parts (step by step). I worked around this by breaking down the questions into parts, so the students were forced to understand the nuances of the method...The temptation for free marks is always high, so avoiding this issue is preferable to ensure student understanding.

In their reflective meeting, the *Lecturer* had the epiphany of the significance of generational differences in exploring blended learning. We have discovered that student partnerships can provide vital insight into using different forms of technology effectively as a medium for learning.

The uncovered difference in the core belief of the *Lecturer* and the *Student* regarding how to enable success for students enrolled in this service course explicate the distinction in their Orientations, which, in turn, dictate the formation of their Goals in the design of the quizzes. The *Student* states she wants 'to develop student intuition and a comprehensive understanding of the mathematics so they make connections and enjoy the mathematics.' Whereas, the *Lecturer* has obvious concerns about student performance on quizzes as a reflection of the course. This concern, perhaps, has shaped the format of the questions she created to match the examples that are covered in class or provided in the coursebook. The *Student* did not have the same pressure and in composing the questions, sought to promote an appreciation of the mathematics, with significantly less regard for students' expectations for quiz questions to match worked examples that have been already provided to them.

Assessment can be a double-edged sword in that, if structured correctly, can provide great incentive for student engagement, but can also mean students may prioritise correct answers over understanding if the option is there. The *Student* approached the quizzes as a further learning opportunity with, ultimately, less consideration for the assessed performance of the students taking them.

Aligned with perspective, we recognise the role of the Orientations as a motivator for the varied responses in the Goals:

Lecturer:

- To develop a bank of quizzes that will be delivered on-line preceding every lecture (to increase learners' frequency of engagement with content)
- To write questions that assess two main learning outcomes from the previous lecture only

Student:

- Write questions that promote 'aha' moments in students / Write questions that allow students to discover the relationships within the mathematics they are studying
- Keep students up to date/refreshed with the course content
- Provide an opportunity for students to practice the method

The approaches of teaching how to do mathematics (skills-based) and teaching in-depth comprehension are a well-known struggle in mathematics education. As previously

highlighted, most of the students in the course major in other subjects that they are trying to understand deeply and simply need to be able to execute the mathematics. The *Student* was a more recent learner of this level of mathematics and argued that deeply understanding the mathematics leads to more consistent results over time. Simultaneously, the *Lecturer* had insight into the types of students taking this course. In particular, the majority of the students will not be engaging with mathematics at this level again once the course is completed. The effectiveness of the different style of questions can only be gauged by trial, which will take place in a future.

CONCLUSION

Benefit to lecturer development

As a consequence of participation in the study, the *Lecturer* reported a major change in her perspective that will affect her Goals in the future design process. It was triggered by becoming cognisant of the *Student*'s Resources - in particular the Internet. The acute realisation of the significance in generational change brought to the fore the extent of use of freely available online resources by students and, most importantly, students' perception of those 'google-able' resources as being first port of call when answering quiz question. In unpacking the *Student*'s design process, the *Lecturer* paid particular attention to the *Student*'s intent to write non-'Google-able' questions and the tactics employed. Through engaging in this analysis, the *Lecturer* was able to internalise these insights. This has altered her Goals for future design processes to actively work around potentially detrimental consequences of the accessibility of ever-growing technological resources.

Benefit to student development

Similarly to The Catalyst Project (Jaworski et al., 2018), we have reviewed how the student partner engaged with the task. In our project, the *Student* reported that engaging with the content of the course on this level clarified aspects of the content and promoted a deeper understanding of the mathematics. Two features of the process stood out as particularly beneficial. The first was composing the 'combination of statements' questions, as illustrated in Figure 7. She found the process of transitioning the concepts between the mathematical notation and normal language cemented her understanding of the concepts and their corresponding applications. The other helpful characteristic of the process was attempting to predict student errors for the possible solutions. In taking time to consider where mistakes could be made, she now feels she developed skills to anticipate errors in her own mathematics.

We offer the following design principle as a result from our study. If you want to design online quizzes for a service mathematics course with an aim to enhance quality of engagement and learning, then you are advised to consider incorporating features deemed valuable by a student in designing the questions. These could be identified through a *Student* and *Lecturer* independently writing their R/O/Gs (Schoenfeld, 2010), devising questions following those R/O/Gs for the quizzes, and then examining the results through the R/O/G lens. The basis for this case is supported by the theoretical considerations on the advantages of student partnerships and by the conclusions of our exploratory study. In our analysis, we have tested a student's construction of questions for a blended learning assessment in comparison to a lecturer's and found that an adaptation of Schoenfeld's R/O/G framework accounted for the differences and allowed insights into question design.

The direct findings for this course may not prove directly transferable to another course as the usefulness of one student on one topic is limited. However, the resulting design principle is useful and transferable to other educational settings, and we suggest, particularly relevant for

courses featuring blended learning. Central to the design is the insights offered by student partnerships in course development. Our analysis yields the benefits of student partnerships in courses incorporating blended learning through student familiarity with modern technological resources in their study. This familiarity can be used to identify and eliminate disadvantages of working online, as well as promote innovative use of the blended learning resources in learning and assessment. It is plausible to suggest that, generally, through understanding the different R/O/Gs of lecturers and students, such collaborations would allow for more assessment conducive to learning at the university level. This project is ongoing and further research will be conducted to explore how the students in the service mathematics course respond and perform to the questions written by the *Student*.

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KEYNOTE ABSTRACTS

WHO IS TASKED WITH MODERNISATION OF MATHEMATICS EDUCATION? RESEARCH MATHEMATICIANS AND MATHEMATICS EDUCATION RESEARCHERS BRIDGING THE DISCIPLINARY GAP

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KEYWORDS: Undergraduate mathematics, developmental research, blended learning, assessment, professional development

In this address, I focus on a systemic issue facing tertiary mathematics education - a lack of research-informed evidence-based approaches to teaching and learning. The issue highlights the disciplinary disconnect as many research mathematicians do not research in mathematics education. A consequence of this is that many mathematicians contribute to the academic inertia by teaching in a traditional transmitting method - the mode of teaching they are familiar with - simply replicating the teaching they received. In contrast - on the other end of the spectrum - there is increasing experimentation with new modes of delivery by enthusiastic innovators who, in some cases, lack the skills required for conducting rigorous educational research as part of their innovative endeavours. Innovations are often based on integration of new technological gadgets for use in mathematics education with only anecdotal evidence about their merits.

Rapidly accelerating advances of emerging technologies are likely to exacerbate the problem. Globally, the higher education sector is challenged to keep up with the times and reassess its sustainability in a technological era. Blended learning, the integration of face-to-face and online instruction, is now widely adopted as the ‘new normal’ in course delivery across tertiary institutions. Yet the people who are tasked with educational modernisation are not generally supported by mathematics education researchers in their attempts to try out new technology-assisted instruction.

In addressing this disciplinary disconnect, I outline a proposal for a field of research in university mathematics education that aims to bridge the gap by focusing on the following research themes:

Theme 1: Developing frameworks for conducting evaluation research in a realistic university setting for testing an innovation that aims to integrate insights from broader educational research. I will talk about a suitable methodology for this type of developmental research, which is gaining prominence as an effective approach in mathematics education – design (-based) research (Bakker, 2018). To demonstrate how this methodology can be used, I draw on my recent work, in which an innovation involving regular online pre-lecture quizzes was designed, developed, implemented and evaluated. The aim of the intervention was to optimise the effect of distributed (spaced) practice on long-term memory retention. At the completion of the first iteration of design research, our findings suggest that this relatively small change

in course instruction can improve efficiency and effectiveness of educational exchange (Evans, Kensington-Miller, & Novak, 2019).

Theme 2: Investigating the mechanisms involved in successful professional development projects in mathematics departments and developing frameworks for dissemination of effective/efficient teaching and learning practices in a realistic departmental setting. This largely unexplored area of research, if developed, can potentially have a major impact on the university mathematics education. I will talk about my current research project with Barbara Jaworski involving a professional development discussion group in the Department of Mathematics at the University of Auckland. This project draws on the pioneering research in this domain that was conducted at the university in the last 10 years (Barton, Oates, Paterson, & Thomas, 2014; Oates & Evans, 2017; Paterson & Evans, 2013; Schoenfeld, Thomas, & Barton, 2016).

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TEACHING CULTURE = DEEP LEARNING

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KEYWORDS: pedagogy, creativity, culture

Embedding Indigenous perspectives in the curriculum, particularly the mathematics curriculum, is often seen as political correctness to appease past wrongdoings. This usually goes hand-in-hand with the perception that Aboriginal and Torres Strait Islander peoples had no form of mathematics and hence, teaching from this perspective is pointless. This presentation will explore the connection between mathematics and culture, how Terra Nullius has shaped our education system and how this maintains the status quo of poor educational outcomes for Aboriginal and Torres Strait Islander students. Prof. Chris Matthews will demonstrate how taking a cultural approach to the teaching and learning of mathematics leads to deep learning for all students. The presentation will then explore the connections between mathematics and Aboriginal culture, how this can be used to transform mathematics education for Aboriginal students and how this education is important for all students.

PLAYFULNESS AND A MATHEMATICS EDUCATION FOR THE TWENTY-FIRST CENTURY

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KEYWORDS: online assessment, learning, conceptual development

For nearly twenty years I have been developing and researching online assessment for mathematics as a contemporary tool through which to engage students and, I hope, improve mathematics education. Online assessment has grown from a cottage industry, to a mainstream activity with almost every major university textbook now accompanied by online assessments. All tools have their strengths and weaknesses, including the paper based exam. Online assessment has been highly successful in providing immediate feedback about the correctness of students' answers. As such, online assessment is proving to be a very useful tool for building basic skills and procedural understanding. This talk will provide some examples of our use of online assessment in Edinburgh with year 1 students in skills-heavy methods-based courses. However, there are some important things still missing. In particular, the assessment of free-form proof is poorly supported in 2019 by online assessment tools. More seriously, these tools risk polarising the nature of what is learned and potentially impoverishing the curriculum as a result. In this talk I will explore these issues by discussing the role of playfulness in mathematics education. Playfulness here is taken in the sense of "playing in the match" or "playing in the concert" rather than childishness. That is to say, I will explore play as a desirable and important end goal.

