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Proceedings of the 14th Delta Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics

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FOREWORD

REFLECT. CONNECT. BE INSPIRED!

The 2023 “Table Mountain Delta” takes us past the 25th anniversary of the biennial Delta series of Southern Hemisphere Conferences on the Teaching and Learning of Undergraduate Mathematics and Statistics. Rotating between Southern hemisphere host countries since 1997, this is the 14th Delta Conference and the fourth to be held in South Africa.

The conferences are dedicated to research-based interaction around improving undergraduate mathematics and statistics education. They have played a crucial role in building a community of like-minded researchers in the global south and we trust that the 2023 event will continue this tradition.

We still tend to reference so many of our interactions as “pre-” or “post-pandemic”, and indeed this is our first post-pandemic in-person Delta. We continue to learn from and reflect on the educational impact of the past four years, and these conference proceedings include some contributions that do just that. However, the collection covers a range of topics from first-year selection and success to the handling of abstract algebra; a good indication of the breadth of research that underpins our communal endeavours as educators in the mathematical sciences, enriched but not defined by the disruptions of recent years.

As for previous Delta conferences, some of the papers to be presented appear in a special issue of the International Journal of Mathematical Education in Science and Technology. The conference proceedings which follow contain a further eight full papers which have undergone double-blind peer-review by at least two reviewers. Additional talks are selected on abstract submission, also subject to peer-review.

We owe a debt of gratitude to those who gave of their time and expertise in reviewing papers and abstracts. The balance of the editorial team extends particular thanks to Anita Campbell who did all the hard work in coordinating this communal effort.

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PAPERS

INTERNATIONAL APPROACHES TO DIAGNOSTIC TESTING FOR FIRST YEAR UNIVERSITY MATHEMATICS STUDENTS

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KEYWORDS: diagnostic testing, mathematics support, first-year students, student success, transition, assessment.

ABSTRACT

In this paper we describe the diagnostic testing processes and purposes for three universities in Australia, Ireland, and the United States, including their design and implementation. We outline the rationale for each process and provide results related to student success (and non-success) and discuss future research agendas in this area. We include summary statistics related to how students perform on certain fundamental mathematical topics, how at-risk and/or under-represented students achieve on such tests, and what actions students are recommended to take after their diagnostic test results. We also discuss pandemic-related impact of diagnostic testing and item classification via factor analysis.

INTRODUCTION AND LITERATURE REVIEW

Diagnostic Testing (shortened to DT from now on), also referred to as proficiency, competency, skills or mastery-based testing, has been widely used internationally for 30 years in university mathematics departments particularly for those entering their first year of study (Tall & Razali 1993). Its purposes, among others, range from supporting at-risk students who may be in danger of failing their first university mathematics course (Mullen & Cronin, 2022), informing faculty and management of student abilities, informing students of gaps in their abilities, requiring students to achieve a minimum standard (Rylands & Shearman, 2022), placing students in their appropriate first course (Reddy & Harper, 2013), and identifying falling standards in mathematics competencies and curricula (Hodd, Shao & Lawson, 2022).

Determining students' mathematical and/or statistical background, their readiness for university mathematics and the appropriate interventions has never been more important given the unprecedented numbers of students accessing higher education. While anecdotal perceptions abound that testing weaker students mathematically in this way simply reinforces their lack of confidence, fear or anxiety towards mathematics, feedback from students who undertake such DT indicates that students find it a good idea and that it is helpful to their learning (Mullen & Cronin, 2023). In this paper we describe the purposes, design and implementation of DT processes at three international universities, namely Western Sydney University in Australia, The University of Illinois Urbana-Champaign in the United States, and University College Dublin in Ireland.

BACKGROUND AND CONTEXT

Australian Context

Mathematics is not a mandatory subject in the Australian upper secondary school system. Since the early 2000s many universities have removed prerequisites for entry to specific programs with students being admitted on the basis of their Australian Tertiary Admissions Ranking (ATAR), a ranking which is derived from results of public exams in each state taken at the end of a student's secondary education. Whilst individual subjects often specify assumed knowledge which a student should possess before studying the subject, there is no requirement for a student to possess this before enrolling. This leads to students arriving for their tertiary study with a wide range of mathematical abilities and difficulties for academics in teaching to this diverse cohort.

WSU Context - Western Sydney University (WSU) in Australia was founded in 1989. It is a multi-campus university based in the western part of Sydney, Australia, with just under 50,000 students. Mathematics & Statistics Support (MSS) is available to all undergraduates and postgraduates, and is offered on six campuses as well as online. Timetable and travel constraints make it difficult for students to attend an MSS service on a campus other than the one where they are based. MSS services are provided by the Mathematics Education Support Hub (MESH) and services include drop-in support (usually one-on-one) and workshops in which students often work in groups facilitated by staff. MESH drop-in services are predominantly offered online with some in-person consultations taking place at a table in the relevant campus library, and in-person workshops occur in classrooms.

WSU uses diagnostic tests in a variety of ways. Due to the lack of prerequisites for incoming students, many new engineering students have a poor mathematical foundation. To help bridge the gap between secondary school and university mathematics, a preliminary mathematics course has been established for such students. All new students are automatically enrolled in this course, but they have the option of taking a 50-question multiple-choice diagnostic test to assess their readiness to move directly on to the mainstream first-year mathematics for engineers course. This test has been shown to be effective in identifying students who are capable of taking the more rigorous course, and it has resulted in a decrease in the failure rate for the course (Rylands & Shearman, 2022).

Diagnostic tests have also been used in two other subjects (one in mathematics, one in computing) to help students reach the required level of numeracy for the subject they are studying (Rylands & Shearman, 2022). Each of these tests are relatively short (10 - 15 questions) and can be taken by students multiple times during the semester with support available for students who fail to meet the threshold requirement. The use of the tests has been beneficial in encouraging students to develop the required level of numeracy and mathematics and again has helped to reduce the fail rate of the subjects (Rylands & Shearman, 2022).

MESH has also been developing a series of diagnostic tests to inform students of the particular areas they need to focus on in a series of refresher courses. Prior to the COVID-19 pandemic these courses were delivered face-to-face, however the lockdowns brought on by COVID-19 forced MESH to redesign these as online courses which could be delivered with minimal interaction. As students entered their study with widely varying mathematical skills, this led to some students disengaging with the content due to a perception that it was too easy for them. The diagnostic tests are designed to direct students to the content that they most need to review as was done in face-to-face classes.

The last group of diagnostic tests were developed using a computer adaptive testing algorithm in the Numbas test platform. A computer adaptive test uses answers provided by students to determine the next question to be asked. If a student answers a question correctly it is assumed that any question on which this question depends would also be answered correctly, so these questions are not presented to the student. Similarly, if a question is answered incorrectly it is assumed that the student would not be able to answer a question which depends on this question, so these questions are marked as incorrect. In this way the overall length of the DT can be significantly decreased making students more likely to complete the test. To develop these tests, it is first necessary to construct a “knowledge map” (Numbas terminology) or a partially ordered knowledge structure (POKS) (Desmarais, Maluf & Liu 1995), which links topics (questions) in the test to allow the adaptive test algorithm to infer which topics have been mastered based on a subset of the questions available. While some authors (Desmarais, Maluf, & Liu 1995; Appleby, Samuels & Treasure-Jones 1997) suggest that this knowledge map should be developed from the planned test questions using various statistical techniques, the MESH team has reversed this process, developing the knowledge map from expert knowledge, then developing questions for each topic. In other words, the MESH team first identifies the topics that need to be covered in the test, and then develops questions that are aligned with those topics. This approach is based on the idea that mathematics subject experts have a deep understanding of the subject matter, and that their knowledge can be used to create a more accurate, reliable knowledge map (see Fig 1).

The output of this test provides students with an indication of their abilities in a number of learning objectives (see Fig 2) which provides them with a guide to which areas of the refresher course they need to focus on. Learning objectives correspond to specific topics within each course.
We are currently analysing data from the first 700+ uses of the diagnostic tool to:

- Assess the appropriateness of the questions developed
- Confirm the validity of the knowledge map
- Assess the effectiveness of the adaptive test algorithm being used.

Results of these analyses will be published in due course.

United States Context

There is no national mathematics curriculum or mathematics examination in the United States (US) high school system. There are also no statewide curriculum, examinations, measurements, or requirements. Secondary mathematics subject offerings vary (widely) from school to school, and are heavily dependent on the wealth of the community. Underserved communities and underrepresented student populations have limited access to learning opportunities inside and outside the classroom. Students enter higher education with mathematical preparation ranging from basic arithmetic to differential equations and/or linear algebra. State legislation (government) often dictates access to, and didactics and pedagogy in, entry level mathematics courses in higher education.

UIUC Context - The University of Illinois Urbana-Champaign (UIUC), founded in 1867, is ranked the 13th public university in the United States and 64th in the QS World University Rankings 2024. The student population is 56,000 with an entering undergraduate class of 8,000 students per year. UIUC employs over 4,000 faculty with a 21:1 student - faculty ratio. UIUC has always viewed proper placement of incoming students as a high priority and found a way to provide effective placement on a large scale. UIUC requires all incoming freshmen (first-year students) and transfer students to take the ALEKS PPL Mathematics Assessment Exam to assess students' mathematical knowledge for advising and course placement. ALEKS PPL uses adaptive, open-response questioning to assess each student on a wide range of course material. Students receive a learning report and individualised learning plan, see Appendix for a page 1 sample.

Students are then advised, correctly placed, and given the opportunity to increase their mathematical knowledge in a targeted learning environment. A seamless transition from assessment to learning empowers students and builds learning trajectories.

Findings from UIUC indicate that mathematical background knowledge (measured by ALEKS PPL) predicts course outcomes and course outcomes improved with higher placement exam scores; findings also indicate that some subpopulations of students at UIUC are disproportionately enrolled in mathematics courses below the Calculus I level. Furthermore, they disproportionately represent the number of unsuccessful students in these courses.

Some supporting data below shows the percentage of students in the Fall 2022 gateway mathematics courses who were underrepresented minorities and/or first-generation appears below. Notice how enrollment continually decreases as mathematical complexity increases. Furthermore, the earlier courses are prerequisites for later courses, e.g. Math 220 is a prerequisite for Math 231, hence the lower the initial placement the more semesters required for STEM degrees.

Table 1 Percentage of Underserved Student Populations Enrolled in Gateway Courses Fall 2022

Math 101	77.1%	Math 115	51.9%	Math 231	19.7%
Math 112	67.6%	Math 220	35.0%	Math 241	19.1%

Table 2 Overall Percentages of Non-Successful Grades (DFW, NC, I, and ABS) Fall 2022

Math 101	12.7%	Math 115	15.4%	Math 231	8.7%
Math 112	13.1%	Math 220	19.9%	Math 241	12.2%

However, students in the underrepresented and underserved groups earned a disproportionate number of those grades, e.g., 4 out of every 5 of the MATH 101 students who earned non-successful grades were from underrepresented and underserved cohorts. The percentages of non-successful grades in these Fall 2022 gateway courses that were earned by underrepresented minorities and/or first-generation students appear below:

Table 3 Percentages of Non-Successful Grades for Underserved Student Populations Fall 2022

Math 101	85.7%	Math 115	65.8%	Math 231	35.4%
Math 112	87.0%	Math 220	56.8%	Math 241	35.3%

Republic of Ireland Context

Though mathematics is technically not a mandatory subject in the upper secondary school curriculum in Ireland, it is taken by virtually all (≈97%) terminating students. Irish universities have requirements in mathematics for courses such as business, engineering, science (including computer science) and agricultural and health sciences. Typically students take six subjects at either higher, where up to 100 matriculation points are available, or ordinary level (where up to 60 points are on offer). Significant mathematics curriculum reform has been happening in the past ten years including the added incentive to take higher level mathematics where 25 bonus points are on offer for taking and passing the higher level mathematics course for matriculation to university. This has resulted in the desired and significant increase in the number of students completing higher level mathematics but this move by students to higher mathematics for the bonus points has led to poorer standards of incoming students due to them studying mathematics at a level that is too high for many. Hence in recent years there has been an increase in the number of Irish Higher Education Institutions offering diagnostic testing to ascertain the extent of this issue and what support can be put in place to mitigate it (Hyland & O'Shea, 2022).

UCD context - University College Dublin (UCD) has approximately 38,000 students, is ranked within the top 1% of higher education institutions worldwide with 35% of undergraduate students from under-represented cohorts. It is considered Ireland's global university with 31% of its student body comprising international students from 144 nations. UCD's School of Mathematics and Statistics (SMS) has used DT for many years with mature[a] students upon their entry to the Access to Science, Engineering Agriculture and Medicine programme. In recent years the SMS under the auspices of the Maths Support Centre (MSC) has used DT on a larger scale. Since 2020, approximately 1,650 incoming first-year students from the four degree programmes of Agriculture and Health Sciences, Business, Engineering, and Science students have been invited to participate in MathsFit. This initiative includes a Diagnostic test (known to the students as a Proficiency Quiz), and is just one part of the suite of in-person and digital supports offered to these students throughout their first year of university study.

Data is collected and analysed from students' results on three sections of the fundamentals of mathematics for university (typically learned by 15-16-year old pupils in Ireland's high school system) on Algebra, Arithmetic and Trigonometry, and Functions and Calculus. There are eight questions on each section, it is taken online through the university's Virtual Learning Environment Brightspace via the open source e-assessment tool Numbas. Students typically take the test within 45 minutes. The multi-factorial rationale for the DT include: alerting students to their present knowledge state, raising awareness of the basic mathematics skills needed for successful completion of their first mathematics course at university, encouraging participation at the university's Mathematics Support Centre (MSC) from those identified most at-risk of underperforming based on the DT results, and predicting performance on their first university mathematics examination. A diagnostic test cannot help students reach the required level of mathematical proficiency required to successfully engage in university mathematics. It can only assist the student and/or the lecturer in identifying issues that need to be addressed. Subsequently, the lecturer's or tutor's input according to data from the test may result in a betterment of students' understanding. Thus key to the taking of the MathsFit DT is the automated feedback email (See Fig 4 in the appendix) students receive within an hour of taking their first attempt at MathsFit. This shows the student their DT results and identifies areas they need to work on further if applicable.

Each test item is linked to an online refresher course and students are highly advised to engage with this refresher course before attempting the Dt again. The questions are randomised and parameterised so that their attempt 2 is not identical to attempt 1. Awarding a small amount of continuous assessment marks (2-3.3%) for participating in MathsFit ensures a high uptake of student engagement. This can be seen from 2023-24 data for one course which had 2% assigned to it that year and engagement was high whereas in previous years(2020-2022) there was very low engagement with attempt 2 of MathsFit. There is evidence that students with weaker mathematical backgrounds, in particular those from the traditionally underrepresented cohorts of HEAR, DARE[b], Mature and International students benefit academically, as measured by the final course grade in their first mathematics examination, by visiting the MSC, with this benefit increasing as the number of MSC visits increases (Mullen, Cronin, Taylor, & Liu, 2021).

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[a] Students aged 23 years or above upon year of university registration

[b] Students who enter university from lower socio-economic status or with a disability

At the time of writing there have been 3,299 participants from the four courses mentioned above over the three academic years from 2020-21 to 2022-23. There were 1,825 survey responses and eight in-depth one-on-one semi-structured interviews[c] conducted with students over these three years. Based on this feedback students reported that the Maths Fit DT process (i) aids them in their revision of basic mathematics skills for their first mathematics course at university, (ii) communicates effectively to them the mathematical expectations of their first mathematics course at university, and (iii) offers them greater awareness, access and use of both the online and in-person mathematics supports available at UCD's MSC. The interviews were conducted via an iterative action research process so that student input could be integrated into the evolving design and implementation of MathsFit in subsequent iterations.

DISCUSSION AND CONCLUSION

In this paper we discussed the implementation of diagnostic testing, how different populations are affected, and the potential ramifications and perceptions of being underprepared or underrepresented. Diagnostic testing programs and policies are high stakes. It is a balancing act between granting access and predicting the probability for success. US data has shown that students are less likely to graduate college if they begin with a developmental / remedial course (CCA, 2012). This has lifelong ramifications on social and economic mobility. We also considered who is primarily affected by diagnostic testing and the campus programs designed to support such students and courses. There is little international data regarding the effectiveness and comparison of such programs, but the necessity nonetheless remains apparent.

Apart from the placement function of the engineering DT at WSU, both academics and students have access to the DT results on a question-by-question basis. This serves to guide students to resources targeting their problems such as MESH support, and also alerts teaching staff to find the areas in which students have gaps. In addition, support staff at MESH are guided by the DT results in creating workshops for these students. The WSU DT experience has given rise to purposes beyond those that the original DT were designed to serve. Thus far, DT at WSU is deemed to be a success, with academics reporting that designing, implementing and grading tests is time well spent. A common feature of the WSU DT is that it empowers students to take action via seeking support at MESH. Support and resources for students following DT are important as these enable students to take action to improve.

UIUC internal studies conducted by the third author provide evidence that data from an internal mathematics placement program is an effective tool for predicting course outcomes independent of additional predictor attributes. In particular, it is possible to connect course outcomes to data from the placement scores as measured by ALEKS. These are key findings for best practices for serving all students. There continues to be a lack of availability of external measures (e.g., ACT, SAT, AP, high school GPA) due to the pandemic and so institutions have an increased reliance on internal measures to assist students with the transition to higher education. Additionally, the findings suggest that lacking a fundamental mathematical concept item can have a substantially negative effect on course outcomes, thus highlighting the importance of providing support to serve students who enter higher education less prepared.

Underrepresented and underserved student populations were, and continue to be, disproportionately affected by the pandemic. They had fewer mathematical learning opportunities and less access to mathematical learning supports. Because these students enter higher education less quantitatively prepared than their peers who attended better-resourced schools,

[c] Only eight students came forward to volunteer their time for the in-depth MathsFit interview

they are generally unable to qualify through the ALEKS PPL for Calculus I, which is ideally the starting point for mathematics coursework at Illinois. It is important to note that the issue of students entering the higher education system with weaker quantitative knowledge and skills has ramifications not only for the UIUC Department of Mathematics but also for many other units on campus that have mathematics prerequisites or corequisites.