WHY CHOOSEMATHS?

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KEYWORDS: mathematics education, careers, teacher content knowledge

At a time when more and more of our lives rely on mathematical skills, fewer Australians study and enjoy mathematics. The CHOOSEMATHS Project has taken a multi-layered approach to addressing the issue. Through a national strategy involving mentoring, awareness campaigns, schools outreach and an awards program, we have worked with parents, students, teachers and the public to raise the profile of mathematics and encourage Australians, particularly girls and young women, to engage with mathematics. We've learnt a great deal, much of which teachers can implement readily in their classrooms.

THE HISTORY OF DELTA: 22 YEARS AND COUNTING

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KEYWORDS: undergraduate mathematics, undergraduate statistics, conference

The series of Southern Hemisphere Conferences on Undergraduate Mathematics and Statistics Teaching and Learning has its origins in Queensland, Australia. It was in November 1997 that a group of mathematicians from the Universities of Southern Queensland, Central Queensland, Queensland University of Technology and Bond University had the bold idea to convene academics around the country who are passionate about mathematics education to engage in discussions around the theme *What can we do to improve learning*? The conference was labelled as DELTA'97 to capture the concept of continuous change experienced by educators.

The first Symposium on Undergraduate Mathematics was held at the Gardens Point campus of Queensland University of Technology in Brisbane (Cretchley *et al.*, 1997). The level of participation showed that many teachers of undergraduate mathematics and statistics students were concerned about teaching matters, eager to share experiences and learn from others. The strong participation allowed for a solid three-day program with two sessions running in parallel. Key to the success of that first gathering was the international flavour given to the event, with keynotes by Deborah Hughes-Hallet of Harvard University and Jerry Uhl of the University of Illinois, as well as a few delegates and contributors from the United States, South Africa and New Zealand.

The success of the first symposium proved the need for an ongoing forum to share good practice and discuss latest developments relevant to the teaching and learning of mathematics and statistics. Education was rarely discussed in corridors and tea rooms in mathematics and statistics departments, so the symposium appealed to many academics from around the country who found their tribe in this emerging community. We are indebted to the organisers of DELTA'97 for their passion for elevating mathematics education and for committing their time and energy to bring to fruition a second symposium. This was held in November 1999 in the Laguna Keys Resort on the Whitsunday Coast in Queensland. *The Challenge of Diversity,* with special reference to catering for differing learning styles, developing dynamic curricula, flexible delivery and the role of technology to support these endeavours, was the integrating theme. Again, the invited international speakers Adrian Oldknow of Kings College at the University of London and David Smith of Duke University, the forums led by international coordinators, and the greater participation of international delegates gave this gathering a stronger international flavour. In addition, all contributed papers were published in a book of proceedings, documenting the breadth and depth of the research and innovation taking place in Australia and elsewhere in addressing the challenges of diversity (Spunde, Cretchley & Hubbard, 1999).

This second symposium cemented the identity of the DELTA community, which grew from strength to strength from then on. The strong participation of and commitment from South Africa and New Zealand delegates, led to continue these gathering every two years, with locations rotating around Australia, New Zealand and South Africa. The informal collaboration

between these Southern Hemisphere countries has grown organically into the formation of the International DELTA Committee who oversees the delivery of the conferences and ensures their continuity. One of the guidelines given to future hosts is to choose venues that are conducive to support network building and continue conversations outside the formal sessions.

The third conference, now re-labelled Southern Hemisphere Symposium on Undergraduate Mathematics Teaching, was held within the world famous Kruger National Park, South Africa, in July 2001, around the theme *Gearing for flexibility.* The organisers decided to call it *Warthog* DELTA, and so commenced a trend of naming the gatherings based on their geographical setting. One of the objectives was to use this international meeting as a platform to gain momentum in African countries to increase cooperation on the topic of mathematics education. This conference was also a first in publishing selected research papers in the Journal of the South African Mathematical Society *Quaestions Mathematicae* (Angelow, Engelbrecht & Harding, 2001), with all other contributions included in the *DELTA'01 Communications* (Engelbrecht, 2001). The symposium was attended by 120 delegates from 24 different countries, with five guest speakers from the US, UK, Tanzania and South Africa.

By the third event, the DELTA series of conferences had already gained a truly international reputation as the forum for the exchange of ideas, challenges and good practices in undergraduate mathematics and statistics teaching and learning. The DELTA community had by now a clear and strong identity. From then on, the conferences continued to be held without interruption on a biennial basis, and have maintained the highest standard. DELTA'03 was the *Remarkable DELTA*, held in Queenstown, New Zealand, set against the imposing *Remarkables* mountains (Holton & Reilly, 2003). It focused on the theme *From all angles*, encouraging discussions about the complexities involved in mathematics education. *Kingfisher DELTA'05* returned to its birth place, Queensland, this time on the pristine Fraser Island and with the theme *Blending beyond the boundaries*. In addition to the Conference Proceedings (Bulmer, MacGillivray & Varsavsky, 2005), selected papers were published in a Special Issue of the *International Journal of Mathematical education in Science and Technology* (iJMEST) (Bulmer, 2005). The partnership with iJMEST continued since then, disseminating the work of the DELTA community to the broader readership of the journal.

In 2007, DELTA reached to South America. The Calafate DELTA was held in the picturesque Patagonian town of Calafate, the gateway to the World Heritage Glaciers Park. The theme *Vision and change for the new century* offered the opportunity to look for new solutions to challenges that persisted over time (Darcy-Warmington *et al.*, 2007; Martinez-Luaces & Varsavsky, 2007). The Gordon's Bay DELTA was held in 2009 in this beautiful coastal town close to Cape Town in South Africa with the theme *Mathematics in a dynamic environment* (Wessels, 2009; Oates & Engelbrecht, 2009). The *Volcanic DELTA 2011* took place on the edge of Lake Rotorua, with the theme *Te Ara Mokoroa*, a Maori phrase which describes "The long abiding path of knowledge" aiming to stimulate all participants no matter where they might be on that path (Reilly & Oates, 2011; Thomas & Hannah, 2011). Back in Australia in 2013, Lighthouse DELTA was set in the tourist coastal town of Kiama, and attempted to shine the light through the fog of issues and challenges we all faced (King, Loch & Rylands, 2013; Matthews, 2013). In 2015 *Elephant Delta* took the community to beautiful Port Elizabeth on the south-eastern coast of South Africa (Blignout & Kizito, 2015; Mofolo-Mbokane, 2015). The eleventh and second last conference was held for the second time in South America; it was hosted in Gramado, a holiday resort "German" town in what is known as the Romantic Route in Southern Brazil, which evoked the truly international and collaborative flavour of the DELTA community. Finally, this year we are gathering by the Swan River in Freemantle, on the Australian Western coast (another first!). This year's theme, *Reflections of Change*, brings to

mind the need to constantly research our practices to ensure we provide high quality mathematics education to our students.

The DELTA community should be proud of the impressive body of work it produced over the years, which is documented comprehensively on the DELTA website (Delta conferences, 2019) and the iJMEST special issues. However, there is so much more work to be done. Mathematics, as a discipline, is being subject to extreme pressures for change: pressure to modernise teaching practice, demands from the serviced disciplines, dealing with changing student profile, bridging the gap for underprepared students, students questioning relevance of what they are taught, and the list goes on. The challenges faced by teachers on a daily basis are complex, and the solutions require patience, creativity, commitment and resilience to cope with trials that do not work. It is not a surprise that many of the topics discussed at the first gathering in Queensland are still being discussed today, but they are situated in a forever changing higher education landscape and the continuously evolving technological environment, and so requiring more nuanced solutions.

DELTA is not a formal organisation. It is a community of practitioners, working at the coalface with students who come from all walks of life and who study mathematics or statistics for many different reasons. Members include university mathematicians and statisticians, educational researchers, tutors and students who share the joy, successes, disappointments and challenges of mathematics teaching at university level, and who collaborate in research projects across the world. The community remained vibrant over 22 years due to the willingness and commitment of their members who take turns to host international meetings every two years to advance the teaching and learning of undergraduate mathematics and statistics so that our graduates are adequately equipped for their next stage in life.

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THE CATCH-22 OF TEACHING

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KEYWORDS: mathematics education, teaching pedagogies, technology, assessment, reflective teaching

Teaching is more than a job. It is a human responsibility and one of the greatest responsibilities in a civilized society. Science education has the opportunity to explain the mysteries of the world. Science educators train young minds to explore, to question, to investigate and to discover. Educators therefore have the opportunity to impact young minds in countless ways. As Albert Einstein said: “Education is not the learning of facts, but the training of the mind to think.”

However teaching, particularly mathematics, at university presents a lecture with several dilemmas that needs to be tackled:

- Classroom environment
- Course content and assessment
- Electronic world

One of the biggest dilemmas facing undergraduate lecturers is teaching large classes. With large classes you typically encounter students with many differently learning styles. Is it possible to cater for all of these learning styles? Also, as educators how can you find the perfect balance of interactivity? It is estimated that 60 percent of people are passive learners. Saryon and Snell (1997) found that students receiving an interactive and student-centered lecturing style are more likely to have a higher impact of learning compared to more traditional styles. So as a mathematics lecturer, how do you solve this classroom catch-22?

Coupled with the classroom environment, we also have to be aware that we are teaching generation Z. They prefer instant gratification. Generation Z wants to see value in content and needs affirmation and recognition. How can this dilemma be resolved? It is a real catch-22, as most mathematics problems needs grit, determination and can take a while to be solved, which clearly clashes with the personality of modern students. This is another classroom catch-22 to take note of and try to solve.

The delivery of content, for example mathematical topics, creates even more catch-22 dilemmas. Are we using effective teaching pedagogies? Is the content organized in a sensible manner? Is there sufficient motivation for tackling the topics in the syllabus? Can we resolve the abstract and hierarchical nature of the material that accompanies the syllabus? Or how important is content really? This is emphasized by a quote by Maya Angelou who said “I’ve learned that people will forget what you said, people will forget what you did, but people will never forget how you made them feel.” So is our content really preparing students for the fourth industrial revolution and equipping them with the skills needed for jobs that don’t exist yet?

Another important educational issue is that of assessment. Assessment refers to a wide variety of methods to evaluate or measure students learning progress or skills acquisition of students. So as an educator you need to ask what kind of assessment is good or necessary. When is assessing enough or too much? What are the costs in terms of feedback, money,

time and human resources? These dilemmas need to be resolved to find the best fit for the course and the students.

Finally, education technology is ubiquitous and becoming more and more present in classrooms across the world. However, is using technology hampering or helping teaching? What are the right educational tools to be used in the classroom and which tools are the wrong ones to incorporate? Does using technology make an impact on students? These classroom dilemmas require reflection and appropriate research to fully understand the impact to help educators finding and using the correct tools for their courses and students.

In summary, my talk will tackle these catch-22 topics of teaching. Especially, why reflecting on these issues are important and how these considerations have influenced and shaped my tertiary mathematics teaching. Teaching is a craft that can always be improved. Reflection is a powerful tool that we can all use to improve our own teaching to help students to reach their full potential.

Lastly, using these examples, I will make the case that all universities should have lecturers in the field of education who not only understand the content but also have the ability to better their respective institutions. That is educators who can suggest and implement changes to improve teaching on a bigger scale. Using reflection as a tool of measurement changes we can create and implement initiatives to improve learning environments that will benefit students, educators and ultimately society.

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ORAL PRESENTATION ABSTRACTS

A PROJECT FOR TEACHING SAMPLE SURVEYS USING A VIRTUAL POPULATION

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KEYWORDS: survey sampling, statistics project, The Islands

Sample surveys is a topic usually taught to students undertaking a statistics minor or major in the latter part of their degree. Ideally, students would gain experience sampling from a real live population; however, the logistics involved, including obtaining approval from the university's ethics committee may not be timely or feasible for a short course. An alternative is to use an online virtual population such as The Islands, which provides the students with experience in setting up a sampling frame, requesting consent from potential participants, and obtaining experience with data collection, data manipulation and analysis using statistical software. Written communication skills and teamwork are highly valued by employers of statistics graduates. This project encourages collaborative learning with the development of a written report, carried out in pairs, which fosters active learning and helps to develop a combination of essential skills for statistical practice. The project can easily be adapted to suit first year students or extended to suit Honours or Masters level students.

REFLECTION, INTROSPECTION, AND TRANSFORMATION

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KEYWORDS: team reflection, teaching philosophy, STEM education

A team of dedicated lecturers teaching on a pre-degree certificate STEM program had the opportunity to reflect on their teaching practice while preparing a portfolio for a tertiary teaching award. Five criteria were used to guide the team's practice: Excellence, Teaching Process, Outcome, Evaluation & Feedback and Leadership & Impact. This process gave us an opportunity to reflect on our own practice and gain insight into each other's strategies, philosophy, and reasons for using myriad techniques in diverse situations.

Overall, it emerged that team-teaching means celebrating and embracing diversity within the group rather than pushing uniformity onto everyone. The team found that the level of support required for students' success goes well beyond conventional expectations. Many activities outside of traditional teaching practise contribute to the success of a pre-degree student. The practice of group reflection was challenging as it involved coordination among teaching team but was reinvigorating; refreshing the groups perception. Transformation is ongoing and appraisal helped to identify areas for change.

This presentation will introduce our pre-degree certificate program and the framework of reflective practice. Specific examples discussed include our student-centric philosophy, student engagement with pen-enabled tablets, and embedded student support. Some ideas of future research directions will be shared to conclude.

EVEN IN THIS DIGITAL WORLD, SIMPLE CONCRETE MANIPULATIVES MAY BE USED TO CEMENT MATHEMATICAL AND STATISTICAL UNDERSTANDING

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KEYWORDS: concrete manipulative, primary, mathematical/statistical understanding

Although educators are advised that today's students are 'digitally aware' and 'computer wise', these 'talents' may only apply to mobile applications such as Instagram and Facebook. Learning mathematical and statistical concepts can still require the concrete touch and movement in the classroom rather than an animation on a screen regardless of the excellent graphical techniques available. In this talk, the author will demonstrate some activities that can be adapted into any level of education though many have been used at tertiary level with great results. Materials are inexpensive everyday items entwined with a touch of imagination!

CHANGING ATTITUDES – HOW ACADEMICS UTILISE TABLETS IN MATHEMATICS

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KEYWORDS: tablets, marking assessment, staff engagement

This presentation investigates how academics within the School of Access Education at Central Queensland University utilise tablets. In particular, it looks at the barriers and motivators for using tablets.

Academics were invited to participate in an online survey. The responses from the survey have been used to inform the development of teaching strategies to encourage the use of tablets as a teaching/learning tool. Academics were most satisfied with the use of tablets in lectures or tutorials when it was combined with face-to-face teaching methods. This allowed greater flexibility in teaching methods and styles. The use of tablets has enhanced efficiency in two areas. Firstly, electronic marking of assessment items has improved the turnaround time for marking of student assessments. Secondly, utilising the Tablet PC, academics have been able to provide assistance to students by creating handwritten solutions in 'digital ink' and small personalised instructional videos.

For most academics the tablets assist with the marking of assessment and provision of extensive resources, such as instructional videos, that can engage students.

REACHING OUT: INTRODUCTION TO CALCULUS

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KEYWORDS: calculus, mathematics, online courses, secondary-tertiary interface, education innovation

The University of Sydney's Massive Open Online Course (MOOC) *Introduction to Calculus* was a huge coordinated team effort, created over several months since July 2018, culminating in an official launch on 10 December. This was in time for prospective local students, in transition from secondary to tertiary study, to use the MOOC to satisfy mathematics prerequisite requirements, so that they may enrol in degrees in science, business and economics at the University of Sydney. The MOOC, however, was conceived more generally and holistically, as contributing towards improving diversity and inclusion, creating opportunities for participants throughout the world, including regions suffering from acute political or economic instability or poverty, facilitating pathways towards higher education through greater awareness of mathematics, and calculus in particular. Extensive themes, historical contexts and threads permeate the MOOC, creating perspectives that students might not see in typical classrooms or from reading textbooks. The MOOC also aims to alleviate frustration from students who have attempted to navigate through the huge corpus of other online material, not necessarily of such high quality or focus, or who have had other prior negative experiences in learning mathematics, either online or in the classroom. The MOOC uses a mastery model of learning, with opportunities for copious practice and interaction using Discussion Forums. Positive feedback from participants has exceeded expectations, including personal stories from teachers or prospective teachers from all over the world, who have used the MOOC to upgrade their skills and resources for teaching calculus with new insights and perspectives.

STUDENT POLLING – IT'S THE TAKING PART THAT COUNTS

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KEYWORDS: active learning, in-class polling, student outcomes, electronic voting systems

The presence of active learning activities in tertiary science, engineering and mathematics classes is known to be a strong indicator of increased student performance in examinations (Freeman et al., 2014). Audience polling via electronic voting systems (EVS) is an example of active learning and is used extensively across tertiary education. The use of an anonymous-at-point-of-use EVS such as TurningPoint clickers has proven popular with tertiary mathematics students (Strasser, 2010), with evidence of increased student engagement in class (King & Robinson, 2012), and grade improvement at the aggregate level when compared with classes that do not use EVS (Simelane & Skhosana, 2012). However, it is unclear whether there is a direct association between individual EVS use in a mathematics class and individual student attainment (e.g. King & Robinson, 2009, could find no such link), although researchers have found this in other disciplines (e.g. Samson, 2018, in the context of Environmental Science lectures).

This talk outlines an exploratory analysis exploring the relationship between student use of the EVS system provided in Echo360 Advanced Learning Platform and student attainment in a medium-sized (n=111) first year mathematics unit for non-math majors. There was a significant correlation between the number of EVS polls a student attempted and student attainment, even when controlling for proxy measures of student engagement such as participation in low-stakes online assessment items. Interestingly, the relationship between correct poll responses and student attainment was – although statistically significant – weaker than that for total participation. It really is the taking part that counts.

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ASPECTS OF A TRANSITIONAL SUBJECT FOR FIRST YEAR MATHEMATICS STUDENTS: A REFLECTION

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KEYWORDS: first year experience, learning community, reflective learning, peer assessment

In this talk we showcase aspects of the subject MATH100 Introduction to Mathematics for mathematics and mathematics education students in their first session of study. We present features of the subject that focus on the transition to university, reflective writing, peer assessment, working in a team, writing research reports and creating a mathematics community. We examine the evolution of the subject, and reflect on student and teacher experience.