At UCD, analysis shows that a large proportion of those students identified as most at-risk of underperforming or failing their first mathematics course at university (i.e. those who scored Bronze in one or more of the three basic areas assessed in MathsFit) take up the in-person or online mathematics support on offer. They also engage with this support earlier in their first trimester at university than those from their pre-covid era counterparts. While the MathsFit suite of supports was largely part of a PhD project, its success at engaging weaker students earlier with their mathematics means the DT elements and follow-on supports will be maintained at UCD for the foreseeable future. Discussions are now underway at a School Teaching and Learning level on aligning the DT with the courses' learning outcomes so summative assessment marks can be assigned to MathsFit participation. The embedding of MathsFit within the course structure both as a diagnostic tool and revision aid is central to its sustainability

This paper, while a summary of three international universities' approach to DT and follow on supports, also encourages scope for further research for improved student learning by leveraging data and insight related to diagnostic testing. It is not unexpected to have a misalignment between secondary school and college course expectations as college readiness as a concept is somewhat poorly defined, but it should be uncommon for students to have to repeat, or enroll in, preparatory coursework once entering higher education. Diagnostic testing and the implementation of supports are strategies to nurture and promote the mathematical strengths of every student. Best practices and educational efficacy for all students has the best chance of improving successful outcomes. We hope that this paper encourages further discussion internationally to better understand the process and benefits of diagnostic testing and best serve our students.

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APPENDIX
WSU Context

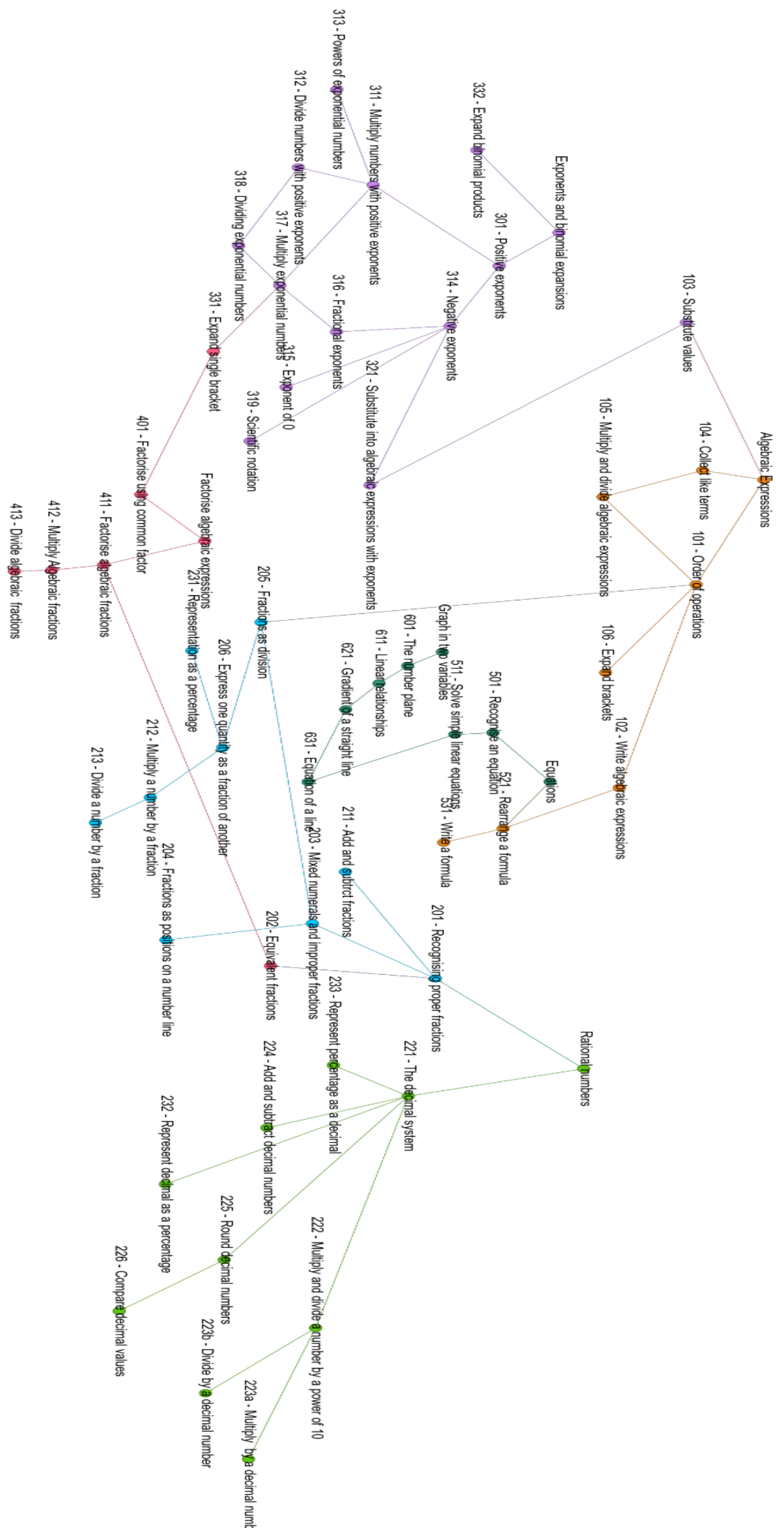

Figure 1: Knowledge map for Algebra 1 Diagnostic Test

Algebra 1 Diagnostic Tool

Learning objective	Score
Algebraic expressions	83%
Rational numbers	73%
Exponents and binomial expansions	46%
Factorise algebraic expressions	0%
Equations	25%
Graph in two variables	75%
Total	**57%**

If you scored less than 80% in any learning objective (topic) you should review that section of the Algebra 1 Module.

The test is over. You need to do some more work on the following learning objectives: Rational numbers, Exponents and binomial expansions, Factorise algebraic expressions, Equations, Graph in two variables.

Figure 2: Sample output for Algebra 1 Diagnostic Test

UIUC Context

Learning Pie Report

Date: 10/13/2023

[redacted]

Cohort Name : Incoming Students, AY 23-24 2023

Institution Name : University of Illinois--Urbana-Champaign

Prep and Learning Module : Prep for Calculus

Initial Knowledge
08/15/2023
140 Mastered | 22 Learned | 119 Remaining

Ready To Learn 91

Equations and Inequalities

Solving a linear inequality: Problem type 4	0/3
Solving a compound linear inequality: Interval notation	0/3

Exponents and Polynomials

Product rule with negative exponents	0/3
Factoring a polynomial by grouping: Problem type 1	0/3
Factoring a product of a quadratic trinomial and a monomial	0/3
Solving a quadratic equation using the square root property: Exact answers, basic	0/3
Applying the quadratic formula: Exact answers	0/3
Multiplication involving binomials and trinomials in two variables	0/5
Solving a word problem using a quadratic equation with irrational roots	0/5
Solving a word problem using a quadratic equation with rational roots	0/5
Power and product rules with positive exponents	0/3
Solving a quadratic inequality written in factored form	0/5
Power rules with negative exponents	0/3

Lines and Systems

Graphing a linear inequality in the plane: Standard form	0/5
Writing the equation of a line through two given points	0/3
Solving a percent mixture problem using a system of linear equations	0/3
Solving a distance, rate, time problem using a system of linear equations	0/5
Writing an equation and drawing its graph to model a real-world situation: Advanced	0/3
Graphically solving a system of linear equations	0/3
Application problem with a linear function: Finding a coordinate given the slope and a point	0/3
Application problem with a linear function: Finding a coordinate given two points	0/3
Finding slopes of lines parallel and perpendicular to a line given in the form Ax + By = C	0/3

Functions and Graphs

Learning Pie Report | Page 1 / 8

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Figure 3: Sample of knowledge map and partial learning report for ALEKS UCD context

Dear {InitiatingUserFirstName},

Congratulations on completing the MathsFit quiz!

- You achieved Gold in the Arithmetic and Trigonometry Section.
- You achieved Silver in the Algebra section.
- You achieved Bronze in the Functions and Calculus section.

You have earnt 1.5% of your MATH10250 assessment marks.

To earn the full 3% available, go for gold in Algebra and Functions and Calculus by engaging with our Refresher course, available in your MATH10250 Brightspace page, and completing the quiz again. The quiz will be available until 9pm on Saturday 30th September.

We also recommend a visit with a Maths Support tutor at the Maths Support Centre (MSC) to look at any specific parts of Functions and Calculus you need help with. Individual and group sessions are available, see the MSC website for times.

The MSC offers one-to-one or small group support sessions with experienced maths tutors both virtually and on campus for first and second year maths and statistics modules. We're here to keep you MathsFit!

Well done again and remember you can only get more MathsFit.

Figure 4: An example of the automated and adaptive feedback email students of MathsFit receive through the Brightspace VLE upon completion of their first proficiency quiz attempt

ANALYSIS OF UNDERGRADUATE MATHEMATICS STUDENTS' FIRST ENCOUNTER WITH EQUIVALENCE RELATIONS IN GROUP THEORY

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KEYWORDS: group theory, commognition, equivalence relations

ABSTRACT

This study investigates aspects of undergraduate mathematics students' learning in their first encounter with Group Theory. Research in the learning of Group Theory proves significant, since various studies have reported that novice students consider it as one of the most demanding subjects in their syllabus. In particular, this qualitative study investigates undergraduate mathematics students' responses to two mathematical tasks on equivalence relations. For the purposes of this study there has been used the Commognitive Theoretical Framework. Analysis suggests that these students' first encounter with equivalence relations is challenging. There have emerged three categories of errors and inaccuracies in students' solutions. The first category of errors is related to the proof of the size of equivalence classes, which is predominantly due to incomplete object-level learning of the form and structure of equivalence classes as well as the notion of bijection. The second category includes several errors regarding the proof of symmetry and transitivity. The third category is related to the distinction between the elements of the set X and the elements of the group $Sym(X)$, when these coexist in the same context.

INTRODUCTION

Equivalence relations is typically part of an introductory course in Group Theory. This study is part of a larger investigation on undergraduate mathematics students' first encounter with this topic. Research in the learning of Group Theory proves significant, since novice students consider this course as one of the most demanding subjects in their syllabus. It is "the first course in which students must go beyond 'imitative behavior patterns' for mimicking the solution of a large number of variations on a small number of themes" (Dubinsky et al., 1994, p268). In particular, this study aims to investigate undergraduate mathematics students' responses to two mathematical tasks that involve the elements of equivalence classes, the proof of symmetry and transitivity of equivalence relations, and the proof that a given relation is in fact an equivalence relation.

This study is part of a larger study (Ioannou, 2012), which investigates the overall learning experience of undergraduate mathematics students in their first encounter with Group Theory, and a development of Ioannou (2018b) where the focus was on undergraduate mathematics students' responses to a mathematical task that involved equivalence relations, and in which students were expected to deal with the newly introduced notion of symmetric group and the notion of set, and their elements, simultaneously. This study uses the Commognitive Theoretical Framework (CTF) proposed by Sfard (2008). Presmeg (2016, p. 423) suggests it is a theoretical framework of unrealised potential, designed to consider not only issues of teaching and learning of mathematics per se, but to investigate “the entire fabric of human development and what it means to be human.” It proves to be an astute tool for the comprehension of diverse aspects of mathematical learning, which although grounded on discrete foundational assumptions, can be integrated to give a more holistic view of the students’ learning experience (Sfard, 2012).

THEORETICAL FRAMEWORK

CTF is a coherent and rigorous theory for thinking about thinking, grounded in classical Discourse Analysis. It involves a number of different constructs such as metaphor, thinking, communication, and commognition, as a result of the link between interpersonal communication and cognitive processes (Sfard, 2008). In mathematical discourse, objects are discursive constructs and form part of the discourse. Mathematics is an autopoietic system of discourse, namely “a system that contains the objects of talk along with the talk itself and that grows incessantly ‘from inside’ when new objects are added one after another” (Sfard, 2008, p. 129). Moreover, CTF defines discursive characteristics of mathematics as the word use, visual mediators, narratives, and routines with their associated metarules, namely the how and the when of the routine. In addition, it involves the various objects of mathematical discourse such as the signifiers, realization trees, realizations, primary objects and discursive objects. It also involves the constructs of object-level and metadiscursive level (or metalevel) rules. Thinking “is an individualized version of (interpersonal) communicating” (Sfard, 2008, p. 81). Contrary to the acquisitionist approaches, participationists’ ontological tenets propose to consider thinking as an act (not necessarily interpersonal) of communication, rather than a step primary to communication (Nardi et al., 2014).

Mathematical discourse involves certain objects of different categories and characteristics. Primary object (p-object) is defined as “any perceptually accessible entity existing independently of human discourses, and this includes the things we can see and touch (material objects, pictures) as well as those that can only be heard (sounds)” (Sfard, 2008, p. 169). Simple discursive objects (simple d-objects) “arise in the process of proper naming (baptizing): assigning a noun or other noun-like symbolic artefact to a specific primary object. In this process, a pair <noun or pronoun, specific primary object> is created. The first element of the pair, the signifier, can now be used in communication about the other object in the pair, which counts as the signifier’s only realization. Compound discursive objects (d-objects) arise by “according a noun or pronoun to extant objects, either discursive or primary.” In the context of this study, groups are an example of compound d-objects.

Sfard (2008) describes two distinct categories of learning, namely the object-level and the metalevel learning. “Object-level learning […] expresses itself in the expansion of the existing discourse attained through extending a vocabulary, constructing new routines, and producing new endorsed narratives; this learning, therefore results in endogenous expansion of the discourse” (Sfard, 2008, p. 253). In addition, “metalevel learning, which involves changes in metarules of the discourse […] is usually related to exogenous change in discourse. This change means that some familiar tasks, such as, say, defining a word or identifying geometric figures, will now be done in a different, unfamiliar way and that certain familiar words will change their uses” (Sfard, 2008, p. 254).

CTF has proved particularly appropriate for the purposes of this study, since, as Presmeg (2016, p. 423) suggests, it is a theoretical framework of unrealized potential, designed to consider not only issues of teaching and learning of mathematics per se, but to investigate “the entire fabric of human development and what it means to be human.”

LITERATURE REVIEW

Research in the learning of Group Theory is relatively scarce compared to other university mathematics fields, such as Calculus, Linear Algebra or Analysis. The first reports on the learning of Group Theory appeared in the early 1990's, following mostly a constructivist approach. The grasp of the newly introduced notion of group is often an arduous task for novice students and causes serious difficulties in the transition from the informal secondary education mathematics to the formalism of undergraduate mathematics (Nardi, 2000). Similarly, the notion of subgroup, with the application of the Subgroup Test is considered another laborious mission to accomplish for novice students. In fact, Ioannou (2018a) suggests that novice students face numerous difficulties regarding both the object-level and metalevel learning of the notion of subgroup.

Students' difficulty with the construction of the Group Theory notions is partly grounded on historical and epistemological factors: "the problems from which these concepts arose in an essential manner are not accessible to students who are beginning to study (expected to understand) the concepts today" (Robert and Schwarzenberger, 1991). Nowadays, the presentation of the fundamental notion of Group Theory, namely group, subgroup, coset, quotient group, etc. is "historically decontextualized" (Nardi, 2000, p169), since historically the fundamental concepts of Group Theory were permutation and symmetry. Moreover, this chasm of ontological and historical development proves to be of significant importance in the metalevel development of the group-theoretic discourse for novice students.

Furthermore, research suggests that students' understanding of the notion of group proves often primitive at the beginning, predominantly based on their conception of a set. An important step in the development of the understanding of the notion of group is when the student "singles out the binary operation and focuses on its function aspect" (Dubinsky et al., 1994, p292). Students often have the tendency to consider group as a "special set", ignoring the role of binary operation. Iannone and Nardi (2002) suggest that this conceptualization of group has two implications: the students' occasional disregard for checking associativity and their neglect of the inner structure of a group. These last conclusions were based on students' encounter with groups presented in the form of group tables. In fact, students when using group tables adopt various methods for reducing the level of abstraction, by retreating to familiar mathematical structure, by using canonical procedure, and by adopting a local perspective (Hazzan, 2001).

An often-occurring confusion amongst novice students is related to the order of the group G and the order of its element g. This is partly based on student inexperience, their problematic perception of the symbolization used and of the group operation. The use of semantic abbreviations and symbolization can be particularly problematic at the beginning of their study. Nardi (2000) suggests that there are both linguistic and conceptual interpretations of students' difficulty with the notion of order of an element of the group. The role of symbolization is particularly important in the learning of Group Theory, and problematic conception of the symbols used probably causes confusion in other instances.

In relation to equivalence relations, Ioannou (2018b) reports on the students' difficulty to distinguish between the elements of the group $Sym(X)$ and the elements of the set X. This study has identified two types of errors. The first was predominantly due to incomplete object-level learning of the definition of group and the difficulty to distinguish between the role of the set X and of the group $Sym(X)$, when these coexisted in the same context. The second was associated with incomplete object-level learning of these notions, which appears to have an unfavorable impact on the application of the routine for proving that a given relation is indeed equivalence.

METHODOLOGY

The present study is a ramification of a larger research project (Ioannou, 2012), which conducted a close examination of Year 2 mathematics students' learning experience in their first encounter with Abstract Algebra. The course was taught in a research-intensive mathematics department in the United Kingdom. It was mandatory for Year 2 mathematics undergraduate students, and a total of 78 students attended it. The course was spread over 10 weeks, with 20 one-hour lectures and three cycles of seminars in weeks 3, 6 and 10 of the semester. The role of the seminars was mainly to support the students with their coursework. There were 4 seminar groups, and the sessions were each facilitated by a seminar leader, a full-time faculty member of the school, and a seminar assistant, who was a doctorate student in the mathematics department. All members of the teaching team were pure mathematicians. The course assessment was predominantly exam-based (80%). In addition, the students had to hand in a threefold piece of coursework (20%) by the end of the semester.