REFLECTING ON THE ROLE OF CENTRALISED MATHEMATICS SUPPORT

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KEYWORDS: flexible learning, mathematics support, unified teaching

In this talk, I will reflect on my time performing the role of centralised mathematics support at an Australian university, highlighting the differences between methods of delivery and available practices compared to standard teaching positions. The role involves consultations and workshops with students from across the university that have a mathematics component in their degree (including education, business, psychology, and engineering students). The reflection will evaluate how the setting allows for easy integration of flexible learning and unified teaching practices, and assists students in their transition into university. Evidence of the success of the practices is provided through data collected from student feedback forms after workshops and consultations in the fifth and eleventh weeks of sessions. I will also reflect on how observing student learning practices in this role has informed teaching approaches and revealed necessary areas of research.

REFLECTIONS ON THE VALUE OF TEACHING MATHEMATICS TO ART AND DESIGN STUDENTS

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KEYWORDS: mathematical art, design, coding

Recently, I designed and delivered a new mathematics course for foundation art and design students, based on geometry and symmetry. It is assessed by portfolio. Students create original mathematical designs by coding in *Processing*. This language is very accessible and allows students to quickly incorporate geometry and animation into their artwork. Transformations such as translation, rotation and scaling are easily implemented. Art and design students are sometimes intimidated by the thought of mathematics and coding, but find *Processing* very easy to use. They enjoy experimenting and the quick results. What is the value of such a course? Mathematical art is beautiful, fun to create and a great addition to the skill set of a graphic or textile designer, artist or architect. Developing the logical thought processes required for coding is helpful for all students. I will present my reflections and those of my students on the value of teaching mathematics to art and design students.

MATHEMATICS LECTURERS' VIEWS ON THEIR MATHEMATICAL MODELLING PRACTICES

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KEYWORDS: mathematical modelling, undergraduate mathematics education, Cultural-Historical Activity Theory

Mathematical modelling (MM) is an important topic in mathematics education. Despite its importance, MM practices at all levels of education are far less prominent than is desirable. Researchers have pointed out that the role of the teacher within MM has not been sufficiently researched, and that there is a lack of understanding of how teachers can develop practices that help foster their students' MM skills. In this presentation, we will discuss mathematics lecturers' views on the aims and teaching practices of MM education in Norway and England, collected from a survey of 154 lecturers and in-depth interviews with 9 of them. By using Cultural-Historical Activity Theory, we aim to expose the tensions that exist within the activity of teaching MM at university, such as those that exist between multiple, sometimes competing aims for teaching MM, or between the lecturers' professional identities and the structure of university degrees. Our conceptualisation of these tensions aims to help lecturers think about ways to resolve their own contradictions and tensions, and to promote the discussion of how they could surpass obstacles in their own cultural-historical contexts. In doing so, we desire to support lecturers in finding innovative ways to facilitate their students' learning of MM.

RANDOMNESS: ITS IMPORTANCE IN STATISTICAL INFERENCE AND HOW TO TEACH STUDENTS ABOUT IT

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KEYWORDS: statistical inference, teaching, Monte Carlo

My experience over 65 years is that many otherwise able tertiary students have difficulty in achieving competence in inferential statistics, and that students are not given opportunities in secondary school to develop an understanding of the notion of randomness. My thesis is that an understanding of randomness provides a foundation for competence in inferential statistics.

The purpose of this presentation is to show how Monte Carlo simulation can be used to help school students, as young as 12, develop an understanding of randomness. The presentation is based on the work of the German mathematician, Arthur Engel (Engel, 1970).

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ANONYMOUS PEER FEEDBACK FOR PROBLEM-SOLVING PORTFOLIOS

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KEYWORDS: peer feedback, problem solving, mathematics presentations

Anonymous peer feedback on student assignment drafts was expanded to problem-solving portfolios in a unit designed for preservice teachers completing a specialization in mathematics. A number of improvements to the process were made after a previous iteration only used peer feedback on the first assessment (a Scratch coding presentation). The use of peer assessment has the potential to develop students' evaluative judgement and, for mathematics in particular, allow students to gain a broader appreciation of the many approaches to solving problems rather than teacher-provided model solutions.

Benefits were observed in terms of reinforcing the learning objectives of the assessments and focusing students' attention to the stages of problem-solving, communication and reflection. While this process was largely facilitated with manual work, there are software platforms that can automate the process which will be trialed next year.

TEACHER AND LECTURER PERSPECTIVES ON SECONDARY SCHOOL STUDENTS' UNDERSTANDING OF THE LIMIT DEFINITION OF THE DERIVATIVE

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KEYWORDS: transition, calculus, derivatives, functions, conceptual understanding, perspectives

The transition of students from studying secondary to tertiary mathematics has been the subject of increasing research interest in recent years. In the first part of a two-year longitudinal study, 750 Year 12 (final year of secondary school) students from a range of public, private and Catholic schools completed eight mathematics questions on pre-calculus and calculus topics taken from the Queensland and Australian school syllabi. The students were comprised of 470 Intermediate Mathematics only students and 280 students studying both Intermediate and Advanced Mathematics. Teachers and lecturers were asked how difficult they thought students would find each question. The Fisher Exact Test was used to determine whether there was a significant difference in perspective between the teachers and lecturers.

This presentation reports on teacher and lecturer perspectives on student responses to a question on the limit definition of the derivative. The results show differences in perspectives within and across teacher and lecturer groups, which have subsequent implications for how tertiary-level mathematics is taught. More discussion between the groups is needed in order to assist students on their calculus journey.

GROUP-WORK VIDEO PRESENTATIONS AS AN EFFECTIVE LINEAR ALGEBRA ASSESSMENT TASK

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KEYWORDS: innovative assessment, collaborative learning, transparency in assessment, student metacognition

In an introductory Linear Algebra course, we wanted to investigate how best to introduce a group-work assessment task to encourage students to help one another learn, in a manner that recognized potential issues relating to academic integrity and fairness in the relative contributions to the submitted work. The cohort comprised 36 students (mainly BSc or BEd). Students working in small groups recorded a video of their solutions to traffic-flow problems. The groups also provided agreed self-assessments of each individual's contribution to the group's submitted work, which were used to moderate individual marks from the overall group mark (adapted from a methodology developed by Heathfield (1999)). The medium of recorded videos was chosen to leverage participation and engagement. As part of the assessment task, students were asked both to rate (on a scale from 1 to 10), and to comment on, the task's effectiveness in helping their understanding of course material, and in establishing a fair moderation process.

While cohort comparison is difficult, we were encouraged by the results. Student responses were positive, albeit with some equivocation. Most positive was the effectiveness of the assignment overall (mean±std 7.8±1.5), followed by the fairness of the method (6.9±2.4), the effectiveness of working in a group (6.5±2.2) and the effectiveness the video format (6.2±2.4). Student comments acknowledged a variety of ways in which the nature of the assignment helped their understanding and engagement with course material, but also helped identify areas for future improvement (particularly with regard to the nature of the assignment questions).

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PRESERVICE PRIMARY MATHEMATICS TEACHERS' PROFESSIONAL LEARNING WITHIN A UNIVERSITY-SCHOOL PARTNERSHIP

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KEYWORDS: mathematics professional learning, pre-service teachers, university-school partnership

This mathematics focused presentation is extrapolated from a larger work that explored the concept of partnership within the Master of Teaching and Learning, an initial teacher education programme, between schools and the university.

School-based teaching experience is fundamental for preservice teachers' professional learning, as it provides the context for theories and practice to amalgamate. However, we are mindful of potential challenges particularly when tensions exist between theory and practice. The initial development of the programme deliberately considered the partnership between teacher educators and schools to bridge the gap between the theory-based university and the practiced-based school. Through an ethnographic approach, this study explored how this partnership between the university and schools supports the professional and pedagogical learning of preservice primary mathematics teachers from the teacher educators and the preservice teachers' perspectives. The data is drawn from four teacher educators' reflections on the various iterations of the mathematics component of the programme and the preservice teacher evaluation surveys from 2015 to 2019 of their mathematics learning. The findings of the study indicate that the university-school partnership in mathematics professional learning of preservice teachers is effective when the roles of members of the professional learning community are clearly defined, and assessments are situated in an authentic school-based context rather than theorised academic essays. In addition, the preservice teachers emphasised that their professional mathematics learning was beneficial when they were centrally positioned as participants in collaborative learning activities rather than peripheral participants.

SUPPORT FOR STUDENTS WITH MATHEMATICS LEARNING DISABILITIES (MLDS) ON BRIDGING OR FOUNDATION PROGRAMMES AT NEW ZEALAND UNIVERSITIES

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KEYWORDS: bridging, repeat, dis/ability

Equality of opportunity where all candidates have equal access to university, is not as secure for those who struggle with making sense of mathematics. An example of a struggle is when students fail the Mathematics 91F course on the Tertiary Foundation Certificate (TFC) programme, and then must repeat in their second semester. Several repeat students have said that they expect to fail mathematics, and their unsuccessful efforts may even lead them to question whether they are suited for tertiary studies (Manalo & Wong-Toi, 2010). The position is exacerbated for those with a Mathematics Learning Dis/ability (MLD) with concurrent difficulties and anxieties with mathematics. This study then, is in the early stages of seeing how bridging programmes in New Zealand universities attend to the mathematical learning needs of students with (MLDs), considering their equity policies for targeted groups.

In 2017, 3% of students at the University of Auckland voluntarily declared they had a disability (Equity Office 2018), well below the 16% of Auckland's 15-44 year-old population (Statistics NZ, 2013) estimated to have a disability. Of this 3%, almost 38.7% identified as having a learning disability. Although the TFC programme has less than 1% of the student population, it accounts for 3% of students registered with Student Disability Services. This begs questions about students with MLDs who are in bridging programmes, for example how they were assessed for MLDs, which students are missing out, and what is done so that students on these programmes are encouraged to share their own mathematical know-how.