The gathered data included the following: Lecture observation field notes, lecture notes (notes of the lecturer as given on the blackboard), audio-recordings of the 20 lectures, audio-recordings of the 21 seminars, 39 student interviews (13 volunteers who gave 3 interviews each), 15 members of staff's interviews (5 members of staff, namely the lecturer, two seminar leaders and two seminar assistants, who gave 3 interviews each), student coursework, markers' comments on student coursework, and student examination scripts. For the purposes of this study, there have been analyzed the data gathered from the thirteen volunteers. The interviews, which covered a wide spectrum of themes, were fully transcribed, and analyzed with comments regarding the mood, voice tone, emotions and attitudes, or incidents of laughter, long pauses etc., following the principles of Grounded Theory, and leading to the "Annotated Interview Transcriptions", where the researcher highlighted certain phrases or even parts of the dialogues that were related to a particular theme. Furthermore, coursework and examination solutions were analyzed in detail, after the data collection period, using the CTF, and mostly focusing on issues such as students' engagement with certain mathematical concepts, the use of mathematical vocabulary and symbolization, and the application of discursive rules.

Finally, all emerging ethical issues during the data collection and analysis, namely, issues of power, equal opportunities for participation, right to withdraw, procedures of complaint, confidentiality, anonymity, participant consent, sensitive issues in interviews, etc., were addressed accordingly.

DATA ANALYSIS

This study investigates the three categories of emerging errors in the students' solutions of two mathematical tasks, which are related to equivalence relations in the context of Group Theory, and the application of the routine for proving that a given relation $\sim$ is indeed equivalence, namely reflective, symmetric and transitive. The formal definition is given below:

> Suppose A is any set. A relation $\sim$ on A is specified by a subset of A^2, we write (for $\mathrm{a}, \mathrm{b} \in \mathrm{A}$) $\mathrm{a}\sim\mathrm{b}$ to mean (a, b) in this subset and $\mathrm{a} \nsim \mathrm{b}$ if not. The relation $\sim$ on A is: reflexive if $\forall \mathrm{a} \in \mathrm{A}, \mathrm{a}\sim\mathrm{a}$; symmetric if $\forall \mathrm{a}, \mathrm{b} \in \mathrm{A}, \mathrm{a}\sim\mathrm{b}$ then $\mathrm{b}\sim\mathrm{a}$; and transitive if $\forall \mathrm{a}, \mathrm{b}, \mathrm{c} \in \mathrm{A}$, if $\mathrm{a}\sim\mathrm{b}$ and $\mathrm{b}\sim\mathrm{c}$, then $\mathrm{a}\sim\mathrm{c}$. A relation $\sim$ on a set A is called an equivalence relation on A, if it is reflexive, symmetric and transitive. If $\sim$ is an equivalence relation on A, then a subset of A of the $[b]^{\sim} = \{a \in A : b\sim a\}$ (for $b \in A$) is called a $\sim$-equivalence class.

The first task was part of the final examination (Figure 1).

(ii) Suppose G is a group and H a subgroup of G. Prove that the relation $\sim$ on G given by

$$g_1 \sim g_2 \text{ if and only if } g_1^{-1}g_2 \in H$$

is an *equivalence relation*, saying carefully what this means. In the case where G is a finite group, prove that all equivalence classes have $|H|$ elements. **[7 marks]**

Figure 1: Examination Task on Equivalence Relations (FEE4ii)

The first category of errors is related to students' incomplete learning of the elements of equivalence classes. An equivalence class is of the form $\{g_2: g_1 \sim g_2\}$ for some $g_1 \in G$. By definition, this is $\{g_1h: h \in H\}$. The map H→ $\{g_1h: h \in H\}$, i.e., $h \mapsto g_1h$ is a bijection, therefore the number of elements in the set is$|H|$. Of the eleven students that chose to do the exercise including this task, four did not attempt this part at all, and only seven (7/11) attempted to solve it, with only one, producing a flawless solution. The main problem with this part of the exercise is related to students' object-level learning of the structure and form of equivalence classes. Consequently, the remaining six students, whose solution was problematic, seemed not to be able to define a bijection that would prove that the number of elements in each equivalence class is $|H|$. In order to be able to solve this part of the exercise, one should have already complete object-level learning of the structure of equivalence classes, objectified all the previously given definitions of the involved notions, and overcome any misconceptions regarding the object of equivalence relation, bijection, order of group as well as group operations. As the analysis below possibly suggests, an important reason of students' difficulty with this exercise is the incomplete object-level learning of the definition of equivalence classes. Again, this error results from the incomplete object-level learning and the distinction between the set and their group and consequently the distinction of their elements. Below, analysis includes examples of the errors that occurred in relation to this issue.

A common error in the proof of the size of equivalence classes is the omission of defining a bijection in order to prove that the size of equivalence classes is $|H|$. For instance, although Student A has successfully solved the first part of the task, regarding the proof of equivalence relation, showing complete object-level and metalevel learning, and consciousness in the application of the related routine, yet she is still not able to prove that all equivalence classes have $|H|$ elements. There is no indication of attempting to define a bijection that would lead her to prove it, albeit the fact that her use of mathematical vocabulary and notation is flawless. She is the only student that uses the more 'sophisticated' notation, used by the lecturer, i.e., $[g_1]_\sim = \{g_2: g_1 \sim g_2\}$. This suggests that Student A has read the lecture notes thoroughly and has endorsed and adopted the group theoretic vocabulary used by the lecturer. The above analysis indicates that Student A has the potential to endorse all the involved object-level rules regarding the equivalence relations as well as the routine's governing metarules, yet she is still in the process of doing so.

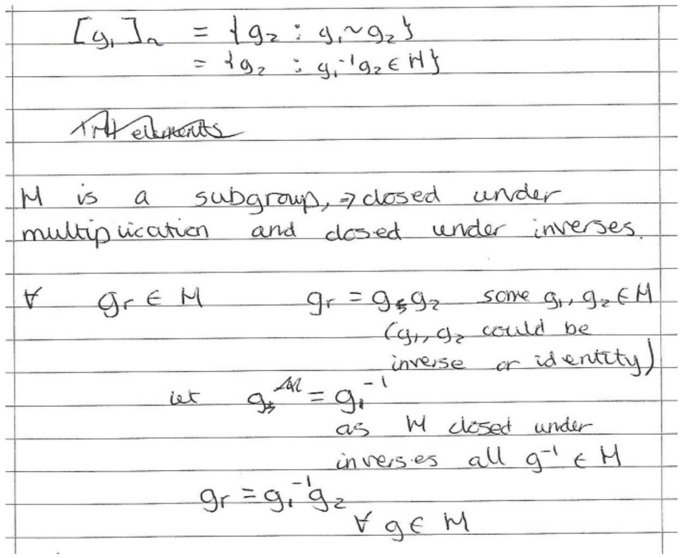


Figure 2: Part of Student A's Solution

Another example, in which object-level and metalevel learning are incomplete, appeared in Student B's solution of the examination task. Student B has not managed to prove that equivalence classes are of size $|H|$. There are indications of incomplete object-level learning regarding the order of the group and the group as such, as the notation in the narratives suggests. For instance, she treats $\left|g_i^{-1}g_j\right|$ as an element of the group, trying to find its inverse. This indicates incomplete objectification of the definition of the inverse of an element. In addition, her attempt to solve this task lacks explicitness and clarity in her solving approach. Her proof indicates incomplete understanding of how she is going to apply the required routine. Similar to Student A, Student B is still not able to define a bijection that would allow her to achieve the expected outcome.

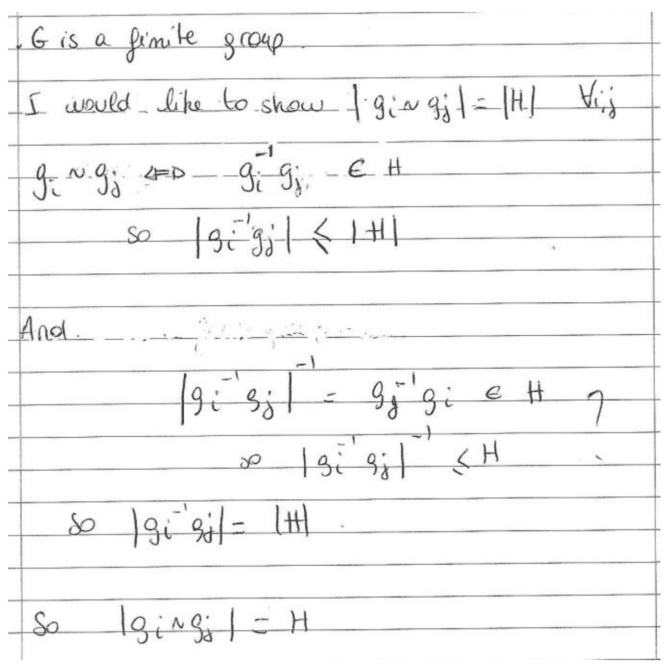


Figure 3: Part of Student B's Solution

A second category of errors appeared in the proof of symmetry and transitivity, but not in relation to the set and group elements, which will be discussed below (also see Ioannou, 2018b). Such minor inaccuracies occurred in three out of the thirteen (3/13) students' solutions of the relevant coursework task.

3. Suppose X is a non-empty set and $G \leq \mathrm{Sym(X)}$. Define a relation $\sim$ on X by:

$$x \sim y \Leftrightarrow \text{ there exists some } g \in G \text{ with } g(x) = y.$$

Prove that $\sim$ is an equivalence relation on X.
(Remark: The equivalence classes are the G-orbits, as in question 2.)

Figure 4: Coursework Task on Equivalence Relations (CS2E3)

Student C has successfully proven that ~ is an equivalence relation in CS2E3. There are indications of a complete object-level learning of the definition of equivalence relations as well as facility in applying the metarules to prove reflexivity, symmetry and transitivity. Her answer was correct, however her narratives regarding symmetry were not particularly explicit. She was expected to include more detailed mathematical narratives as the markers' comments suggest. These omissions were related to her metalevel learning, and the necessity to justify in a precise and rigorous way her mathematical claims. These errors indicate problematic application of the metalevel rules.

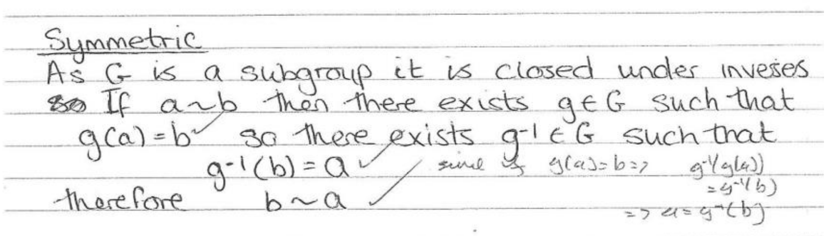

Symmetric
As G is a subgroup it is closed under inveses
If a~b then there exists g ∈ G such that
g(a) = b so there exists g⁻¹ ∈ G such that
g⁻¹(b) = a
therefore b ~ a

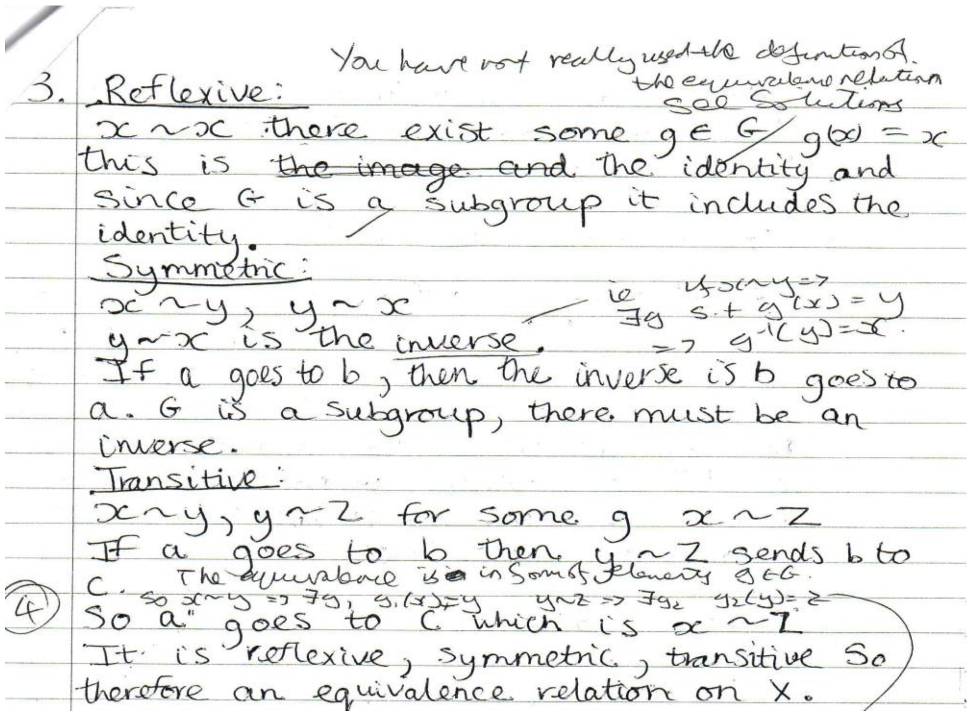
Figure 5: Part of Student C's Solution

Her initial problematic encounter with proving equivalence relations was also reported in her interview, as the following excerpt suggests.

Interviewer: Which part of the course was the most challenging for you so far?

Student C: Um... the equivalence relations stuff… It's not – what it is, like… like that, it's just like proving it, like when they ask on the new question sheet that we've got, when it's asking to prove that, I find that – I found that quite hard…

Another representative example of problematic proof of symmetry occurred in Student D's solution of the coursework task.



3. Reflexive: You have not really used the definition of the equivalence relation See Solutions

x ~ x there exist some g ∈ G / g(x) = x

this is ~~the image and~~ the identity and since G is a subgroup it includes the identity.

Symmetric:

x ~ y, y ~ x ie if x~y => ∃g s.t g(x) = y => g⁻¹(y) = x.

y ~ x is the inverse.

If a goes to b, then the inverse is b goes to a. G is a subgroup, there must be an inverse.

Transitive:

x ~ y, y ~ z for some g x ~ z

If a goes to b then y ~ z sends b to c. The equivalence is in some of elements g ∈ G. So x~y => ∃g₁ g₁(x)=y y~z => ∃g₂ g₂(y)=z

4 So a goes to c which is x ~ z

It is reflexive, symmetric, transitive So therefore an equivalence relation on X.

Figure 6: Part of Student D's Solution

As seen in Figure 6, he proves reflexivity correctly, demonstrating excellent use of words, syntax, and quantifiers, e.g., $\exists g \in G \; such \; that \; g(x) = x$, as well as awareness of the metarules that govern the relevant routine. However, regarding symmetry, although he correctly states the end result, he does not use the appropriate mathematical narratives i.e., $if \; x \sim y, \exists g \in G: g(x) = y \implies g^{-1}(y) = x$. His narratives are not complete, possibly showing some level of uncertainty. Student D's narratives regarding equivalence relations indicate an incomplete object-level and metalevel learning at this stage. His solution lacks explicitness, suggesting uncertainty regarding the how of the routine. This claim is reinforced by the fact that he asked the seminar leader (SL) for guidance in the seminar. As the following seminar vignette shows, he had not objectified the definition given in the lecture and needed further explanation.

> Student D is asking the SL about the task on equivalence relations. Student D says that he is not entirely sure what is reflexive, symmetric and transitive. SL did not understand what the student asked, and Student D repeated the question. SL said that what he has to understand is what is the equivalence relation between x and y. SL says that there is some g that sends x to y. SL says that he should consider the relation stated in the exercise and that this relation exists because of some element g, which sends from x to y. SL says that he should check the reflexive equivalence relation and asked him to find such an element. Student D replies the identity and follows a short discussion. Student D sounds confident. SL tries to explain to Student D the three elements of equivalence relations using quite formal language.

The third category of errors that occurred is related to the distinction between the elements of the set X and the elements of the group $Sym(X)$, when these coexist in the same context. Six out of the eleven students' solutions (6/11) indicated serious problems in their object-level learning of the d-objects of group and set, since they applied the group axioms on the elements of the set. For instance, they were trying to define and use the inverse of a set element. Naturally, this is impossible since there is no defined operation in the set. This category of errors suggests that students have not yet objectified the d-object of group and have fully identified the differences between the set and the group in the object-level learning. Furthermore, they have possibly not yet fully objectified the dual character of the d-object of group. As suggested below, there are indications that students' object-level learning in relation to the binary operation is incomplete.

An example of errors that indicates combination of both incomplete metalevel and object-level learning was seen in Student E's solution of CS2E3. Although initially her solution indicates good object-level learning by stating the three characteristics of equivalence, and that she has well objectified the definition of equivalence relations, yet her attempt later on indicates incomplete object-level learning of the d-object of map and certain issues regarding the application of metalevel rules. She successfully proves reflexivity using the identity element. Regarding symmetry she does not use the group element correctly and her notation shows an incomplete object-level learning of the elements of $X$and $Sym(X)$. Similar problems occur in her attempt to prove transitivity.