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TIGHT CONNECTION TO SUCCESS: THE COLLEGE CREDIT PLUS PROGRAM

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KEYWORDS: post-secondary initiatives, cooperative learning, mathematics pathways

The state mandated College Credit Plus (CCP) program in Ohio has grown significantly in the last several years. Many general education core courses, in fifteen different disciplines, are being offered to high school students, on-premises or in a partnering university. In mathematics, students are able to complete pre-calculus courses while in high school. In forming the partnerships with high schools, universities have applied remediation free standards for initial placement and improved efforts to enhance the school classroom learning experience. The partnerships have facilitated a smoother transition from high school to college and universities have invested additional resources to ensure the success of CCP students while in college. The high school teachers have completed advanced graduate coursework to qualify and become credentialed in the CCP program. The CCP initiative has successfully bridged the gap that existed between K-12 and universities by tightening the collaboration among high school teachers and university faculty. As a result, Kent State University has seen an increase in freshman registration which we posit is due to CCP. Students who follow the CCP program are significantly better prepared for a meaningful college experience. Technology has been used to provide a previously inaccessible experience in mathematics for CCP students. We will highlight how *ALEKS*, *Notability*, and *Explain Everything* platforms have been used to build the mathematics component of the CCP program and the follow-up mentoring program. Data on the success of the overall CCP program will be highlighted in the presentation.

EXAMINING PRE-SERVICE TEACHERS' MATHEMATICAL CONFIDENCE

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KEYWORDS: pre-service teachers, mathematical confidence, mathematics anxiety

Richardson and Suinn (1972) described mathematics anxiety as "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations". Inge Koch (2018) highlights that "higher levels of maths anxiety in teachers are related to lower mathematics achievements of their students".

Ashcraft (2002) wrote, "Highly math-anxious individuals are characterized by a strong tendency to avoid math, which ultimately undercuts their math competence and forecloses important career paths." With competency testing for pre-service teachers a standard practice both here and abroad, mathematics anxiety is indeed foreclosing career paths for some prospective teachers.

In this project we were interested in evaluating the efficacy of approaches taken to increase students' confidence in their mathematical ability and their ability to teach mathematics at the primary school level in two units, one offered by the Department of Educational Studies and one offered by the Department of Mathematics and Statistics.

Students were asked to complete the Maths Anxiety Rating Scale – Revised (MARS-R) at the beginning and end of each of the two units. In addition, we surveyed the students about their levels of confidence regarding teaching mathematics in the future. In the final survey we asked the students which aspects of the units they felt were important in developing their confidence to learn and teach mathematics. This presentation will examine some of the results from the data collected and provide some insights into developing learning activities that build mathematical confidence.

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ARE ADVANCED MATHEMATICS COURSES VALUED BY WESTERN AUSTRALIAN MALE AND FEMALE SENIOR SECONDARY SCHOOL STUDENTS?

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KEYWORDS: mathematics, value, gender

This presentation will include findings from the first two studies in a project aimed at identifying motivational factors on secondary students' Year 11 mathematics course choices. The impetus for this research is the consistently low enrolments of female students, in comparison to male students, in advanced mathematics courses in secondary schools, reported both nationally and internationally. Adopting Expectancy Value Theory (EVT) as a theoretical framework, the first study gathered perceptions on influential factors on course choice from capable Year 10 students from three schools via focus groups (*n=20*). In the second study, a pilot survey was used to obtain the opinions on the self-perceptions, perceptions of the domain of mathematics, and sociocultural influences on course choice from Year 10 students in one school (*n=84*). This presentation will focus on the findings in relation to the subjective value of mathematics in accordance with four facets of value: *intrinsic* value (level of enjoyment in mathematics); *utility* value (the importance of undertaking mathematics courses); *attainment* value (the value of doing well in mathematics); and *cost* value (the sacrifices made in taking mathematics courses). Results will be described regarding the salience of facets of subjective value on mathematics course selections for girls and boys. The next stage of this project will also be outlined. This presentation will include a series of intellectual provocations for teachers to consider in order to elicit ongoing discussions around potential levers for change in Western Australian schools.

RATIOS, GEOMETRY AND HEALTH MESSAGES

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KEYWORDS: geometry, health promotion, secondary education

Before we begin, it is important to note that this author's intention is to promote safe behaviours while providing real-world mathematics problems for high school students to contemplate. At no stage is the consumption of alcohol to be glamorised or lauded but rather it is to be understood as an activity that many Australians of drinking age engage in. By comparing models, students can investigate a number of questions pertaining to multiples of 4, how we visualise percentages, how one volume of liquid containing 40% alcohol has as much alcohol contained in it as ten times the volume of 4% alcohol and the implication this has towards measuring alcohol consumption.

In terms of specific learning goals, students who have attained an adequate understanding of choosing appropriate units of measurement for area and volume can apply this Year 8 content to calculating the areas of composite shapes, solve problems using ratio and scale factors in similar figures and to start developing the skills required for solving problems involving surface area and volume for a range of composite solids. And while the lesson content may not reach those who are most at risk from harm as effectively as it might reach those who are less at risk, there is tremendous value in developing a calm discussion around alcohol consumption where friends of many dispositions can support each other to lead healthier lives.

DEVELOPING SKILLS AND ENHANCING FEEDBACK THROUGH ONLINE ASSESSMENT WITH WEBWORK

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KEYWORDS: assessment, feedback, online

WebWork is a free open-source online homework system maintained by the American Mathematical Society (AMS) which can be integrated into Learning Management Systems. The School of Mathematics and Statistics recently introduced *WebWork* for online assessment and learning tasks in five first-year level mathematics subjects. *WebWork* is used for online formative assessment, non-assessed revision tasks, and a summative computer lab test. The introduction of online assessment provided an opportunity to re-evaluate the assessment in each subject and to focus each assessment mode – online and written – on what it is most suited for. We aimed to use the online assessments to target known skills deficiencies and common conceptual difficulties in new material. As part of the project evaluation, we investigated the impact of the online assessment on student engagement, perceptions of feedback, and academic performance. Data were collected from student surveys, analytics from online systems, and past assessment results. Overall, students were positive about the online assessment with *WebWork*, though felt that it did not provide as much useful feedback on their learning as written assignments. In this presentation, we will give a brief overview of the *WebWork* system. We will show examples of different styles of questions targeting a variety of skills and subject content. We will reflect on some different approaches for providing feedback to students. We compare our own experiences with data from student evaluations.

PROBLEM-BASED LEARNING IN AN INTRODUCTORY ALGEBRA CLASS AT THE UNIVERSITY OF WISCONSIN-MILWAUKEE

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KEYWORDS: project-based learning, developmental, algebra

The Department of Mathematical Sciences at the University of Wisconsin-Milwaukee has recently made extensive changes to its developmental course offerings, with the aim of increasing student engagement and active learning and, ultimately, overall student success in mathematics. In this talk, we will first describe an initial study carried out to assess the effectiveness of project-based learning (PBL) in a developmental algebra course, and then discuss the initial stages of the full-scale implementation of PBL activities in the course.

A total of 9 sections of the Introduction to College Algebra course were selected for the study: 4 control sections, in which instructors delivered content in the usual way; and 5 experimental sections, in which the usual delivery of content on simultaneous linear equations was replaced with a group project addressing the same material in an open-ended, real-world context. The project required each student group to create and analyze a real-world example that could be modeled with a system of 2 linear equations, and to present their solution to the class. The effectiveness of the intervention was assessed qualitatively with a survey given to students in the experimental sections, and quantitatively by comparing the scores of students in the experimental and control sections on a question on their final exam.

The results of the pilot study were sufficiently encouraging that several PBL activities have been included in the revised curriculum for the course, starting in the Fall 2018 semester. Our next step will be to analyze the effectiveness of the full-scale implementation.

MODELLING MATHEMATICS SUPPORT UPTAKE WITH RUMOUR MODELS

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KEYWORDS: mathematics support, rumour models, SI model

A challenge for mathematics support services is ensuring that students who could benefit from the service are aware of it. Demand for mathematics support services is known to be associated with proximity to assessment tasks (Edwards & Carroll 2018), with word-of-mouth advertising between students an important driver of attendance (Lawson, Croft & Halpin, 2003). Such spread of information, such as rumours, in a population by word-of-mouth, has been modelled using ideas from epidemiology (Daley & Kendall 1964). In an attempt to investigate the importance of word-of-mouth advertising and assessment in driving student usage of mathematics support services, we fitted a series of ordinary differential equation (ODE) models to the attendance data from a mathematics support drop-in center at a large metropolitan technical university. We used susceptible-infective (SI) models (Keeling & Rohani 2008), with 'susceptibles' representing potential users of the mathematics support service who are not yet aware of the service, and 'infectives' representing active users of the service. 'Infection' represents a student becoming aware of the mathematics support service. Our aims were to investigate whether such models are appropriate for modelling in this context, and whether they can give insight into the relative importance of different drivers of student uptake of support services. We found that the SI models fitted the data well over all semesters considered, suggesting this could be a fruitful approach for future work. Our preliminary results give some insight into support uptake; such as indicating that advertising does not reach all potential users of the service.

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PRE-CLASS TASKS USED IN A FLIPPED LINEAR ALGEBRA COURSE FOR STUDENT TEACHERS

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KEYWORDS: flipped learning, linear algebra, student perceptions, mathematics achievement, Singapore, student teachers

In recent years, as flipped learning becomes more frequently used in teaching undergraduate mathematics, instructors need to collect data to identify practices that promote student mathematics achievement and data to identify favourable perceptions of this new learning mode. In this presentation we will describe six different types of pre-class tasks for a flipped Linear Algebra course in a Singapore university: short videos narrated by the instructor, synopses, summary sheets, worksheets of problems and activities, and online quizzes.