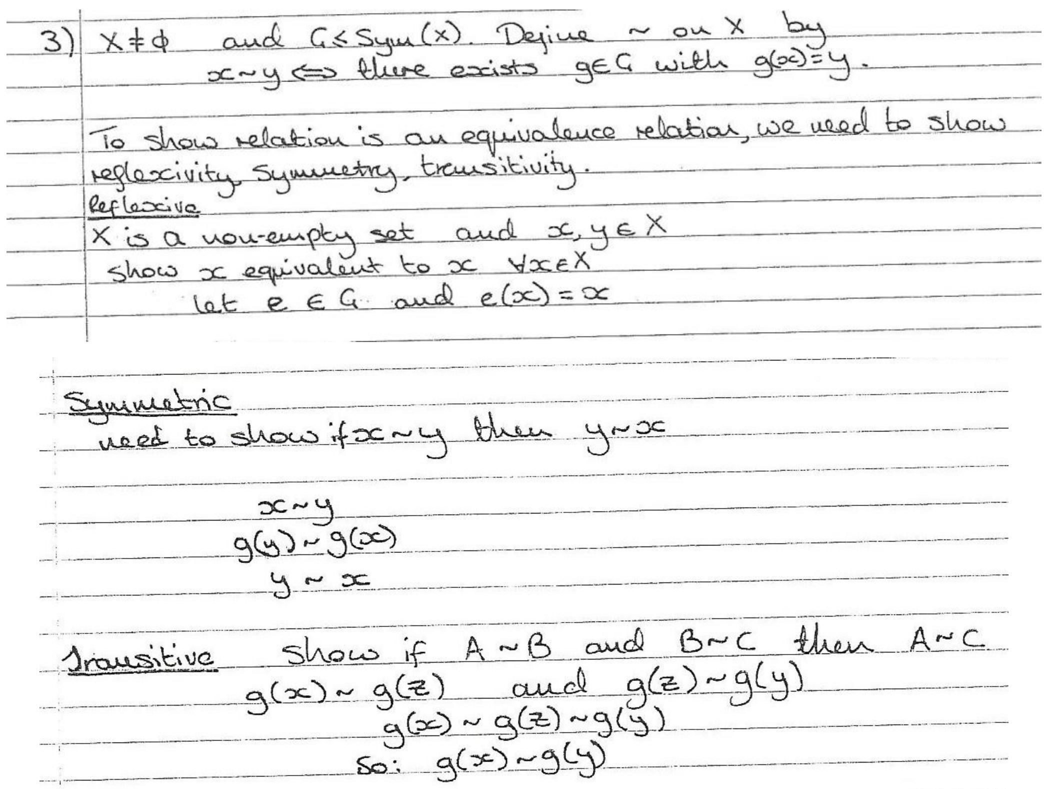


Figure 7: Part of Student E's Solution

Another example of problematic encounter with equivalence relations appeared in Student F's solution of CS2E3. Her object-level learning of equivalence relations appears to be incomplete, with negative consequences in the application of the governing metarules for proving that a relation is in fact equivalence. Her object-level learning seems to have problems from various aspects. For instance, she does not seem to have objectified the Orbit-Stabilizer Theorem, given as a hint in the problem sheet. The reason of this problematic objectification is that she has possibly not grasped the definitions of the involved d-objects, namely what the stabilizer and orbit mean, as well as the d-object of subgroup, as her solution later reveals. Her use of notation, in particular her use of $g's$, and the fact that she has omitted to distinguish the different $g's$, it probably indicates that her object-level learning of the elements of the group is not yet complete. This misunderstanding is obvious in the entire solution. Incomplete object-level learning is linked in this case with problematic application of the amenable metarules. In particular, in order to prove reflexivity, she used the data in the previous exercise i.e., $a \in X$ *and* $H = \{g \in G: g(a) = a\}$ as suggested by the lecturer, but she tried to use it to prove symmetry and transitivity as well, which is not acceptable by the marker. Again, Student F has still not a clear view of the how of the routine, which involves the use of group and set elements in order to prove that $\sim$ is an equivalence relation on X. It would be interesting to mention that Student F was the only student that regarding equivalence relations had shown progress in her performance in the exams, having resolved all problems in her object-level learning and the application of the metarules that occurred in the coursework.

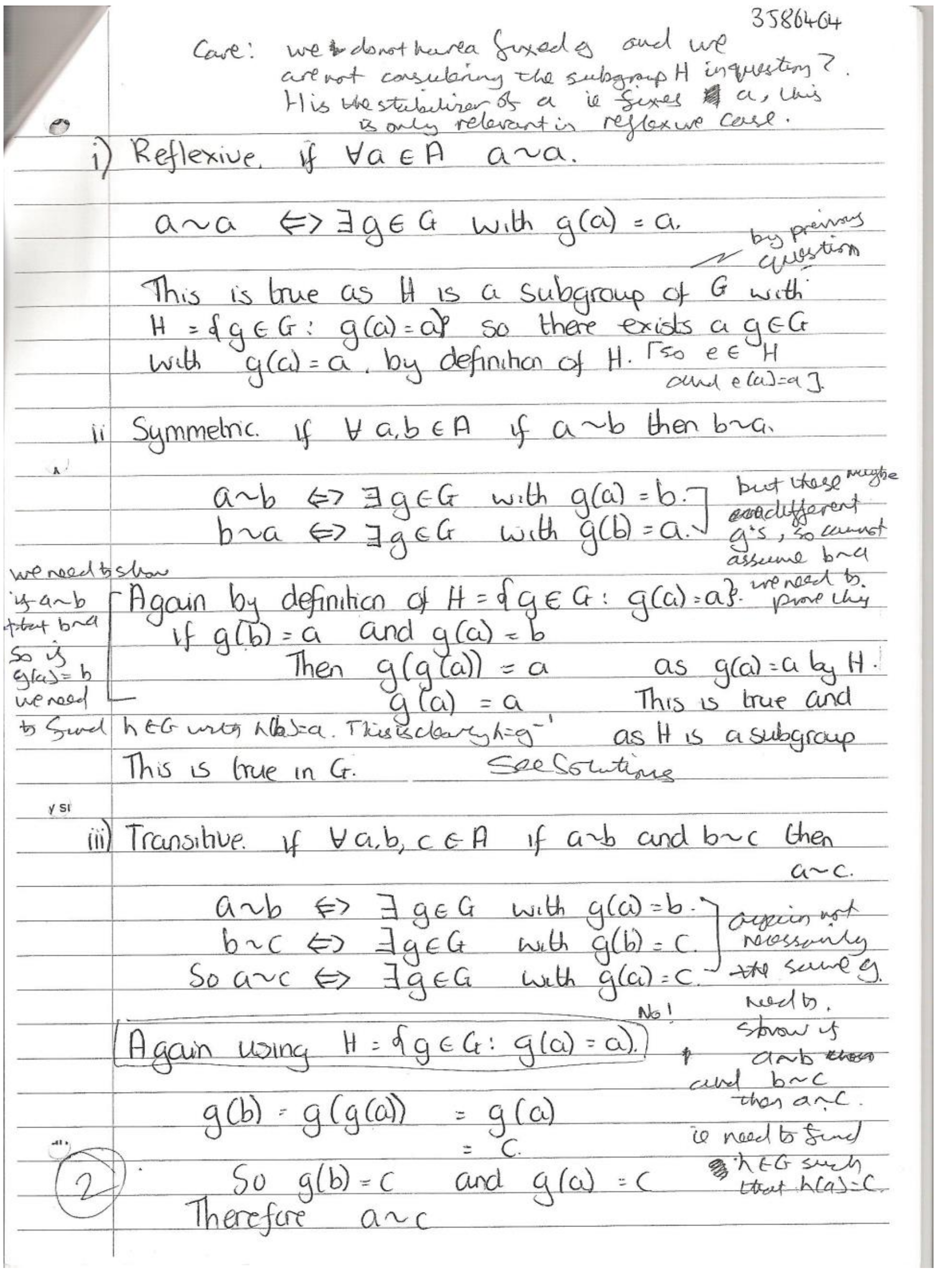


Figure 8: Student F's Solution

Incomplete learning with negative consequences in the application of the governing metarules appeared also in Student G's coursework and examination solutions. In CS2E3 there are indications of an incomplete object-level learning regarding the definition of equivalence relations as well as the difficulty to distinguish the elements of X and the elements of $Sym\,(X)$. As the excerpt below suggests, although he seems to know the various steps of the routine to be applied, namely prove reflexivity, symmetry and transitivity, there are particular problems with the d-object of permutation g and how this acts on x. Moreover, there are problems on a metadiscursive level resulting problematic application of the governing metarules. For instance, throughout his proof he assumes certain overgeneralized claims that are not necessarily true in certain instances. In addition, regarding the application of metarules, he assumes the truth of statements without adequately proving them. In general, his proving strategy seems to be partial since, according to the marker, there were steps missing in his proof. For instance, he assumes that $x \sim y = g(x) = z$, a narrative which should be proven. These errors indicate that Student G has not yet completed his metalevel learning regarding equivalence relations. In fact, his regressive performance in the final examination indicates that Student G has not overcome these difficulties.

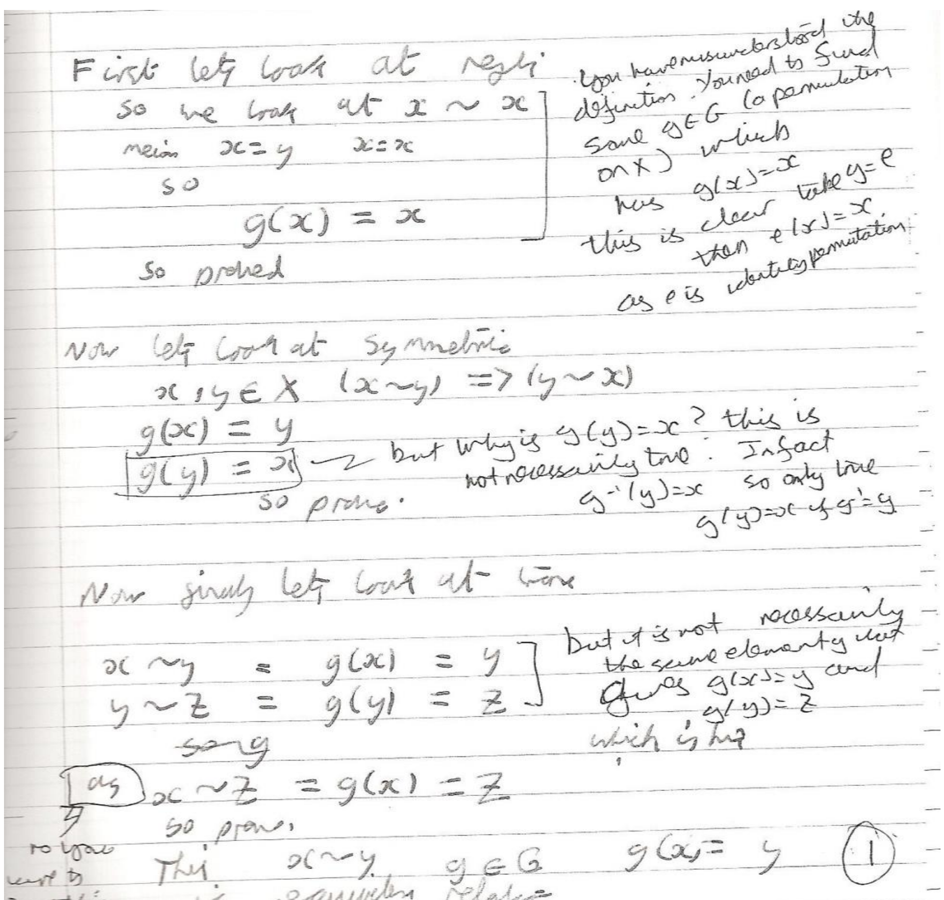


Figure 9: Part of Student G's solution

The above issues are in accordance with the comments of the two seminar assistants about the coursework of the 78 students.

Q3 - Again at times a poorly done question. Most recognized the need to check that $\sim$ is reflexive, symmetric and transitive. The main problem again was the confusion between the set X and the elements of $Sym(X)$. Elements of X are not elements of $Sym(X)$ and thus if we have $x, y \in X$ to say x has an inverse is not defined and neither is the composition xy, since no operations are defined on the set. We only have operations defined for elements of $G \leqslant Sym(X)$. So for each property (reflexive, symmetric and transitive), the idea was to show that there was always an element of G that fits the bill.

Figure 10: Excerpt from the Seminar Assistants' Report

In sum, this study has investigated three categories of errors in relation to undergraduate mathematics students' first encounter with equivalence relations. The first category of errors is related to the proof of the size of equivalence classes, which is predominantly due to incomplete object-level learning of the form and structure of equivalence classes as well as the notion of bijection. The second category includes several errors regarding the proof of symmetry and transitivity. The third category is related to the distinction between the elements of the set X and the elements of the group $Sym(X)$, when these coexist in the same context.

CONCLUSION

This study has focused on undergraduate mathematics students' difficulties with two tasks on equivalence relations. Although in this strand of research there have emerged a list of errors and difficulties these students have faced (see also Ioannou (2012, 2018b)), this study highlights three general categories of such difficulties and consequent errors. The first category of errors is related to the proof of the size of equivalence classes. This error appeared in seven students' solutions and was due to incomplete object-level learning of the structure and form of equivalence classes as well as the notion of bijection. Also, as the analysis suggests, an important reason of students' difficulty is the incomplete object-level learning of the definition of equivalence classes. Again, similarly to the results in Ioannou (2018b) this error results from the incomplete object-level learning and the distinction between the set and the group and consequently the distinction of their elements. The second category includes errors, related to the proof of symmetry and transitivity, as these appeared in three students' solutions. Moreover, the incomplete object-level learning of these notions has a negative effect on the application of metarules and consequently the quality of the produced proofs is occasionally problematic. Finally, the third category is related to the distinction between the elements of the set X and the elements of the group $Sym(X)$. This error appeared in six students' solutions. The analysis suggests that the application of metarules is a dynamic and multilevel process, in which novice students need to overcome malpractices such as overgeneralized claims, missing steps and ungrounded assumptions. A more extensive study will follow focusing more holistically on students' learning experience with equivalence relations in the context of Abstract Algebra.

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INNOVATIVE AND ALTERNATIVE EXAMINATION FORMATS IN MATHEMATICS AND STATISTICS

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KEYWORDS: online examinations, alternative assessments, open-book examinations, take-home examinations, graduate attributes

ABSTRACT

In the Covid era, online and open-book assessments were forced on universities. While this was intended to be a temporary and undesirable measure, several advantages of online, open-book examinations became apparent. These include resource savings as examination papers do not need to be printed, and the ease of sharing the scripts among a team of markers. This research is on appropriate types of questions to include in such examinations. We first discuss a survey of students and staff regarding their experience of online examination during the Covid lockdown. We then consider online, open-book, timed examinations for a first-year mathematics examination, and take-home, open-book, open-internet, untimed but set submission time examination for two higher level applied statistics units. Our experience dictates that such examinations are not only feasible but desirable. In particular, several graduate attributes for the applied statistics units cannot be tested in a traditional, timed, closed-internet examination.

INTRODUCTION

The Covid era caused a major disruption to higher education and caused a rethink of several aspects of teaching and learning. All of mathematics and statistics teaching and learning was forced to the online mode with very short notice. In some institutions the change happened over one weekend as nations went into shutdown. Very little support—technical, resource or training—was provided to staff. In particular, all assessments were forced to be online in some form, again with very little support to staff.

The final examination is the largest single assessment in most mathematics and statistics units. Several modes of the final examination were implemented at various institutions. Most seemed to be online based around some software. Most were open-book, with an attempt at some form of supervision. The majority were timed examinations. Examplify, Examsoft and FlexiQuiz are some of the online testing platforms. Most of these platforms were not suitable for mathematics and statistics. The examination paper could not be readily imported into the software. Most mathematics and statistics examinations are prepared using LaTeX which produces a pdf file. However, most of the platforms allowed only Word documents to be uploaded. In addition, solutions could not be readily written into the software. This presented a challenge for mathematics and statistics examinations. Other common problems were loss of internet connection and all kinds of issues with video monitoring of students—privacy, cheating, no camera available.

After the examination, there were major issues with scanning and uploading solutions, even though students were allowed extra time (twenty minutes at my institution) for this process. Some of the issues are listed below.

- The recommended mobile phone-based scanners did not work.
- Uploading did not work, especially for students with inadequate internet connections, which is common in remote locations.
- Unclear scans.
- Incomplete scans.
- Late submissions.
- Huge number of emails to unit coordinator!
- Stressed and frustrated students, which then affected their performance in subsequent examinations.

Several consequences of the online format followed. Videos of students were perused for any evidence of cheating. In the event of late, incomplete, unreadable or no submissions, students had to be contacted to pursue any possible recovery actions. Often such issues were only discovered at the marking stage, and often student, especially international, were difficult to contact.

Traditionally most large stakes assessments in mathematics and statistics are supervised, timed and closed book (Iannone, 2020; Trenholm, 2007). In our institution for example, most units have two short tests in weeks 4 and 10 and a mid-semester examination in week 7 (in a 12-week semester). Several advantages of online examinations, listed below, were apparent during the COVID era. These are true also for other large stakes assessments.

- Traditional resources are not required, such as venues, invigilators and printing. These save costs.
- Individual requirements can be easily set, such as session times for individuals and extra time for special needs students.
- Online assessments readily cater for online students.
- The assessment can be easily graded using an online rubric, so marking is easy. Rubrics can be re-used or easily modified.
- There is no need to carry around and share bundles of scripts between the marking team.
- Several markers can grade a script simultaneously, so marking is faster.
- No lost scripts!
- No need to return scripts to students.
- The scripts are available online for perusing and discussing with students.
- Appeals are easy to handle. Most appeals for the final examination occur when staff may be away.
- Most students do not pick up their semester assessment scripts after grading, leaving the lecturer of a large unit with several hundred scripts. In online assessments this is not an issue.

Recent research on online assessments focuses mainly on the format of online examinations and the various platforms used during Covid. Fitzmaurice & Eabhnat (2021) conducted an online survey on assessments by university lecturers in mathematics during Covid. At total of 257 responses from 29 countries (mainly from Europe) were obtained. They found that various formats for summative assessments had been adopted, with both timed and untimed open-book examinations being the most popular. Valdez & Maderal (2021) surveyed second year engineering students regarding their perception of online assessments. They found that students indicated a high level of motivation towards learning mathematics and viewed online assessments positively.

Khan (2015) reported an experience with open-book examinations in a first-level statistics unit. The format of the examination was writing short reports for each question based on the given analysis of data. His experience was that surface learning by students was apparent in semesters that followed, and while the mean mark increased the highest marks were lower than previously under closed-book examinations. However, he further concluded that well-designed open-book examinations often assess for higher levels of understanding and may not be appropriate for first-year units. Baily et al. (2020) also state that open-book examinations are more appropriate for testing higher order skills.

Iannone (2020) suggests the types of questions that are suitable for open-book assessments, including explanation type questions and totally unseen questions. Iannone (2020) also suggests oral examinations. Bailey et al. (2021) also discuss how to modify closed-book examinations to make them appropriate as open-book examinations.

Craig and Akkaya (2021) used open-book examinations for a vector calculus course during Covid, when all assessments were compelled to be online and remotely administered. Their premise was that open-book remotely proctored examinations necessarily implied open internet as well, with a suite of powerful digital tools. Consequently, they re-designed the questions to remove any advantage afforded by such tools. Similarly, Jungic (2020) included explanation-based questions using real data on public health events.