The sample comprised 15 student teachers, who had good mathematics backgrounds. Being in-service teachers, their participation in this project would prepare them to implement flipped learning in school mathematics in the future. On average, they spent about an hour to complete these weekly pre-class tasks, but the higher-ability ones reported spending less time on these tasks than the other students. Almost all the students rated these tasks very highly in terms of helping them to learn and enjoyment at mid-semester and end-of-course surveys. These perceptions had weak correlations with the course grade.

The Calculus II course grade in the previous semester for the same sample was used as a predictor of the time spent on pre-class tasks; a negative but non-significant correlation was found. The time spent on pre-class tasks was then used as a predictor of the Linear Algebra II course grade; again a similar, negative correlation was found. These results suggest that stronger students could complete these pre-class tasks much quicker than the other students.

TRANSFORMATION OF PRE-SERVICE MATHEMATICS TEACHER SPECIALISED CONTENT KNOWLEDGE THROUGH ERROR ANALYSIS

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KEYWORDS: error analysis, pre-service mathematics teachers, specialized content knowledge, school mathematics

This presentation explores a relatively under-focused strategy for addressing the widespread concern about the quality of pre-service mathematics teacher (PSMT) education. At the heart of the debate regarding the delivery of professional mathematics teacher education curricula has been the reported lack of development of PSMTs' mathematical knowledge for teaching. However, the discussion of what mathematical knowledge for teaching is needed by PSMTs and how it should be developed has been uneven. Subject matter knowledge embodies two strands of mathematical knowledge for teaching, that is, common content knowledge and specialized content knowledge (SCK). In this presentation, we draw from literature that emphasizes the development of PMSTs' subject matter knowledge (SMK) of school mathematics topics. This presentation explores how the attention to SCK within a pre-service teacher education curriculum could potentially influence deeper quality mathematics learning and teaching among PSMTs through using error analysis. For the purpose of this study, a qualitative design was used. Aligning with the design, data was collected by means of written tasks from 61 third-year PSMTs enrolled in the B.Ed programme. Data was analysed inductively to generate themes about PSMT development process of SCK, which could potentially influence the evolution of their SMK. Findings suggest that attention to SCK has the potential to evoke school mathematics concepts and evolution of subject matter knowledge. Based on the findings, the researcher concluded that error analysis has the potential to improve PSMT subject matter knowledge of school mathematics and therefore recommends its inclusion in the curriculum.

CODING TUTORIALS: USING JUPYTER AND SCRATCH TO TEACH CONCEPTS IN BLOCKCHAIN

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KEYWORDS: blockchain education, *Jupyter*, *Scratch*

This presentation investigates *Jupyter* and *Scratch*; two contemporary tools that can be used for coding-based tutorials. A blockchain course has been developed for undergraduate and master's students called Applied Blockchains and Cryptocurrencies. The aim of the course is to provide students with exposure to blockchain development and methods. Coding experience was not required as a pre-requisite because it was a new course that did not fit within any specific subject pathway.

Programming tutorials were developed to guide students through concepts related to blockchains such as the Poisson distribution and hash functions. The *Jupyter* Notebook is an open-source web application developed for interactive data visualization. Students without programming experience found the *Jupyter* tutorials to be too abstract and struggled to complete the course project. One-third of the students did not complete a coded blockchain implementation. This has prompted looking into *Scratch* – a block-based visual programming language aimed at teaching kids to code – to help ease the transition for students without programming experience.

The Poisson distribution is an important topic for understanding block-intervals in a blockchain. A tutorial on the Poisson distribution using *Jupyter* is presented and compared to a version created with *Scratch*. Early indications are positive that students without programming experience feel more comfortable starting with *Scratch*.

CAPSTONE COURSES – WHAT, WHY AND HOW

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KEYWORDS: capstone, statistics, transition

The name capstone comes from the word used to describe the final stone placed on a building. Thus a capstone course should represent the culmination of a student's journey and support their transition to future destinations. Compulsory capstone courses have been introduced at the University of Auckland (UoA) for all undergraduate Science students enrolled from 2019 onwards. The catalyst for their introduction at UoA was a desire to provide a vehicle through which students can demonstrate the characteristics of our graduate profile, as well as to improve their employability. Globally capstone courses have been in existence since the 1950s, originally in the USA. Since then the number of capstone courses has spread to other countries and to other disciplines. They are most prevalent in disciplines such as Architecture, Creative Arts, Engineering and Business but least common in Law and Science.

This presentation will consider historical factors that have contributed to the increased demand for capstone courses from the perspectives of the main stake-holders in Higher Education. One aim of a capstone course is to support the transition from student towards enculturation into, for example, the statistics practitioner community. From a review of the literature, I will discuss why and how the theory of cognitive apprenticeship can be used to inform the development of teaching and learning activities, using a hypothetical statistics capstone as an example, to support this transition. Provisional analysis of feedback from my research on staff and student participants involved in a capstone-like Geography course will be given.

TEACHING DECISION THEORY IN THE PC LAB

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KEYWORDS: teaching with technology, decision theory, computer algebra system

In the Faculty of Business and Economics at Schmalkalden University, the Decision Theory course in the bachelor program was routinely taught in a traditional classroom setting. This course is actually one half of the subject “Mathematics II”, the other half is Matrix Algebra, which has been taught in the PC lab, using a Computer Algebra System (CAS), for many years. As the teacher of the Decision Theory course is currently on maternity leave, I took over teaching of this course from her for two years.

I was curious if topics from the Matrix Algebra portion of “Mathematics II” were useful in the Decision Theory portion, particularly as a lot of matrices (or tables) are used in Decision Theory, for example payoff matrices, results matrices, opportunity loss matrices. However, the answer is No.

Nevertheless, having a CAS readily available is not only useful for matrix operations, but also for finding the perfect alternative, or action, in a decision problem, using other mathematical methods. This will be demonstrated in several examples from different areas, including decisions under uncertainty, and decisions under risk.

As the students learn to work with the CAS in the Matrix Algebra portion, and have access to it during the final exam in the PC lab, using it (in addition to a spreadsheet program) also in the Decision Theory portion comes without a steep learning curve. Note that the CAS license of our faculty covers also the private PCs of our students.

THE BEARING THAT THE SOUTH AFRICAN MATHEMATICS CURRICULUM DESIGN HAS ON UNDERACHIEVEMENT

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KEYWORDS: national curriculum statement for mathematics, curriculum design, scope, link/ progression, spiral curriculum, achievement

The objective of this paper is to investigate the perceptions of teachers and subject advisors on the bearing that the South African mathematics school curriculum design has on enhancing or inhibiting achievement in mathematics at the Grade 12 level. Data was obtained from interviews which were conducted with the mathematics educators of selected schools, and the mathematics subject advisors of the district to find out about their perceptions with regard to the impact of mathematics curriculum design on students' underachievement in Grade 12. Also, the contents of the National Curriculum Statement for mathematics documents were analyzed.

The findings revealed that the scope (contents coverage) is too large, and that the skills which are supposed to be developed in learners each term are not well developed. Time allocation is unreasonably limited; as a result students are unable to cover the curriculum each semester. Revisiting of topics done every year in a spiral fashion is good but the balance between superficiality and depth is not achieved, and that it is done without checking and understanding learners' previous knowledge of the topic. The sequencing of certain topics has to be looked into and revised. The study also revealed that mathematics curriculum design, link, and progression from primary to secondary school is good, but not all details are covered, which leaves students with content gaps that impede the learning of certain topics.

REFLECTIONS OF CHANGE: ADDRESSING CHALLENGES IN THE TRANSITION TO SECOND-YEAR MATHEMATICS

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KEYWORDS: mathematics, academic needs, transition, pedagogy, learning communities

In South Africa, access to higher education in mathematics and science is often facilitated through foundational support programmes. This study reports on the challenges that students face in the transition from first-year to second-year mathematics. An academic development practitioner collaborated with academic staff in mathematics to conduct classroom observations and interviews with students, and subsequently analyse this data.

Based on the findings from classroom observations, the study reflects on the challenges that students face in the epistemic transition from first year to second year. In the interviews, students shared their experiences about studying second-year compared to first-year mathematics: the rapid increase in pace, limited time to absorb unfamiliar concepts, and high levels of abstraction.

The evidence from findings indicated an urgent need to develop teaching and learning communities that would support the academic needs of students and staff. The findings also suggested a fresh look at pedagogical practices in the classroom and the modes of assessment.

After reflection on the findings of the study, the academic development practitioner recommended pedagogical practices that have led to changes in student attitude and learning habits, greater focus on variation in representational modes, and more holistic interactive engagement. The change in student perception of mathematics emanating from this new approach has fostered academic identity so that students take ownership of learning and has demystified second year mathematics. Students' awareness of themselves as part of learning communities has stimulated interest in personal progress, and spurred willingness to engage at a deeper academic level that shows curiosity and creative energy.

HOW DO WE TEACH MATHEMATICAL MODELLING?

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KEYWORDS: mathematical modelling, teaching approaches, tertiary

Gaining useful insight into real world problems through mathematical modelling is a valued attribute across many disciplines including mathematics, biology and engineering. This being the case, in what ways can mathematical modelling be taught to first time modellers? A New Zealand study was carried out involving three case studies. Each case study comprised of a mathematical modelling course, lecturer participant and student participants. For student participants, it was the first time they had taken a mathematical modelling course during their tertiary study. Data was collected through participant interviews and classroom observations to address the question “How do your lecturers create student learning experiences in mathematical modelling?” Lecturer participants all had a different approach to teaching modelling. For the first case study, the lecturer taught modelling techniques and processes during lectures, followed by an open-ended modelling day. For the second case study, modelling techniques, including mathematical tools, where taught during lectures, with students experiencing the modelling process through modelling case studies. For the final case study, modelling techniques were taught during lectures and students used computer programming to explore how these modelling techniques could be applied. Reflective thematic data analysis was used to reveal insights into the student experience for these three different approaches. Preliminary results show that providing opportunities for students to discover their own process for modelling allows for the learning of modelling to occur. Due to the change in culture experienced by the students, ways of providing reassurance need to be explored.