Not much research is available on take-home examinations, and we found very few in mathematics and statistics, mostly reporting the experiences of forced online examination during Covid. Bengtsson (2019) conducted a review of research on take-home examinations. He concluded, based on a review of 35 articles, that take-home examinations are not suitable for assessing lower taxonomy levels, but that more research was required. However, the actual formats of the take-home examinations he considered are not clear, that is, the types of examinations, the style of questions and the time period allowed.

Spiegel & Nivette (2023) compared take-home (open-book) and in-class (closed-book) assessments in two social science courses. They found no difference between the two in performance and knowledge retention measured four to six months later. Trenholm (2007) states that all unproctored assessments are by their nature take-home. Dekkers et al. (2022) reported on take-home examinations during Covid in first-year mathematics units for engineering students. Students downloaded and printed the examination paper from the unit LMS. Solutions were written in spaces provided and uploaded within 24 hours. Dekkers et al. (2022) found that the pass rates were higher compared to supervised examinations and were concerned about the increased opportunities for cheating. Lancaster and Clarke (2017) also discussed the susceptibility of take-home examinations to cheating. They considered such examinations to be “coursework assignments in disguised format”.

In a psychology class, Rich (2011) compared the performance of students in an in-class test and a take-home test. The students performed significantly better in the take-home test. A survey of the students indicated that students preferred the take-home examination. In addition, students studied harder for the take-home examination.

A systematic search of the literature for take-home examinations was conducted using various databases (ProQuest search of eighteen databases, Eric, Education source, A+ Education and Scopus). No research was found on take-home examinations in mathematics and statistics by conscious and deliberate choice of the lecturer.

The research question of interest in this study is: Is it feasible to allow online open-book, open-internet examinations in mathematics and statistics? Further, particularly for higher level statistics, are take-home, untimed, open-book and open-internet examination feasible? That is, can we learn from the Covid experience which forced online examinations to rethink and improve how we assess?

In this research we focus on two major aspects of assessments. Firstly, we were interested in the attitudes of students and academic staff regarding online examinations. Secondly, we report on alternative examination formats that avoid the issues of online examinations as practiced during COVID. In particular, we investigate open-book examinations and take-home examinations, the first for mathematics and the second for statistics units. We also discuss the pros and cons of these examination formats. We are especially interested in gauging any evidence of cheating by students.

This is action research and a descriptive paper reporting on authentic assessment and lessons learnt from our experience.

THEORETICAL FRAMEWORK

The take-home examination aligns with various learning theories. Khan & Watson (2018) provide a good summary of various learning theories. The learners' need for autonomy is addressed, a feature of the self-determination theory proposed by Deci & Ryan (1985). Cognitive load (Sweller, 1988) is reduced since the examination is not timed. Learners are empowered to have control over their assessment, satisfying the humanist and self-theories of Maslow (1943) and Rogers (1946). Students have ownership of learning and assessment.

The take-home examination is strongly based on the theoretical framework of learner-centred assessment (Duncan & Buskirk-Cohen, 2011; Rich et al., 2014). This form of assessment incorporates elements of problem solving. It tests the higher level of Bloom's revised taxonomy: analyze, evaluate and create (Anderson et al., 2001; Krathwohl, 2002). In addition, it is more realistic and more closely resembles a real work situation; as such it includes aspects of work integrated learning (WIL).

METHODOLOGY

There are two parts to this research.
Part I. We conducted a survey of staff and students on their attitude towards online examinations. The survey was sent to students taking any mathematics units and also to staff in the Department of Mathematics and Statistics at The University of WESTERN AUSTRALIA. The link to the online survey questionnaire was shared with students and staff of the Department of Mathematics and Statistics at The University of WESTERN AUSTRALIA. We focused on first-year units as these were the largest. A total of 372 students enrolled in seven units (some were enrolled in more than one) responded to the survey, from a total of approximately 3,000 enrolled students. Thirteen out of 19 fulltime staff responded to the survey.

The student and staff surveys each contained twelve questions (see Figures 1 and 2). Demographic details are given in the Results section. Descriptive statistics and summaries of the data were produced.

Part II. We report on our own experience with alternative examination formats during the COVID and the period following, in particular, online and open-book assessments for a first-year mathematics unit. We also report on take-home, untimed, non-supervised and open-internet and open-resource examinations for higher level applied statistic units. Finally, we discuss the pros and cons of these alternative examination formats.

This survey is on your experiences with assessments in the Mathematics and Statistics units you have taken at The University of Western Australia. Your responses are anonymous, and participation is voluntary. Your participation in this survey implies consent.

Q1 What degree programme are you enrolled in (BSc., BEngSc, B.A., etc.)
Q2 Which mathematics or statistics unit(s) have you taken at UWA (list all by code)?
Q3 Other than the final exam, what was the format of the assessments in these units? (Select all that apply.)
Fully online Multiple Choice
Fully online, numerical and short answer
Online delivery, written and scanned solutions uploaded
Written, submitted online of hard copy
Q4 Do you believe that this assessment format allowed you to show what you had learnt in the unit? Select all that apply.
Fully online, Multiple Choice Yes/No
Fully online, numerical and short answer Yes/No
Online delivery, written and scanned solutions uploaded Yes/No
Written, submitted online of hard copy Yes/No
Q5 Do you think that the assessment format helped you to learn the content for this unit? Yes/No
Q6 What was the format of the final examination in these units? (Select all that apply.)
Fully online Multiple Choice
Fully online, numerical and short answer
Online delivery, written and scanned solutions uploaded
Written, submitted online or hard copy
Q7 Do you believe that the final examination format allowed you to show what you had learnt in the unit? Select all that apply.
Fully online, Multiple Choice Yes/No
Fully online, numerical and short answer Yes/No
Online delivery, written and scanned solutions uploaded Yes/No
Written, submitted online of hard copy Yes/No
Q9 Do you think the final examination format was an appropriate way to test this unit? Yes/No
Q10 Do you any other comments regarding assessment format in the mathematics units you have taken?
Q11 What was your age at your last birthday (enter just a number)?
Q12 What is your gender? Male/Female/Prefer not to say

Figure 1: Student survey questionnaire

This survey is on your experiences with assessments in the Mathematics and Statistics units you have taught at The University of Western Australia. Your responses are anonymous, and participation is voluntary. Your participation in this survey implies consent.

Q1 Which level mathematics or statistics unit(s) have you taught this year and in the last two years?
First/Second/Third/Honours/Other (please specify)
Q2 What is the format of the semester assessments in these units? Please list separately for each unit.
Fully online Multiple Choice
Fully online, numerical and short answer
Online delivery, written and scanned solutions uploaded
Written, submitted online
Written, submitted hard copy
Q3 Other than the final examination, do you believe that the assessment format allowed your students to show what they had learnt in the unit? Select from the drop-down list for each assessment type that applies.
Fully online Multiple Choice Yes/No
Fully online, numerical and short answer Yes/No
Online delivery, written and scanned solutions uploaded Yes/No
Written, submitted online Yes/No
Written, submitted hard copy Yes/No
Q4 Do you think this assessment format helped your students to learn the unit material? Yes/No
Q5 What was the format of the final examination in these units? (Select all that apply.)
Fully online Multiple Choice
Fully online, numerical and short answer
Online delivery, written and scanned solutions uploaded
On campus written examination
Q7 Do you believe that the final examination format allowed your students to show what they had learnt in the unit? Select all that apply.
Fully online Multiple Choice Yes/No
Fully online, numerical and short answer Yes/No
Online delivery, written and scanned solutions uploaded Yes/No
On campus written examination Yes/No
Q9 Do you think the final examination format was an appropriate way to test this unit? Yes/No
Q10 What was you age at your last birthday (enter just a number)?
Q11 What is your gender? Male/Female/Pref
Q12 What is your level of appointment? Professor/Associate Professor/Senior Lecturer/Lecturer/Other (sessional, casual, etc., please specify)

Figure 2: Staff survey questionnaire

RESULTS

Part I: Survey results

All the examinations in both semesters in 2020 were online. Examinations took several formats. In two units the examinations were entirely multiple-choice, but this was the usual format for these units. In the rest, the questions were a combination of multiple-choice and written component. The multiple-choice sections of the examinations were fully online and automatically marked. The written components were scanned and submitted online. The institution examination package was Examplify (https://examsoft.com/).

The ages of the staff were between 26 and 62. Three were females and the remaining ten were males, and the level of appointments were: 6 lecturers, 3 senior lecturers, 2 associate professors and 2 professors. They taught units from first-year to honours. The student survey contained 161 females, 198 males and 13 did not specify their gender. The age range was 17 to 61, with most students (78%) in the 17 to 22 age group. Most of the students were enrolled in a BSc (75%), BEngSc (10%) and BCom (9%).

A summary of the examination types is given in Table 1. Staff overwhelmingly agreed that the final examination format allowed them to test student knowledge and was an appropriate way to examine their unit, and this was independent of staff level or age.

Regarding the appropriateness of the examination format, student responses were as follows.

- 82% agreed the examination type was appropriate.
- 76% agreed that the multiple-choice section/format allowed them to show what they had learnt.
- 68% agreed that numerical and short answer format allowed them to show what they had learnt.
- 84% agreed that the online delivered scanned solutions examination format allowed them to show what they had learnt.

Table 1. Examination types.

MATH1720	MATH1721	MATH1722	MATH1011	MATH1012	STAT1400	STAT1520
Fully online, MCQ	Online, written solutions, scanned	Online MCQ section, written solutions, scanned.	Online MCQ section, written solutions, scanned.	Online MCQ section, written solutions, scanned.	Fully online, MCQ and numerical answers	Fully online, MCQ and numerical answers

However, the student comments provided more information on student views.

- Generally, comments were unfavourable for online assessments. Several students expressed dissatisfaction with the process of scanning and submission of solutions. Particular issues were the recommended mobile phone scanner application did not work; internet issues; and it took longer than the allowed time to scan and upload the solutions.
- Multiple-choice format was not liked, as there is no possibility of follow through marks. Similar comments were made regarding numerical answer format.
- It was difficult to read the question online and write the answers on paper.
- Some units had multiple-choice for formative assessment during semester and written format for the final examination, and this mismatch was not appreciated by some students.
- Some examinations required solutions to be typed into the computer. Solving questions on paper first and then transferring to the computer was a problem for several students in this examination format. This format also took extra time compared to paper-based examinations, but not extra time was allowed.

Part II: Alternative examination formats

Supervised, timed and closed-book examinations are largely the norm for universities. This is true of mathematics and statistics examinations in particular, but also for other disciplines. Table 2 shows the mean weight (%) of final examination over all units in each school. It also shows the mean percentage of all assessments over all units that are supervised.

The final examination is the largest single assessment in most mathematics and statistics units. Traditionally and even in the COVID era most examinations in mathematics and statistics were closed-book and supervised to the extent allowed by the software.

Table 2. Mean weight (%) of all supervised assessments and final examination over all units in each discipline.

School	Mean weight of All Assessments (%)	Weight of Final Exam (%)
Computer Science	46	58
Humanities	24	10
Math and Stat	76	49
Physics	53	37
Science	40	40

Under the examination conditions imposed by Covid, we needed to design a paper that would discriminate between the top students. At the same time the paper would test standard procedures to conform with learning outcomes, and thus provide all students with an opportunity to pass. Post Covid, we reconsidered our approach to assessments in other units, in particular for higher level applied statistics units. Our approach and experiences for both are described below.

Online open-book examinations

Our focus was a first-year introductory calculus unit (which we refer to as CALC). We were interested in designing as examination that could be conducted as an open-book and potentially an open internet examination. This required redesigning the questions to eliminate the advantage afforded by any resources, both online and hard copies.

Since the examination is open-book, the traditional types of questions such as statements of theorems and standard proofs cannot be asked. Standard problems that mimic classroom examples test standard techniques need to be included, since content and learning outcomes must be tested. But overall, the types of questions need rethinking and innovation.

We stayed within the syllabus brief. Besides some standard questions, we included some that, while within the syllabus, were unseen and required a higher level of understanding. Three examples are given in Figure 3 for the online, open-book, timed and supervised examination in 2020 during COVID.

Example 1: Log laws
Let

$$\int_1^a \frac{1}{x}\,dx = A \text{ and } \int_1^b \frac{1}{x}\,dx = B.$$

Evaluate the following in terms of A and B.

(1) $\displaystyle\int_1^{ab} \frac{1}{x}\,dx$

(2) $\displaystyle\int_1^{\frac{a}{b}} \frac{1}{x}\,dx$

(3) $\displaystyle\int_1^{a^2\sqrt{b}} \frac{1}{x}\,dx$

Example 2: Integration and FTC Let $f(x)$ be a differentiable function such that $f(-2) = 4, f(-1) = 0, f(0) = -1, f(1) = 0$ and $f(3) = 2$. Further, $f'(x) < 0$ for $-2 \leq x < 0, f'(0) = 0$ and $f'(x) > 0$ for $0 < x \leq 3$. Evaluate the following integrals.

(1) $\displaystyle\int_0^3 f'(x)\,dx$

(2) $\displaystyle\int_1^3 f'(x)\,dx$

(3) What is the area bound by the graph of $f'(x)$ and the x-axis between $x = -2$ and $x = 3$?

Example 3: Differentiation rules The function $g(x)$ satisfies $g(0) = 2, g'(0) = 1$.

(1) Put $f(x) = x\,g(\ln x)$. Evaluate $f'(1)$.

(2) Put $h(x) = \dfrac{e^{-x}}{g(1-e^x)}$. Evaluate $h'(0)$.

Figure 3. Example examination questions for the open-book examination in Introductory Calculus.

Table 3. Summary of marks and pass rates for CALC. Note that the new assessment paradigm was held only in 2020.

		Sem 1	Sem 2
2019	Min (%)	12	3
	Mean (%)	78.5	73.6
	Max (%)	100	100
	Pass (%)	88.0	84.0
2020*	Min (%)	18	14
	Mean (%)	64.1	60.9
	Max (%)	100	98
	Pass (%)	82.5	70.5
2021	Min (%)	7	8
	Mean (%)	55.6	57.3
	Max (%)	99	98
	Pass (%)	64.0	71.2
2022	Min (%)	5	10

	Mean (%)	68.2	51
	Max (%)	100	96
	Pass (%)	82.6	53.6

Table 3 shows summary statistics of the final mark and pass rates for CALC. Note that the open-book paradigm was only implemented in 2020.

Take-home examinations

The two applied statistics units, a second-year unit which we refer to as STAT2 (in semester 2) and a third-year unit which we refer to as STAT3 (in semester 1), are core for the majors in statistics and data science. In previous years the assessments in these were two assignments and a supervised, timed and open-book but closed-internet final examination held in a computer laboratory. Both forms of assessment contained a mixture of theoretical questions and data analysis. However, an increase in the number of data science students with limited mathematics backgrounds necessitated a change in content and assessments. In addition, the laboratory-based final examination suffered from software issues, machines crashing and students losing work. The examiner was consequently required to be present at the venue throughout the examination. Students also became flustered and frequently had to be given extra time. A further issue was that with increase is student numbers no laboratory was large enough to accommodate the class for the examination, so several venues were required.

Our view was that timed, supervised, closed-book and closed-internet assessments were not appropriate for these types of units. Graduate attributes for statistics and data science courses place a strong emphasis on report writing and interpretation of data analysis. However, these attributes are neither developed nor assessed in most such courses. Further, these skills cannot be tested in a timed, supervised and closed-internet examination. Our focus was to mimic a real job situation and develop skills that will prepare these graduates for a career in statistics and data analytics.

For both units the assessments were modified as follows. Both assignments and final examination were modified to take-home, open-book and open-internet. The submissions were required to be the student's own work without collaboration or assistance from anyone. Students were allowed four weeks each for the assignments and three weeks for the final examination. For each of the two assignments, students were required to analyse a given data set with the context and aims of the analysis. The submission was in the form of a report on their analysis. Guidance for the report style was provided, as well as an example report. The assignment report consisted of sections with the descriptions as below.

- Executive summary: Briefly introduce the data, the context, the aims of the analysis and the main findings (not more than half a page).
- Introduction: Setting the context of the data and research aims. Present a literature review on the topic. Then provide a more detailed description of your data and context, and the aims of the analysis (between 1 and 1.5 pages).
- Methodology: Describe the statistical methods that you use. Roughly speaking, provide enough information to enable a professional statistician to reconstruct what you did (half a page).
- Results: Describe the results of your analysis. You will include appropriate exploration of data (tables and graphs) and describe your findings. You will also describe the model fitting procedure (such as steps in arriving at the final model). No R outputs or code should be given here. Note that this section contains results only, not interpretation.

- Discussion: Draw conclusions based on your results. Discuss any issues in the analysis that may affect the conclusions, such as potential weaknesses of your analysis and alternative analyses (about half a page).
- References: A list of references that you have used and cited in your report. Any standard form of referencing and citing may be used but be consistent in the format that you use.

The final examination had a similar format to the assignments, except that the submission was in the form of a more substantial journal article.

Table 4. Summary of mark and pass rates for STAT2 and STAT3. the new assessment paradigm began in 2021. Note that STAT3 was not taught in 2020.

		STAT2	STAT3
2019	Class size	57	14
	Min (%)	6	37.1
	Mean (%)	66.6	62.1
	Max (%)	95	91
	Pass (%)	92.6	84.6
2020	Class size	155	-
	Min (%)	3	-
	Mean (%)	59.1	-
	Max (%)	92	-
	Pass (%)	76.9	-
2021*	Class size	169	18
	Min (%)	1	66.8
	Mean (%)	73.2	78.6
	Max (%)	97.9	90.0
	Pass (%)	91.4	100
2022*	Class size	222	25
	Min (%)	6.0	57.9
	Mean (%)	65.9	72.0
	Max (%)	95.2	97.4
	Pass (%)	87.0	100
2003*	Class size	203	27
	Min (%)	18	62
	Mean (%)	70.8	78
	Max (%)	95	97
	Pass (%)	93.1	100

Table 4 shows the summary statistics and pass rate for the two units from 2019 to 2022. The class sizes are also given. STAT2 is taught in semester 2 and STAT3 in semester 1.