CUSTOM SOFTWARE TOOLS FOR ACTIVE LEARNING IN HIGHER EDUCATION MATHEMATICS

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KEYWORDS: digital image processing, *Excel*, software tools, mathematics, active learning

Active or experiential learning is a well-recognised and successful method for helping students contextualise and apply their knowledge. Higher Education Mathematics is often highly theoretical, and an element of active learning embedded in a programme of study can be helpful in preparing graduates for the world of work.

A useful method for the active learning of many theoretical concepts is through computer simulation or modelling, especially if the learner is able to either create or control elements of the simulation. Depending on the mathematical topic being covered, it may be possible to create the simulation using existing software; in some specialist areas however, custom-designed software will be needed. This also of course allows the developer to create a tool tailored to the delivery of a particular topic.

In this presentation, particular examples of the development, use and benefits of such software will be discussed. Although they address specific topics, and therefore have a necessarily narrow focus, they are representative of the type of tools that can be provided to enable students to apply their knowledge, more effectively engage with a topic and hopefully gain a deeper understanding of it. Examples will include the generation and manipulation of digital images, the selection and analysis of time series and the illustration of number bases.

All software presented is freely available for unrestricted use, which may in itself be useful to attendees. The examples are intended to be indicative however, and hopefully the presentation will raise ideas for new applications of this kind.

THE INFLUENCE OF STUDENT ENGAGEMENT AND SELF-REGULATION ON THE PERFORMANCE OF FIRST-YEAR NATURAL SCIENCE MATHEMATICS STUDENTS

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KEYWORDS: student engagement, self-regulation, first-year mathematics performance, higher education

The aim of this research was to determine the influence of student engagement in mathematics and corresponding self-regulatory factors on the performance of first-year mathematics students in their first semester in a faculty of natural sciences. Student engagement (in learning mathematics) is a complex concept and according to the National Survey of Student Engagement (NSSE) it can be divided into five components namely level of academic challenge, active and collaborative learning, supportive campus environment, enriching educational experiences and student-staff interaction. In this presentation the focus will be only on the first two components.

A quantitative design in the form of a survey was used, where the NSSE questionnaire was administered to a study population of 304 students. This presentation will only report on the results of the 107 students who registered for a variety of BSc degrees (the natural sciences students). Descriptive statistics, confirmatory factor analyses and linear regressions were done to analyse the quantitative data.

Results from this research indicate that *Level of academic challenge* and *Active and collaborative learning* emerged as the most prominent components of student engagement. The presence of self-regulatory factors in these components was also evident. Perseverance, motivation and time management have emerged as significant factors that impact on mathematics performance.

Further studies may seek to gain more insight into how these factors need to be addressed in order to improve mathematics performance. The construct *seeking help from peers* also needs to be investigated, since it emerged as a factor influencing mathematics performance negatively.

FIVE WAYS COMPLEX NUMBERS GIVE INSIGHT INTO REAL NUMBERS

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KEYWORDS: complex numbers, real numbers, multiple proofs.

Problem-solving and proofs are the heart and soul of mathematics. Providing students with several proofs of the same result, students get to appreciate the beauty and elegance of mathematics. As a teacher, you will be armed with more insight into the content you are teaching. Consequently, you will be enhancing students' understanding of mathematics. The author will demonstrate five results that are typically proven using with real numbers but that can also be done with complex numbers. The aim is to promote creativity, build more connections in mathematical content and to teach students that multiple proofs or solutions exist to some problems.

Students typically start learning mathematics through integers. This is their first contact with real numbers. They notice problems like 2+3 or 2x3 only have one answer. Their continued exposure to mathematics through primary and high school often ends up being problems with only one answer. Therefore, most students enter university without seeing multiple ways of solving or proving results. The author feels that this exposure strangles their problem-solving creativity as it is not being demonstrated.

In my presentation, I have picked five results from the world of real numbers which I re-prove using complex numbers. My aim is to show that it is possible to provide students with a feeling to look for multiple ways for solving a problem. In so doing, I aim to encourage students to appreciate the connections between various branches of mathematics and that it is acceptable to tackle problems from different points of view.

DESIGN-BASED INSTRUCTION INTEGRATING COMPUTATIONAL THINKING FOR UNIVERSITY STATISTICS – FROM PROBABILITY TO ERRORS IN HYPOTHESIS TESTING

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KEYWORDS: computational thinking, design-based instruction, probability & statistics

This study developed design-based in-class instruction integrating computational thinking for second-year probability and statistics for applied mathematics students at a university in Taiwan. The goal was to link theoretical theorems and real-world applications.

For probability, a four-step instruction cycle motivated students with the QUESTION: “Generation of lucky-draw”. Then an EXPERIMENT followed to engage students. Zero-to-one random numbers were generated by *Excel*. In the COMPARISON and ANALYSIS step, students were guided to the generating algorithm by relating random numbers and drawing results. Finally, in the REFLECTION step, the Bernoulli distribution was introduced. Instruction cycles for the binomial distribution and other discrete distributions were developed and deployed in 2018.

For type I testing errors, students were asked to reply with decision rules in the QUESTION step. *Geogebra* tools were developed for students to do EXPERIMENT on errors of their rules. In the COMPARISON and ANALYSIS step, students provided their “optimal” rules. In the REFLECTION step, type II error was introduced. Design-based instruction for testing errors with accompanying *Geogebra* learning tools were developed for discrete and continuous distributions, and will be deployed in 2019 for a statistics course.

Students studying probability in 2018 were tested using paper-and-pen tests with the same items, with numbers changed, as for students in 2017. Results showed that scores of students in 2018 were significantly higher than that of the previous year. Meanwhile, in a course questionnaire, over 90% of students recognized that probability had practical applications.

POSTER PRESENTATION ABSTRACTS

PERSPECTIVES FOR THE TEACHING AND LEARNING OF PROBABILITY INVOLVING BAYESIAN SITUATIONS

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KEYWORDS: Bayesian reasoning, tree diagram, unit square

The 21st century is steeped in a complex, data-driven technological world. Citizenship-fostering literacy and effective training for the labor market require students to be able to make data-based decisions, analyze, infer, and predict. These skills demand probabilistic knowledge, particularly on conditional probability and Bayes' theorem. Both topics are involved in the mathematical component of judgment in situations of uncertainty and risk, crucial for decision-making in medicine, law, and other professions. Investigations have highlighted the need for teachers to be better prepared for classroom work on activities involving Bayes' theorem, as well as concepts such as conditional probability. Research on didactics has revealed, at various educational levels, numerous shortcomings concerning these topics, despite their major relevance to present-day life. In this report, an approach is proposed for organizing didactic situations aimed at improving teachers' mathematical and pedagogical knowledge in this regard, along with a discussion on probability teaching and learning based on a sequence of activities developed in the light of didactic engineering. The sequence was applied to teaching-degree students who are training as mathematics teachers. To address concepts involving Bayes' theorem and conditional probability in these activities, the questions articulate several representation types, including natural and symbolic language, tree diagrams, and unit squares. These last two representations provide support for discussions from an educational perspective, in which visualization serves not only to solve specific tasks, but also to elucidate the structure of mathematical concepts. The report concludes with an analysis of the didactic adaptation of this knowledge for teacher training.

RITUAL VS. EXPLORATIVE TEACHING OF UNIVERSITY MATHEMATICS TEACHERS

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KEYWORDS: discourse, explorative, ritual

There is a large body of research on teaching practices at the K-12 level, but very little research on the teaching practices of university teachers exists (Speer et al., 2010). Examining the teaching practices used by university mathematics teachers when lecturing, a topic within university mathematics education research is gaining an increasing interest. This paper reports on a discursive analysis of mathematical discourse on the derivative concept through the lens of the commognitive framework (Sfard, 2008).

The empirical data in this study consists mainly of videotaped lectures given by three teachers in calculus classrooms at a university of technology in Taiwan. It took all teachers 35 lessons to teach the concept of derivatives. The transcribed lectures were then analyzed, using Sfard's Commognitive Framework (2008) with its four components of mathematical discourse (words, visual mediators, narratives and routines) to try to distinguish the discursive patterns characterizing the teachers' respective discourses of derivative.

A categorization of routines was found in the pedagogical discourses of the teachers. The construction routines include stipulation, naming, motivation, and exploration routines. The substantiation routines include proof, auxiliary proof, and making contradiction routines. There are significant differences in the way the three teachers' pedagogical discourse is used in their lectures. These differences present themselves on the level and kind of discursive routines, and on the "ritual-explorative" continuum.

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PROSPECTIVE MATHEMATICS TEACHERS' VISUAL THINKING IN SOLVING MATHEMATICAL ANALYSIS PROBLEMS INVOLVING THE CONCEPT OF INFINITY

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KEYWORDS: infinity, mathematical analysis, visual thinking, semiotic representations, visual-analytic strategy

Undergraduate mathematics education students have difficulties in understanding the concept of infinity. This presentation explores undergraduate mathematics education students' visual thinking in solving mathematical analysis problems involving the concept of infinity. The study was qualitative research conducted with second-year undergraduate students at Great Zimbabwe University who had completed a Calculus course in the previous year. The participants were 10 Bachelor of Education in-service training students who majored in mathematics. The students were subjected to a cognitive test on the infinity concept. The data was qualitatively analysed and guided by Duval's theory of registers of representations. The result showed that the students had difficulties with the concept of infinity. Only one out of ten students was able translate correctly between countably infinite and uncountably infinite registers. It was also found that four students produced visual representations that approximated the limit of an infinite process. The results showed that visual thinking in mathematical analysis is not reliable when used to investigate the nature of the limit of an infinite function. The study can act as an eye-opener that may lead to further rigorous proof on the concept of infinity.