DISCUSSION

From Table 3, the first aspect to note for CALC is that there is no evidence of grade inflation for the open-book assessments. The summaries of the marks are comparable between different years. The pass rate is also comparable.

Perusing examination scripts showed that even standard questions in open-book exams were not done well. These included standard questions on differentiation, integration and constrained optimisation. Further, the examination scripts indicated strong evidence of “surface” or “spot” learning.

Nonetheless, this particular class had some very bright students, and we had also set high expectations from the class. Several students had marks in the high 90s. About 5% of students answered all three questions in Figure 1 correctly and 15% answered at least one of these questions correctly. We feel that if students are encouraged and challenged then they will lift their efforts and are capable of higher level of understanding.

Note also that the pass rates in semester 2 is generally lower than in semester 1. This is because the second semester class comprised students who had insufficient mathematical background and took a prior bridging unit, as well as failures from semester 1.

For STAT2 and STAT3 the new assessment style began in 2021. From Table 4, the mean for STAT2 is higher in 2021 for the new assessment regime. However, for 2022 the mean is comparable to those in 2019 and 2020 under the traditional examination format. The pass rates across the years are comparable under the two examination formats. The maximum marks are also comparable across the years.

For STAT3 (Table 4), the mean marks are higher for the new assessment regime The maximum marks are comparable across the years. However, the pass rate is 100% each semester under the changed assessment style, unprecedented under the traditional examination. We believe this is a direct result of the more realistic assessment paradigm.

A major point of concern is the opportunity for cheating by students in take-home examinations. For STAT2 the results do not show any evidence of cheating since there is no grade inflation compared to the traditional proctored examination format. For STAT3 the mean and pass rates are higher. However, the examination scripts show no evidence of cheating. The more demanding task in the examination is not the data analysis but the writing of the article. A professionally written journal article style submission is easy to pick up from the style, the language and the English expression. It will be difficult for anyone not familiar with the unit material to use the appropriate language and terminology.

CONCLUSION

Assessments motivates learning and focuses students on the key aspects of the subject. Many important skills and graduate attributes cannot be tested in the traditional, timed and proctored examinations. The advantage of the take-home examination is that it can test a wide range of skills, more closely mimic the real work environment, and are more realistic for some subjects. They do test the higher taxonomy levels and may not be suitable for all study levels. In particular, for higher year units in applied statistics, take-home examinations are very appropriate. However, the style of the examination needs to be suitably modified and cannot be simply a traditional style paper.

For lower levels in mathematics and statistics, open-book online examinations provide several advantages. These can be proctored, timed examinations. However, the style of questions need some imagination, originality and ingenuity. We found that even standard questions are only accessible to students who know their material, but the more imaginative question types are necessary to separate the best students. Examination scripts provided evidence of surface and spot learning by some students, but this is not restricted to open-book examinations.

Given the advantage of online examinations and our research, we recommend open-book proctored examinations for lower-level mathematics and statistics units, and take-home, open-book and open internet examinations for higher level units in applied statistics.

ETHICS APPROVAL

This research was conducted under ethics approval 2019/RA/4/20/6514 from The University of Western Australia.

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A REFLECTIVE PRACTICE APPROACH ON ISSUES ARISING AFTER THE PANDEMIC ERA

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KEYWORDS: reflective practice, action research; digital literacy; mathematical literacy

ABSTRACT

The advent of the COVID-19 pandemic and the consequent move to online education have impacted students enrolling in universities from 2021 onward. Many students in South Africa found themselves unable to engage with online education, resulting in a significant portion of the incoming first-year cohort lacking the essential digital skills required for their university learning, leaving them ill-equipped for their academic endeavours. The lecturers of two first-year courses became aware that several students had difficulties using a computer and could not solve problems involving mathematical content in real-life contexts. This paper presents the first stage of the action research cycle to diagnose the problematic situation experienced in the computing classroom. The diagnosis of the problem was done using reflective practice to gain an understanding of the experiences of the students. It was found that the students lack digital literacy such as keyboard and mouse skills as well as mathematical literacy. The paper concludes with potential strategies for the action planning phase to alleviate the literacy gap.

INTRODUCTION

The first-year students who entered universities from 2021 onward were part of the COVID-19 pandemic during their last years in secondary school. In South Africa, the government temporarily closed all educational institutions in an attempt to suppress the spread of Covid-19 and all face-to-face activities were prohibited. This had a huge influence on teaching and learning in schools and universities where all previous face-to-face courses had changed to emergency remote teaching. South Africa has a diverse population with numerous learners and students coming from disadvantaged backgrounds with little or no access to the Internet and electricity which resulted in some learners and students not having access to online education.

With the rapid pace of technological advancements, students are expected to be proficient in various digital tools and platforms, including cloud-based applications, social media and online learning management systems. In addition, the COVID-19 pandemic accelerated digitalization, especially in economies or industries that had been lagging and in South Africa, an urgent call for digital transformation has been made (Mhlanga & Moloi, 2020). The ability to effectively use technology is becoming increasingly essential for academic and professional success. Today's students were born in the digital age and therefore the idea prevails that they will be digitally literate. Although today's students are often referred to as "digital natives" this may not be the case in South Africa where numerous learners and students come from disadvantaged backgrounds. As a consequence, several students are unable to access online education, leading to a situation where a significant portion of the incoming first-year cohort lacks the necessary digital literacies, rendering them unprepared for their university learning experiences.

Reflective practice (RP) presented by Schön (1983) promotes the process of reviewing the actions taken by seeking improved approaches. The action research (AR) method allows the practitioner and participants who are involved in the AR process to apply self-reflection and consider interventions to improve present practices (Burns, 2015). This paper presents the first stage of the AR cycle to explore or diagnose the problematic situation experienced in the classroom. The diagnosis of the problem was done using RP in order to gain some understanding of the experiences of the students before selecting appropriate strategies and ideas to guide the action-planning phase.

BACKGROUND

Digital Literacy

Spante, Hashemi, Lundin, and Algers (2018) reviewed higher education research where the concepts of digital competence and digital literacy are defined and found that there is a range of definitions used. The variations depend on whether the concepts are defined by policy, research, or a combination of both, as well as whether the emphasis is on technical skills or social practices. The term digital literacy was first introduced by Gilster (1997) as: “the ability to understand and use information in multiple formats from a wide range of sources when it is presented via computers”. In more recent publications, definitions of digital literacy point towards cognitive skills and competencies with digital literacy defined by Martin and Grudziecki (2006) as “the awareness, attitude and ability of individuals to appropriately use digital tools and facilities to identify, access, manage, integrate, evaluate, analyse and synthesize digital resources, construct new knowledge, create media expressions, and communicate with others, in the context of specific life situations, in order to enable constructive social action; and to reflect upon this process.”

Ferrari (2012), p. 3 defined Digital Competence as ”the set of knowledge, skills, attitudes (thus including abilities, strategies, values and awareness) that are required when using ICT and digital media to perform tasks; solve problems; communicate; manage information; collaborate; create and share content; and build knowledge effectively, efficiently, appropriately, critically, creatively, autonomously, flexibly, ethically, reflectively for work, leisure, participation, learning, socialising, consuming, and empowerment”.

In the context of this paper, the term digital literacy will be used and will be defined as: “the set of knowledge and skills that are required to appropriately use digital tools and facilities to identify, access and manage digital resources in the context of higher education.”

Digital literacies are becoming increasingly relevant to meet the demands of present and future learning, working and socialization environments (Ehlers & Kellermann, 2019; González Vázquez et al., 2019). In the National Digital and Future Skills Strategy, prepared by South Africa’s Department of Communications & Digital Technologies (2020), a society of digitally skilled South Africans is envisaged. The strategy document describes a set of initiatives aimed at enhancing South Africans' abilities to handle the difficulties brought on by the increased deployment and uptake of digital technologies in the economy and society, while also recognizing that the digital revolution takes place within the context of the broader Fourth Industrial Revolution. The strategy document further states that the combined effects of these technological trends are having a significant impact on the workplace, educational institutions, research, people, and communities. It outlines a vision of a South Africa where all citizens can gain from improved digital capabilities, resulting in a markedly improved quality of life, better education, and greater economic growth (Department of Communications & Digital Technologies, 2020).

Furthermore, the European Union acknowledges through their Digital Education Action Plan (2021-2027), the key role of higher education institutions in supporting students in gathering a set of digital competencies that enable them to succeed throughout their tertiary education and facilitate their integration and advancement in the labour market (European Commission, 2020).

Portillo, Garay, Tejada, and Bilbao (2020) measured the perception that teachers had about their own performance during emergency remote teaching due to the COVID-19 pandemic. The teachers reported limitations in their digital competence as the greatest challenge which caused a perceived increased workload along with negative emotions. In addition, lower digital competencies were reported with the groups that are the most vulnerable in remote teaching and the authors call for measures to improve equity, social justice, and the resilience of the educational system.

Coldwell-Neilson (2017) conducted a study to examine the information that Australian universities offer to prospective students for the purpose of communicating the anticipated level of digital literacy required upon enrollment in a course. It was found that minimal information is being provided to new students and that there is no shared understanding of what digital literacy entails. Coldwell-Neilson (2017) concludes that it poses challenges for students who are expected to have an ill-defined or unknown set of digital skills. Furthermore, it creates tensions within the staff-student interactions, given the misalignment between expectations and comprehension of digital literacies.

The study at a South African university by Leonard, Mokwele, Siebrits, and Stoltenkamp (2016) demonstrates that despite a significant number of students commencing their first year with prior technological exposure, a need persists to educate students on the basic digital literacy skills essential at the university level. Despite their ability to use technological devices and tools every day, the new generation has difficulty using them to enhance their personal and professional lives (Lucas et al., 2022; Santos, Lucas, & Bem-Haja, 2022). Consequently, higher education institutions should commit to supporting students in gathering a set of digital literacies and it should start with an assessment of students' training needs. Based on such an assessment, an intervention strategy could be developed to assist students in overcoming their challenges and decreasing digital competency gaps (Santos et al., 2022).

Mathematical Literacy

The Program for International Student Assessment (PISA) of the OECD defines mathematical literacy as: "*an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts, it includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena"* (OECD., 2019).
The UK's National Numeracy charity (National Numeracy, 2015) states the following essential aspects of numeracy:

- Being numerate: incorporating problem-solving, reasoning, and decision-making.
- Numbers: being confident with straightforward number handling, including fractions and decimals in real-world contexts
- Handling information: understanding straightforward graphs and charts, and different presentations of numerical data
- Shape, space, measure: being able to use straightforward measures for space, volume, distance, time, weight, and everyday problem-solving using such measures.
- Operations and calculations: fundamental operations of mathematics (addition and subtraction, multiplying and dividing, exponents, the effective use of calculators)

In the 2015 OECD global school ranking of 76 countries which is based on a combination of international assessments - the OECD's PISA test, the TIMMS tests and TERCE tests - Australia ranked 14th and New Zealand ranked 17th. However, South Africa ranked 75th globally – beating only Ghana.

Thomson, Hillman, and De Bortoli (2013) reported on Australian students' performance in PISA, and provided an overall picture of achievement in mathematical literacy, to enable teachers to gain a deeper understanding of PISA, and to use the results of the assessment to inform their teaching. The authors reported:

- Students who have an interest in and derive enjoyment from mathematics tend to perform better in the subject compared to those who do not.
- The data underscores that while enjoyment is a significant factor, it is not the sole determinant of high achievement in mathematics. Understanding the relevance of mathematics in a student's future also plays a pivotal role, and educators and schools can contribute by connecting students' learning to real-world applications.
- There is a need to address gender disparities in mathematics education. A smaller proportion of female students achieve proficiency at higher levels, while a larger proportion perform at lower proficiency levels.
- Teachers can enhance students' mathematics learning by offering clear and explicit guidance on strategies for comprehending mathematical concepts and problem-solving.

Spaull and Kotze (2015) argue that mathematics education in South Africa is in a grim state. This sentiment is widely shared among academic researchers and members of civil society and it is strongly corroborated by numerous local and international assessments of mathematical achievement. Many of these studies have identified that students accumulate learning gaps early in their educational journeys, and these deficiencies are the primary culprits behind later academic underperformance.

At the beginning of 2022, the lecturers of two first-year courses at North-West University (NWU) became aware that several students had difficulties using a computer and could not solve problems involving mathematical content in real-life contexts. That gave rise to the lecturers' reflection on the problem at hand. In the next section, the context of the study is described.

CONTEXT

The School of Computer Science and Information Systems at NWU offers two first-year courses in the first semester – "Introduction to Computing and Programming" for the BSc students and a service course for BCom students called "Introduction to End User Computing". Several BCom programmes such as Agricultural Economics, International Trade, Risk-, Business-, Marketing-, Tourism- and Safety management include this course and there is a large number of students taking a two-year Diploma in Coaching Sciences

For studies at the NWU, prospective students need to adhere to certain requirements. One of these is an Admission Point Score (APS) based on their grade 12 matriculation results. The APS score is calculated by using the marks obtained in 6 subjects – excluding the score of the subject Life Orientation. For the BSc students an APS of 26 and above is required and a minimum Grade 12 Mathematics mark of 50%. For the BCom programs, an APS of between 20 and 26 is required. However, an APS of only 18 is required from the Coaching Sciences students. This means that if a prospective student obtained between 40% and 49% for 6 Grade 12 subjects, the APS score will be sufficient for taking this diploma.

Table 1: Admission Point Score system

NSC (National Senior Certificate) marks	APS score
90 – 100	8
80 – 89	7
70 – 79	6
60 – 69	5
50 – 59	4
40 – 49	3
30 – 39	2
0 - 29	1

In order to obtain the National Senior Certificate in Grade 12 in South Africa, learners must have seven subjects – four compulsory and three which are chosen at the end of Grade 9 for Grade 10 – 12. The four compulsory subjects are two official Languages, Mathematics or Mathematical Literacy and Life Orientation. Learners must also choose three optional subjects from a list of 25 approved subjects.

The subject Mathematical Literacy is described as: "Mathematical Literacy allow individuals to make sense of, participate in and contribute to the twenty-first century world - a world characterised by numbers, numerically based arguments and data represented and misrepresented in a number of different ways. Such competencies include the ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology" (Department of Basic Education, 2011). The interplay between content, context and solving problems is illustrated in Figure 1.

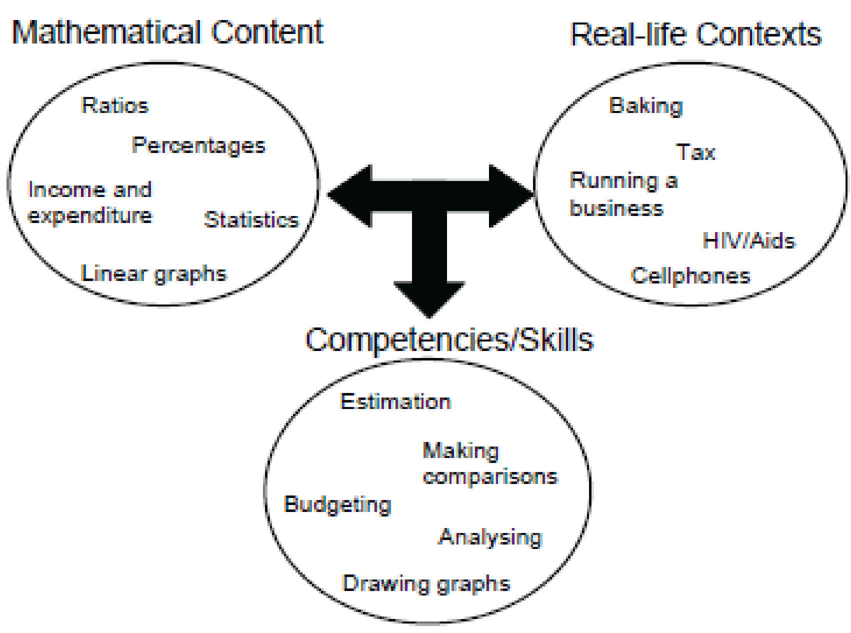

Figure 1: The content, context and problem-solving skills in Mathematical Literacy

Both first-year courses make intensive use of the LMS to for instance manage the submission of assignments. The content of both courses consists of basic computer concepts and the use of MS Excel. The outcomes of the courses cover topics such as the binary number system; the storage, manipulation, analysis and presentation of data using spreadsheets; problem-solving by using formulas and functions; and the presentation of information through charts. The course for the BSc students also includes programming in Python. The BSc programmes include courses like mathematics and statistics and software such as Matlab, SPSS and Excel are used in those courses. The BCom programmes include subjects like accounting and again, Excel and accounting software are used.

The students in the BSc programmes are expected to write a mathematics test before the start of the first semester, and if they fail they have to attend a bridging course which is offered before the classes commence. The BCom programmes however do not have any required readiness assessments or transition courses.

THEORETICAL FRAMEWORK

Action Research

The five iterative phases of action research as illustrated in Figure 2 are diagnosing, action planning, action taking, evaluating, and specifying learning (Baskerville, 1999).

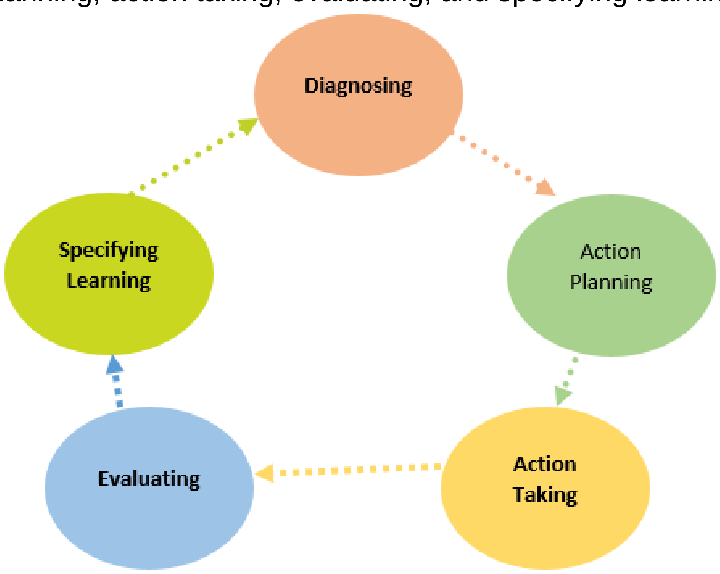

Figure 2: The action research cycle (Baskerville, 1999)

This paper presents the first stage of the action research (AR) cycle to explore or diagnose the problematic situation experienced in the classroom. The diagnosis of the problem was done to gain some understanding of the experiences of the students before selecting appropriate strategies and theories to guide the action-taking phase. The methodology used in the diagnosis phase was reflective practice.

Reflective Practice

To reflect means to engage intentionally in a mental process of thinking about things that have happened. Reflective practice is a professional development strategy where the primary purpose is behavioural change in order to improve practice (Argyris & Schon, 1974; Osterman, 1998; Schön, 1983). By using past experience when performing a task, one is in a position to criticize, examine, and improve their experience (Zeichner & Liston, 2013). According to Schön (1983), the experience can be developed through reflection. Reflection is regarded as a significant component of professional development through learning by challenging our beliefs and understandings (Graham, 2017).

Experiential learning theory maintains further that learning is a dialectic and cyclical process consisting of four stages: experience, observation and reflection, abstract reconceptualization, and experimentation (Kolb, 1984). While experience is the basis for learning, learning cannot take place without reflection. Conversely, while reflection is essential to the process, reflection must be integrally linked with action. Reflective practice, then, integrating theory and practice, thought and action, is an interaction between thinking and doing and through this process, skills are improved (Osterman & Kottkamp, 1993).

According to Larrivee (2000), as a reflective practitioner, one moves beyond basic understandings and skills to the point where one can adapt those skills to different situations, and finally to the point where new approaches are developed using those tools.
It is crucial that higher education instructors regularly reflect on their teaching and provide dynamic and relevant instruction (Lubbe & Botha, 2020). Zwozdiak-Myers (2011) captured the inter-related dimensions of RP to make it easier for teachers to provide evidence to inform their own teaching practices as displayed in Figure 3.

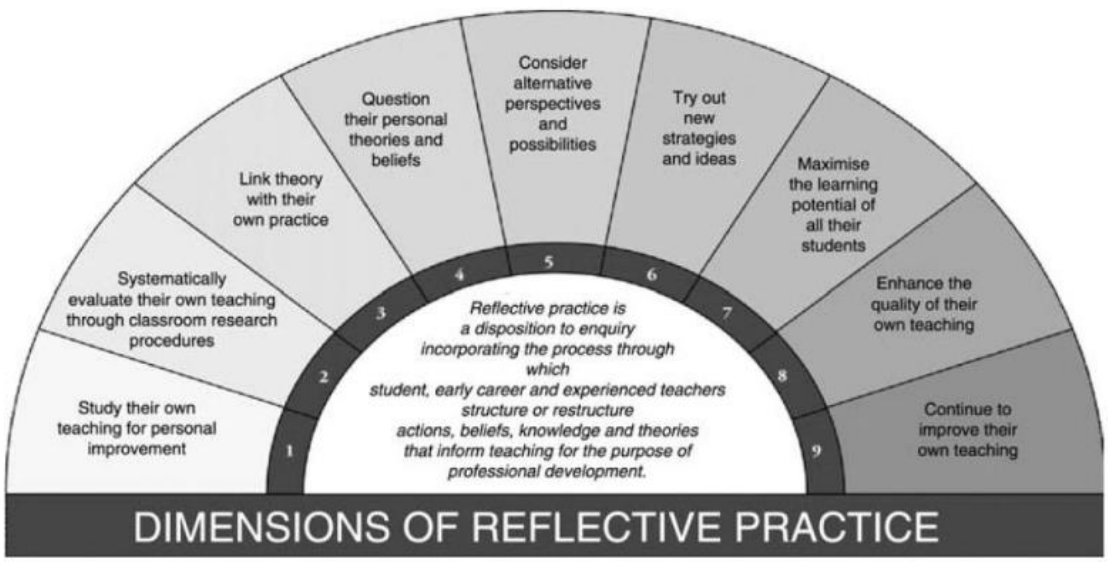

Figure 3: Dimensions of reflective practice (Zwozdiak-Myers, 2011)

RP is not restricted to the process of reflection itself - it extends to managing the situations, locations, and the mind/body condition in which reflection takes place. Lawrence-Wilkes proposes the following during the process of RP:

- Reflect at the right time.
- Balance subjective and objective reflection.
- Understand how and why you think in the way you do - generally and about specific things.
- Consider your personal role and responsibilities - examine your strengths, skills and development needs.
- Seek external clarifications - refer to external references, advice, information, clarifications, facts, figures, etc.
- Be mindful of the place where you reflect - a person's location can have a dramatic effect on thinking, and aid the stimulation of different attitudes or unlock feelings.

- Activities such as walking, swimming, yoga, and exercising can also produce dramatic shifts in thought patterns.

Lawrence-Wilkes and Ashmore (2014) represented the stages of RP by using the backronym of the word 'REFLECT" as shown in Table 2.

Table 2: The 'REFLECT' model of Reflective Practice

R	1. Remember	Look back, review, ensure intense experiences are reviewed 'cold'. (Subjective and objective)
E	2. Experience	What happened? What was important? (Subjective and objective)
F	3. Focus	Who, what, where, etc. Roles, responsibilities, etc. (Mostly objective)
L	4. Learn	Question: why, reasons, perspectives, feelings? Refer to external checks. (Subjective and objective)
E	5. Evaluate	Causes, outcomes, strengths, weaknesses, feelings - use metacognition. (Subjective and objective)
C	6. Consider	Assess options, need/possibilities for change? Development needs? 'What if?' scenarios? Refer to external checks. (Mostly objective)
T	7. Trial	Integrate new ideas, experiment, take action, make change. (Repeat cycle: Recall...)

In the following subsections, the stages of RP as presented by Lawrence-Wilkes and Ashmore (2014) applied to this study will be discussed.

Remember

RP begins with an unexpected incident which is normally a negative experience or a surprising event. The lecturers became aware that several students had difficulties using a computer and their basic mathematics skills were lacking.

Experience

The lecturer steps back and reflects on the situation at hand – what happened?
At the university, COVID-19 measures were still in place at the start of the semester and students had the choice of coming to the computer labs to do their work or to work online. Within the first week, some students who elected to work online notified the lecturers that Excel does not work on their cell phones. The lecturers had to put an announcement on the LMS notifying students that they wouldn't be able to successfully complete the course if they only had access to a cell phone. The lecturers had to make it clear that the students have to come to campus and use the university computers or they have to obtain an appropriate laptop or computer to complete the Excel assignments.

In the class for the BCom students, they had to use an application called SAM (Skills Assessment Manager) and they had to log in with a specific username and password. The username was studentnumber@sam.com. One of the students called for assistance because she didn't know how to type the @. She didn't know about the Shift key and how it works. The lecturer had to explain more than once how to keep the Shift key in while typing the @.

This was not the only student who had difficulty using the keyboard. One student pressed the Shift key but did not hold it in, then pressed the @ and then a 2 appeared in the cell. The lecturer explained that the Shift key should be kept in, while simultaneously pressing the @ key. The student then kept in both the Shift and @ keys which resulted in @@@@@@@@. After the third try, the student understood what to do and succeeded in typing @.

Another student had such long nails that it hindered her from typing - at the end, she started typing with her knuckles. In addition, the typing speed of many students was so low that it caused a major stumbling block in the progress of the students.

A student called for assistance because he did not succeed in using the fill handle to copy a formula to the cells below. It transpired that the student could not use the mouse – he did not know the difference between click and double-click. Dragging and positioning the pointer on the fill handle at the right-hand corner at the bottom of the cell, proved to be an extremely hard activity for him. The courses cover the digital presentation of data and consequently, the binary and hexadecimal number systems are studied. Especially the BCom students, but even the BSc students found the comprehension of these “new” number systems challenging and it seemed that they were conditioned to only use and understand the decimal number system.

The Excel part of the courses contains mathematical and statistical functions. The formulas and functions used include financial calculations such as monthly payments, the future value of an investment and the present value of a loan. In addition, functions for calculating for example frequencies, minimum, maximum and means are also included. These calculations in Excel need a thorough understanding of the mathematics behind the Excel functions. A lot of students seemed to lack mathematical litaracy and they found it difficult to use these functions.

Focus

These students and the lecturers had expectations in terms of their roles and responsibilities. The lecturers usually expect students to be able to use a computer and a mouse in order to solve problems using Excel. In addition, the lecturers expected after COVID-19 and the schools’ move to online learning that these students would be competent in using a computer and a mouse. Furthermore, the lecturers expected an adequate level of mathematical literacy from these first-year students, since they have all completed their matriculation and met the admission requirements of the university.

The students on the other hand expected to learn about computers and how to use them. In addition, the students did not expect the level of mathematics literacy required in the courses. There is thus a difference between the expectations of the lecturers and the actual knowledge and skills of the students.

Learn and Evaluate

When asking “Why did this happen”, one can only assume that some of these students had little or no exposure to the use of computers before entering the university. When one looks at the prerequisites for some of the programs taking these modules, it is apparent that some of the students are low achieving and come from disadvantaged backgrounds. We learned from this experience that a possible weakness is that the lecturers are not aware of the skills and knowledge of these first-year students entering the class.

Consider and Trial

At this stage, a problem has been identified through observation and reflection and the lecturers understand what happened and why. At this point, an active search for new ideas and strategies on how to modify the existing situation and how to solve the imminent problem should be conducted. The objective is to develop new strategies and ideas that may address the problem. A comprehensive approach to the digital literacy of students in these courses is advisable. The first step could be to evaluate students' training needs using instruments based on descriptors of digital literacy. As a result of this assessment, an intervention strategy aimed at bridging the gap can be outlined to help students overcome their handicaps and diminish disparities in their digital literacy.

Applying a comparable strategy to enhance the mathematical literacy of students enrolled in the BCom course is suggested. Similar to the requirements of the BSc programmes, an initial assessment of the students' mathematical literacy can be conducted and an intervention strategy aimed at lessening inconsistencies in their mathematics literacy can be drafted. In the final stage of RP, a new plan should now be set in place and the new ideas and strategies should be tried and tested. In this paper, we reported on the diagnosis phase of AR with the use of RP and the next phase of the AR cycle - action planning - will be discussed in future research reports.

DISCUSSION AND CONCLUSION

The goal of this study was to diagnose the problematic situation experienced in the computing classroom in order to gain some understanding of the experiences of the students before selecting appropriate strategies and ideas to guide the next phase of action planning. University lecturers' expectations for these students were to be naturally proficient in digital competencies and it was expected that they received online teaching during COVID-19. The results suggest that some of the students do not have basic keyboard skills and it might be explained by the fact that they did not receive any online teaching, or they might have used their cell phones and never had access to a computer and consequently a keyboard.

Excel requires some complicated combined actions such as point, click and drag when using the fill handle which necessitates the use of a mouse, but some of the students were not proficient in the use of the mouse. This might be explained by the fact that these students only used devices such as cell phones and tablets where only touch screens and touch pads are available and the use of the mouse is not necessary.

One of the outcomes of the course is to: "solve certain problems by using formulas, functions…". The students seemed to lack mathematical literacy and did not understand the mathematics behind those formulas and functions. They consequently struggled to solve seemingly simple problems in different contexts such as calculating percentages in school grade books or interest rates. This struggle might be justified by the fact that a large group of these students only had the subject Mathematical literacy in secondary school and furthermore, some of these students were enrolled for the Diploma in Coaching Sciences where the only prerequisite is an APS of 18, which is very low. The study diagnosed that a significant number of students lacked digital literacy which can cause negative emotions such as anxiety and a lack of self-confidence. Consequently, the students might give up or fail, causing the university to face low retention and throughput rates.

The students cannot be blamed for their lack of digital literacy and mathematical literacy. Due to their disadvantaged backgrounds, many South African students are vulnerable, and universities need to put measures in place to improve equity and social justice. The students need to be empowered to capitalize on the potential of digital technologies for their employability, work prospects, and personal, academic and professional development.

Having diagnosed the problem, we can now proceed to the next phase of AR namely action planning. Possible strategies to consider are to let students do a literacy test at the beginning of the semester and to develop a bridging course to fill the gaps in their knowledge and skills.

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A REFLECTION ON TEACHING THE INVERSE FUNCTION IN THE CONTEXT OF A LINEAR ECONOMICS MODEL OF THE MARKETS

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ABSTRACT

This paper is the result of a reflection on students' understanding of inverse functions, in pursuit of a teaching sequence to broaden and challenge their limited understanding of this topic. Although the commonly used procedure of switching-and-solving is effective for determining and sketching a one-to-one function and its inverse function on the same set of axes, the deeper conceptual understanding of the inverse function needed for contextualized problems is troublesome. The contextualized problem described in this paper is a simple linear economics model of markets. The students are first-year economics students, registered for the service module MTHS112 (Mathematical Techniques for BCom) at the North-West University. This paper reflects on students' prior knowledge of inverse functions and unpacks the influence of their limited understanding of a contextual problem. This is augmented by examples from a teaching sequence. The paper concludes with directions for future research.

INTRODUCTION

The purpose of this paper is to reflect on students' understanding of inverse functions and to develop a teaching sequence that will expand and challenge their limited understanding. Traditional questions on the topic of inverse functions are focused on displaying a one-to-one function and its inverse function on the same set of axes. However, inverse functions can be understood far better conceptually when teaching is not limited to determining the inverse function in an x-y function notation context, and then representing it accordingly. Wilson et al. (2011) state that as they brought real-world applications into their mathematics classrooms, they discovered that the traditional way of teaching inverse functions puts obstacles in the way of the students' understanding of inverse functions. A more productive inverse function understanding should be developed (Paoletti, 2019), to make it useful in contexts beyond the x-y function notation context. Wilson et al. (2011) give numerous examples that show misconceptions about inverse functions in different contexts. This paper contributes towards a better understanding of students' learning difficulties of inverse functions in the context of a linear economics model of the markets.

BACKGROUND

At the North-West University (NWU), mathematics modules specifically designed for students from other faculties are referred to as service subjects (Benadé & Liebenberg, 2019). Such modules are included in BCom curricula to provide students with a mathematical foundation to support learning in economic and financial modules. As for different professional domains, the language and techniques of mathematics enable economists to frame and solve problems that cannot be modelled effectively in other ways (Anthony & Gibbs, 2000). Subsequently, a semester module of supporting mathematics is included in first-year economics students' curricula. The service module MTHS112 (Mathematical Techniques for BCom) focuses on helping students to apply previously acquired mathematical concepts, to an economics context. The module prerequisite for MTHS112 is a mark of at least 40% in matric mathematics. This prerequisite module includes the concept of a function and the applications of linear functions. One specific application of linear functions is described in the next paragraph.

CONTEXT

The context described is a very simple linear economics model of the markets. The model describes supply and demand (SAD) in the market for a single good and is concerned with the relationship between two things: the price per unit of the good (usually denoted by p), and its quantity on the market (usually denoted by q) (Anthony & Gibbs, 2000). Fundamental to conventional economics for much of the last 100 years is the concept of "equilibrium" price as the unique price that ensures or supports a balance between planned market supply and planned market demand (Aspromourgos, 2020). Three unknowns are at play in this model, namely p (price), q^S(quantity supplied) and q^D(quantity demanded). In the theoretical mathematical model of the markets, it is assumed that a market price is given, and it is simply up to the individual to decide how much to buy or sell at that price (Renshaw, 2012). In other words, price determines the quantity demanded or supplied. Therefore, price is taken as the independent variable. Quantity demanded or supplied is then a function of price – $q = f(p)$. With this in mind and following the mathematical convention of displaying the independent variable and dependent variables on the horizontal and the vertical axes respectively, we will plot the dependent variable – quantity supplied (q^S) or demanded (q^D) – on the vertical axis and the independent variable – price (p) – on the horizontal axis.

However, in the graphical representation of the intersecting supply-and-demand apparatus (SAD) – known as the "Marshallian cross" – price is displayed on the vertical axis. This specific graphical representation is named after Alfred Marshall (Aspromourgos, 2020). In Marshall's SAD diagram, quantities and prices are displayed on the horizontal and vertical axes respectively. This resulted in a long-established convention among economists to plot price on the vertical axis when graphing supply and demand functions (Renshaw, 2012). However, key contributors prior to Marshall (1890), include the reverse of the later convention. Cournot simply posits demand (quantity) for a commodity as a negatively sloped function of its price and displayed price on the horizontal axis (Aspromourgos, 2020). Following Marshall, price (p) should be displayed on the vertical axis (dependent variable) and quantity demanded/supplied (q^D/q^S) on the horizontal axis (independent variable) (Anthony & Gibbs, 2000; Renshaw, 2012). Thus, the graphical representation of this linear economics model of the markets should be price as a function of quantity, $p = f^{-1}(q)$, that is, the graph of the inverse of the supply and demand functions (Anthony & Gibbs, 2000; Renshaw, 2012).

This greatly stimulated the need to reflect on students' prior knowledge of inverse functions. How students' knowledge of inverse functions is constructed is important for understanding the relationship between price and quantity, $q = f(p)$, on the one hand, and on the other hand the graphical representation of the supply and demand functions, $p = f^{-1}(q)$. This calls for unpacking of subject matter from school mathematics (prior knowledge) and translating it into representations that students can understand (Fennema & Franke, 1992). Being aware of the understanding that students may hold and their likely consequences could inform teaching.

PRIOR KNOWLEDGE

Prior knowledge has long been acknowledged as an important factor in teaching and learning. "If I could reduce all of educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the students already know." (D. Ausubel as quoted by Hiebert & Carpenter, 1992). Prior knowledge that is well understood influences learning differently than prior knowledge that is less understood. As students make the transition from school to university mathematics, it is important to be aware of prior knowledge of key concepts, and then to make those mathematical concepts useful for students in their different professional domains of study. First-year students should have encountered inverse functions during their final year of school mathematics.