USE OF COMIC STRIPS IN TEACHING MATHEMATICS

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KEYWORDS: comic strips, teaching and learning mathematics, stimulate reflective thinking

“A picture is worth a thousand words.” Sometimes, it is difficult for students to comprehend newly introduced definitions or theorems in mathematics. The reasons vary and are often due to the fact that students may already have their own understanding of terms which however do not apply or carry the same meaning in the new definitions or theorems. The traditional way of explaining new or different concept of these terms can be challenging and not effective, especially for a large group of students. Comic strips can be used for this purpose. The recommended approach is to stimulate interest in reflective thinking so that students will search for new meaning and interpretation of these terms in all new circumstances, through proactive self-review and questioning. In general, students show interest and are enthusiastic about comic strips which presents a different and stimulative form of delivery in mathematics lectures.

BOOSTING STATISTICAL EDUCATION AND SKILL WITH FREE DESCRIPTIVE STATISTICAL SOFTWARE AMONG EMERGING YOUNG STATISTICIANS

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KEYWORDS: statistical software, students, free software, Sourceforge, statistical education

The inability of pre-tertiary and tertiary institution students to implement statistical software for learning and use in Statistics has been associated with the inability to use statistical software and the cost of proprietary software. This study has investigated the use of free statistical software as against proprietary, available on the Sourceforge online software repository, to improve statistical skills among pre-tertiary students and tertiary students. This study had investigated twenty free statistical software. The What, Where, When, How about each of the twenty software had been intensively, presented in order to improve the statistical software skill among the concerned class of students. Similarly, quantitative comparison of these free statistical software had been presented. It was concluded that this study will increase awareness of free statistical software, Statistics as a discipline and as skill among the students prior to the tertiary education admission.

WORKSHOP ABSTRACTS

DESIGNING MATH THAT MATTERS: LEVERAGING INTERDISCIPLINARY PARTNERSHIPS ON YOUR CAMPUS TO MOTIVATE LEARNING

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KEYWORDS: general education, contextualization, math modeling, statistics, non-mathematicians

Teaching mathematics to students whose majors do not require calculus can be challenging. Historically, many universities taught precalculus or statistics to fulfill general education requirements, but these topics were often divorced from application. In lieu of these typical non-calculus university mathematics courses, the Virginia Military Institute developed Math that Matters, a two course sequence grounded in evidence-based pedagogy and leveraging topics of interest at a local, national, and international level. These new courses were designed to develop learners who appreciate the value and usefulness of mathematics and feel empowered to engage in quantitative problem-solving in their careers and lives. In these courses, students learn to harness the power of basic statistics, data science, mathematical modeling, and technology to frame, solve, and communicate the answers to contextualized problems that were suggested by faculty specializing in diverse fields including art history, physical education, and biology. In this session, participants will learn about our courses and how we created them. We will also demonstrate a lesson and facilitate the work of participants as they take a module (set of lessons) and learn how to engage their local faculty to personalize Math that Matters lessons for their own school.

ARE YOU READY TO PLAY THE SCIENCE GAME? MELDING MATHEMATICAL SKILLS FOR TRANSITION INTO UNIVERSITY SCIENCE

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KEYWORDS: transition, science, mathematics skills

Students come to university to study science with varying educational levels and backgrounds. The role of a transition unit is to prepare them for their science studies and to introduce them to a challenging academic culture. Mathematical skills are used in all areas of science, but often students come ill-prepared for the mathematical requirements of their courses. Studies have shown that mathematics educational level is a strong indicator of future success for students embarking on science courses.

Designing a transition subject for science students provided an opportunity to incorporate mathematical activities which allow an authentic experience of science writing. It was seen as imperative to meld mathematics with science in a meaningful way to assist students to develop enough mathematical skills so they can perform at their best as they enter the science ‘game’.

This workshop centres on the skills students enter university with as they begin their studies in science. Examples of transition activities will be provided for participants to work through and discuss. It may be surprising to learn how tertiary students cope with mathematical activities set in different science and communication contexts.

STATISTICS AND ORIGAMI

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KEYWORDS: origami, statistics, recycle, authentic data, squareness

Origami is used as a source of authentic data for the statistics component of a first-year mathematics subject. Students work in groups to make their own origami paper and construct a polyhedron using unit origami. Making your own origami paper is harder than it sounds. Students also have to decide what data to select to statistically evaluate the “squareness” of the origami paper.

ALGEBRA USING POST-IT NOTES

Michael Rutkay

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Curtin College, Perth, Western Australia, Australia

KEYWORDS: algebra, manipulatives, teaching, solving, equations

The workshop demonstrates the use of Post-it notes in algebra as a way of keeping signs and coefficients with variables. The Post-it notes are used as a framework for students to build up a visualisation for correct algebraic techniques when rearranging or solving equations.

ENGAGING MATHEMATICIANS IN ADDRESSING THE NEEDS OF THE 21ST CENTURY LEARNER

Cami Sawyer

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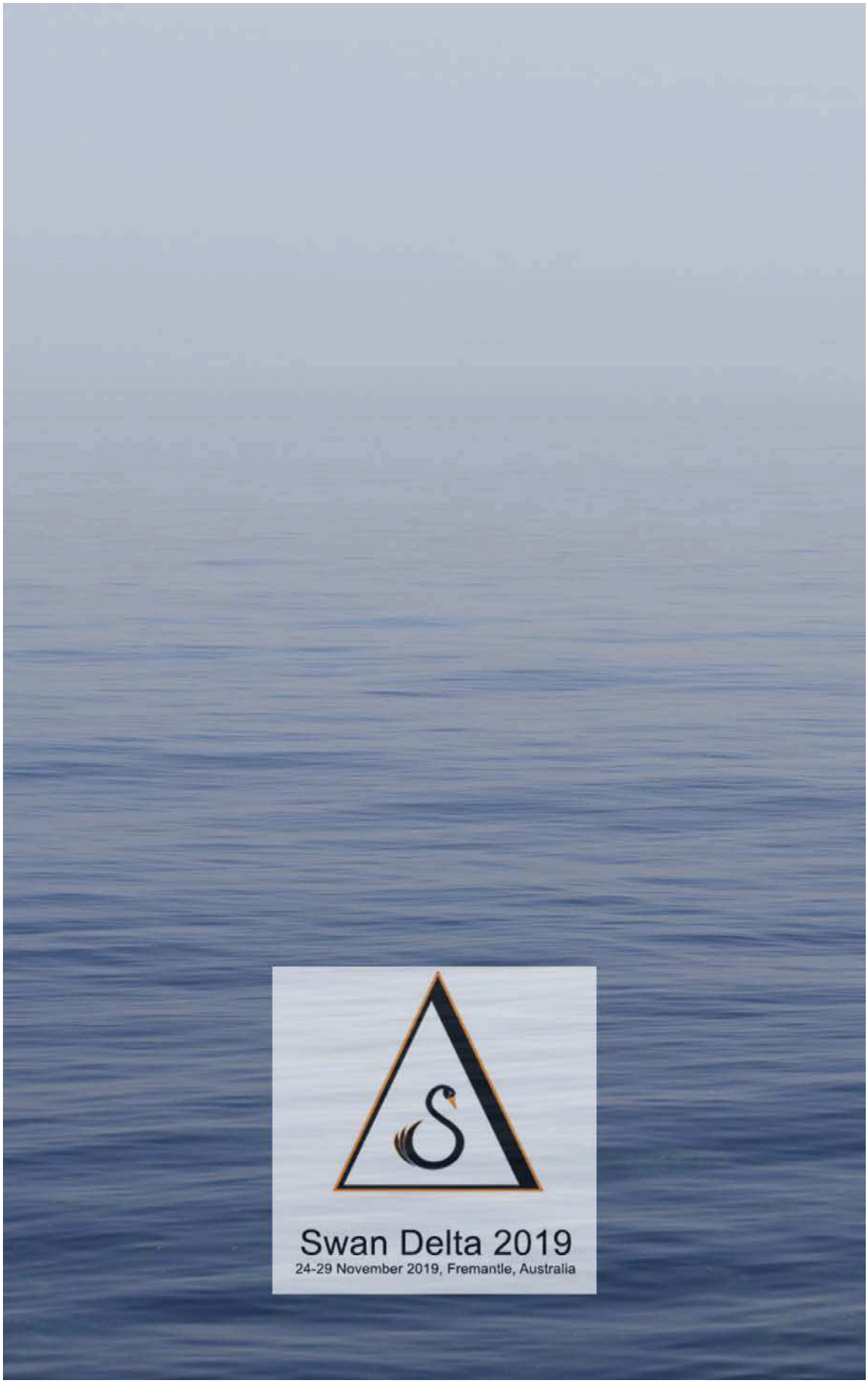
KEYWORDS: curriculum change, teaching and learning change, mathematical modelling

Being aware that the NZ education system was overdue for a curriculum review and that secondary Mathematics had very little change in the last review, a small group from the NZ Mathematical Society has been engaging mathematicians across NZ in rethinking teaching and learning mathematics. We have been holding local and national discussions with secondary and tertiary teachers. We have also developed an online discussion group to further the discussions. How do we engage people who were successful in traditional mathematics, in thinking about new approaches?

Through this process, we have engaged with the Ministry of Education as it begins to undergo a large-scale change in how secondary students will be assessed. A particular concern that we are helping with is promoting an equitable common mathematics pathway. Can our ideas help you address changes in your area or do you have experiences that you can share to help us?



Swan Delta 2019

24-29 November 2019, Fremantle, Australia