Profiling students' prior knowledge starts with a useful distinguish between two types of mathematical knowledge, namely procedural knowledge and conceptual knowledge (Hiebert & Lefevre, 1986). Procedural knowledge of mathematics refers to the conventions, rules and procedures that we use in carrying out routine mathematical tasks, as well as the symbolism that is used to represent mathematics (Van de Walle, 2004). In the context of inverse functions, it will be the procedure to determine an inverse function, commonly known as switching-and-solving.

Procedural knowledge is usually bound to the initial context in which the procedure was learned. Conceptual knowledge, on the other hand, releases a procedure from the surface context in which it was learned and encourages its use in other structurally similar problems. For Hiebert and Lefevre (1986), conceptual knowledge is characterized most clearly as knowledge that is rich in relationships. Well-developed conceptual knowledge can assist in identifying the underlying features of the specific problem context, which in turn can lead to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods**.** Liang (2016) describes conceptual conflict as the conflict between students' new knowledge and their prior knowledge, and states that the process of learning by connecting prior knowledge to new knowledge is often related to conceptual change. Tall and Vinner (1981) state that a concept image consists of all the cognitive structure in the individual's mind that is associated with a given concept. They use the term "concept image" to describe the total cognitive structure that is associated with the concept. It includes all the mental pictures and associated properties and processes. This influences a student's evoked concept image, the portion of the concept image that is activated at a particular time (Tall & Vinner, 1981). Learning more about students' progress in developing their concept images could inform our teaching preparation and help us to provide greater opportunities for students to gain valuable insights into the concepts of a function and inverse functions (Breen et al., 2015).

A procedural understanding of inverse functions is well-documented (Even, 1992; Wilson et al., 2011; Breen et al., 2015; Paoletti et al., 2018; Ikram et al., 2020; Weber et al., 2020, Okur, 2013). According to Weber et al. (2020), many scholars report that students' understanding of inverse functions is compartmentalized and that the essential reason for misconceptions is the fact that the concept of an inverse function is generally taught based on a memorization procedure (switching-and-solving). Wilson et al. (2011) state that this problematic procedure, as taught in secondary school, obscures the basic inverse function concepts. The paper of Even (1992) illustrates a naïve conception of the essence of an inverse function and a lack of adequate relationships between conceptual and procedural knowledge. The following memorized procedure (Procedure A) can be described as "in-school acquired knowledge" (Fennema & Franke, 1992). Therefore, it forms part of students' prior knowledge of inverse functions.

Step 1:	Write down $y = f(x)$.	
Step 2:	Switch the x's and y's.	$\boldsymbol{x = f(y)}$
Step 3:	Solve for y in terms of x.	You find $\boldsymbol{y}$ as a function of $\boldsymbol{x}$.
Step 4:	The result is $y = f^{-1}(x)$.	

<u>Procedure A</u>: Switching-and-solving (Switching x and y and solving for y).

In traditional exercises on the topic of inverse functions, Procedure A is commonly used in school textbooks and learning material. Typical exercises focus on this procedure and will ask you to determine the inverse function $f^{-1}(x)$ of a one-to-one functions $f(x)$, and to then plot the function and its inverse function on the same set of axes. Mathematically there is nothing wrong with this procedure, as it will allow you to represent the function and its inverse function on the same set of axes, with the independent variable $\boldsymbol{x}$ on the horizontal axis, which is a graphing convention throughout the school years. This procedure can thus be used for a specific purpose, in decontextualized situations to maintain our mathematical graphing conventions.

However, Wilson et al. (2011) argue that the common procedure of swapping x and y to find the inverse can lead to the meaning of the result being obscured, especially for contextual or real-world problems. The authors contend that swapping the variables and plotting the graph as a reflection in the line $y = x$ does not consider the important aspect of the domain of the inverse function being the range of the function, and vice versa. This causes problems when the dependent and the independent variable of the function are in different units. A study by Paoletti et al. (2018) examining students' inverse function understanding in relation to contextualized applications of inverse functions, highlights the difficulties students have with interpreting inverse functions in context. This becomes crucial when you start to work in a context beyond the x-y function notation. It has a direct influence on students' ability to recognize similar core features in pieces of information that are seemingly different. With Procedure A, the true conceptual understanding of an inverse function is lost, and the perception arises that a function and its inverse function should be displayed on the same set of axes. It negatively affects understanding of the graphical representation in the context of very simple linear economic models.

To bring about some conceptual change, I use different representations of functions during my teaching sequence. Dossey et al. (2002) state that different representations of functions extend a person's understanding of a concept and shed light on an idea not fully understood in another form.

REPRESENTATIONS

In order to develop a comprehensive understanding of mathematics one should focus on the cognitive processes through which students of mathematics acquire and use mathematical knowledge (Mamona-Downs & Downs, 2002). Various cognitive processes through which students of mathematics should acquire and use mathematical knowledge have been described. These processes include problem solving, reasoning and proof, communication, connections and representations. These cognitive processes become knowledge per se (Plotz et al., 2012).

A relevant cognitive process, specific to the context unpacked in this paper is representations. Functions can be represented – in a specific context – as equations, graphs, tables and words. A student's ability to develop and interpret various representations increases the ability to do and understand mathematics. Some students develop a stronger understanding when they see graphs (visual aspects); others prefer algebraic or symbolic representations, such as equations. It is my strong belief that knowledge of different representations of functions and connections between these representations is necessary for students to be able to use prior knowledge in new or different ways.

Thompson (1994), as quoted by Tall et al. (2000), suggests that an appropriate initial focus builds not from the various representations, but from a meaningful context that embodies the function concept:

> "I agree with Kaput that it may be wrongheaded to focus on graphs, expressions, or tables as representations of function. We should instead focus on them as representations of something that, from the students' perspective, is representable, such as aspects of a specific situation" (Thompson, 1994; p.39).

The perspective that different representations of functions are particularly meaningful when set in a specific context is important for an understanding of the link between the theoretical economics model that describes quantity as a unique function of price, and the graphical representation of the inverse function. Students come to realize that a function and its inverse function represent the same relationship, but from different perspectives.

A RENEWED FOCUS ON INVERSE FUNCTION UNDERSTANDING IN AN X-Y FUNCTION NOTATION CONTEXT

From the definition of a function, it follows that the inverse of a function will only be a function if the original function is a one-to-one function (Stewart, 2015). After the unique relationship between the independent and the dependent variable for a one-to-one function has been revised, students are presented with the formal definition of an inverse function. This definition part of the teaching sequence is included to provide a stimulus that requires them to question their current understanding (prior knowledge) that has been formed from previous experiences (Mezirow, 2009) and thereby creating a stimulus for transformation.

Definition: Let f be a one-to-one function with domain A and range B. Then its inverse function f^{-1} has domain B and range A and is defined by $f^{-1}(y) = x$ if and only if $f(x) = y$ for any y in B.

Given this definition (Stewart, 2015), we observe that the independent variable for the inverse function $x = f^{-1}(y)$ is y and the dependent variable is x, but for the function $y = f(x)$, the independent variable is x and the dependent variable is y. Falling back on the jargon of our profession, we could say that we are solving for x in terms of y. Thus, the inverse function is determined by rearranging the terms of the function. In doing so, you create a new rework function rule. It is important to understand that this reworked function rule – the inverse function rule – provides an alternative way to express exactly the same relationship between the two unknowns x and y (Renshaw, 2012). Students' evoked concept image of an inverse function has to be transformed to include the realization that a function and its inverse function represent the same relationship, but from a different perspective.

To connect this realization with their prior knowledge of inverse functions, they are presented with Procedure B, given below, which is based on Stewart's steps (Stewart, 2015) for determining the inverse function of a one-to-one function f. With this **procedure, students are challenged to rethink their set and fixed understanding of inverse functions. Changing the procedure they know can help them realize that inverse functions may not be as fixed as they thought.**

Step 1:	Write $y = f(x)$.	
Step 2:	**Solve this equation for x in terms of y (if possible).**	**Inverse function $x = f^{-1}(y)$.**
Step 3:	To express f^{-1} as a function of x, interchange x and y.	
Step 4:	The resulting equation is $y = f^{-1}(x)$.	

<u>Procedure B</u>: Stewart's steps (Stewart, 2015) for finding inverse functions.

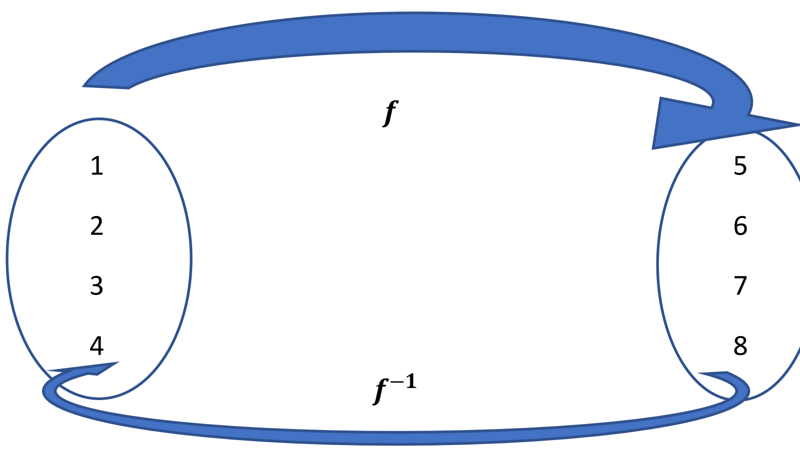
Procedure A and Procedure B are slightly different, but both can be used to determine the inverse of a one-to-one function f. The only difference between the two procedures is the order in which Step 2 and Step 3 are performed. In Step 2 of Procedure B, the subject of the formula is changed to create a reworked function rule, the inverse function f^{-1} of a one-to-one function f.

Equation (Symbol) representation of a function and its inverse function	
$\boldsymbol{y = f(x) = x + 4}$ **f – function rule**	$\boldsymbol{x = f^{-1}(y) = y - 4}$ **f^{-1}– inverse function rule that maps y back to x**

This reworked function rule works with a new input variable; the old output variable becomes the new input variable. Therefore, this reworked function rule f^{-1} gives the same information as the original function, f, but from a different perspective. It should be noted that one needs this reworked function rule for use if one wants to switch input and output values. That is the switching that takes place. If f maps x into y, then f^{-1} maps y back into x (Stewart, 2015). The domain of f^{-1} is the range of f and the range of f^{-1} is the domain of f.

There should be some hesitation before using the word "interchange" out of concern that it may imply switching/swapping the x's and y's**. It is true that switching takes place, but input and output values are switched. Therefore, a reworked function rule representing the same relationship is needed.** With this in mind, and to reinforce students' understanding of the formal definition of an inverse function, the following table representation of a function and its inverse function is given. For the function f, the input variable is x and the output variable is y. For the inverse function f^{-1}, y is now the input variable and x is the output variable.

Table representation of a function and its inverse function



During the next phase, students are shown a very simple contextual example of the workings of a function and its inverse function. For a function $M = f(h)$, the input value in Table 1 is the hours worked, and the output is the amount of money earned. Students must determine the function rule f, given that money earned is a function of hours worked, for values given in Table 1. The input variable should be hours worked and the output variable should be money earned. It does not take them long to figure out that the function that gives you money earned in terms of hours worked is $M = f(h) = 50h$.

Table 1: M as a function of h	
h (hours worked)	$M = f(h)$ (Money earned M as a function of hours worked h)
1	50
2	100
3	150
4	200
Input	Output

Table 2: h as a function of M.	
M (Money earned)	$h = f^{-1}(M)$ (hours worked h as a function of money earned M)
50	1
100	2
150	3
200	4
Input	Output

However, when a person is saving to go on a trip, one may think of the money earned as the input variable: If the person needs R1000 to go on the trip, how many hours does the person have to work? One needs a reworked function rule to determine the number of hours the person should work (output) if he or she must earn R1 000 (input). Having the proper conceptual understanding of an inverse function, one can quickly and easily make the connection that one should work with the inverse function. From Table 2 it is clear that if one simply swaps M and h in equation $M = 50h$, one will not get the same table of values. As soon as students realize that it is not the case, I ask them to solve the equation for h, and then try again. The reworked function, that is, the inverse functions, that give the hours of work in terms of money earned, is $h = f^{-1}(M) = \frac{1}{50}M$.

Step 3 in Procedure B is performed to plot the original function and its inverse function on the same set of axes in an x-y function notation context. But if one strips the conceptual understanding of an inverse function from an x-y function notation context – that is, outside the x-y function notation context – it might not be necessary to display a function and its inverse function on the same set of axes.

To develop a more productive inverse function understanding, it might be useful to expose students to Procedure B. The reason for each step in the procedure should be emphasized and well understood. In that way, the concept of an inverse function can be useful for the x-y function notation context and contexts beyond the x-y function notation context. For Wilson et al. (2011), the approach of switching x and y leads to a mathematical misunderstanding of the inverse function concept. They argue that beyond the notion of inverses, the idea of switching x and y does not make sense conceptually, as becomes clear when a mathematical function is used to model real-world data. They state that in these situations, the meaning of the variables is essential to understand the context being modelled, and switching variables is nonsensical.

INVERSE FUNCTION UNDERSTANDING AND FUNCTION REPRESENTATION IN THE CONTEXT OF AN ECONOMICS MODEL OF THE MARKETS

Functions arise whenever one unknown depends on another (Stewart, 2015). The mapping from price to quantity supplied/demanded is unique, therefore this relationship is an example of a function. Given that price determines quantity demanded/supplied and given the long-established economics conversion to display price on the vertical axis, the following representations (Figure 1) of an economics model of the markets emerge.

The functional relationships for a supply and demand function are represented respectively in words, in a table, as equations and graphically. Each of these representations has its advantage: with graphs the behaviour of the respective supply and demand function and changes over different input values can be displayed visually; in a table it is possible to recognize the pattern for generating the functional value (output) of a specific input value; and an algebraic equation, the function rule, can be used to compute functional values and make predictions. These different representations are all connected and illustrate the same relationship between price and quantity, for supply and demand respectively.

Context

A mathematical model in economics – the description of supply and demand in the market for a single good

Words

Demand ($\boldsymbol{D}$):
The quantity demanded is 9 units when the price is R12 per unit, and 8 units when the price is R14 per unit.

Supply ($\boldsymbol{S}$):
The quantity supplied is 4 units when the price is R12 per unit, and 5 units when the price is R14 per unit.

Algebraic equation (Function rule)

$\boldsymbol{D = f(p)}$: $\boldsymbol{q^D = -\frac{1}{2}p + 15}$ **..... (1)**

$\boldsymbol{S = f(p)}$: $\boldsymbol{q^S = \frac{1}{2}p - 2}$ **...... (2)**

$\boldsymbol{D = f^{-1}(q^D)}$: $\mathbf{p = -2q^D + 30}$ **From (1)**

$\boldsymbol{S = f^{-1}(q^S)}$: $\mathbf{p = 2q^S + 4}$ **From (2)**

Table

Price	Q_D	Q_S
12	9	4
14	8	5
16	7	6
18	6	7

Graph

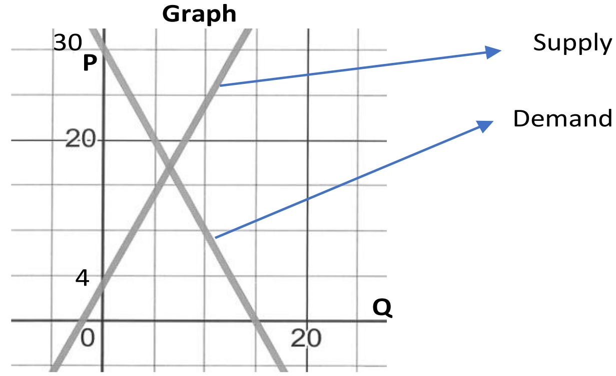


Figure 1: Different representations of a linear economics model of the markets

The supply and demand schedule is typically provided in words or a table. By inspection, a linear relationship between price and quantity for the respective supply and demand schedule can be derived. The supply function $q^S = \frac{1}{2}p - 2$ and the demand function $q^D = -\frac{1}{2}p + 15$ can then be determined by writing the information in the table as ordered pairs. Students then apply their prior knowledge of determining the equation of a linear function.

The table representation in this model can be expanded to include the notion of the relationship between the supply/demand function and the inverse supply/demand function.

Table 3: q^S as a function of p	
p (price)	$q^S = f(p)$ (quantity supplied at price p)
12	4
14	5
16	6
18	7
Input	Output

Table 4: p as a function of q^S	
q^S (quantity supplied)	$p = f^{-1}(q^S)$ (price at quantity q^S)
4	12
5	14
6	16
7	18
Input	Output

Table 5: q^D as a function of p	
p (price)	$q^D = f(p)$ (quantity supplied at price p)
12	9
14	8
16	7
18	6
Input	Output

Table 6: p as a function of q^D	
q^D (quantity supplied)	$p = f^{-1}(q^D)$ (price at quantity q^D)
9	12
8	14
7	16
6	18
Input	Output

In Tables 4 and 6, the input and output values were switched for the respective supply and demand functions. For this to work one needs a reworked function rule that represents the same relationship between the unknowns p and q. We use the second step in Procedure B to determine these reworked functions that will represent the same relationship between the unknowns. It also works when input and output values are switched.

Step 1:	Write $q^S = f(p)$.	$q^S = \frac{1}{2}p - 2$	
Step 2:	Solve this equation for p in terms of q^S:	$p = 2q^S + 4$	**Inverse supply function**

Step 1:	Write $q^D = f(p)$.	$q^D = -\frac{1}{2}p + 15$	
Step 2:	Solve this equation for p in terms of q^D:	$p = -2q^D + 30$	**Inverse demand function**

There is no need to perform Step 3 and Step 4 of Procedure B as we do not need to display the supply and demand functions and their respective inverse functions on the same set of axes. To graphically display these two linear inverse functions, the intercepts with the axes can be determined using the supply and demand functions or the inverse supply and demand functions, as they respectively represent the same relationship between quantity and price. There is therefore no need to first determine the inverse supply and demand functions.

Students are asked to determine demand and supply functions in the form $Q = aP + b$, where quantity is a function of price, to reinforce the understanding that price (P) determines quantity

(Q) demanded or supplied. To follow the economics convention of displaying price on the vertical axis, they have to provide a graphical representation of the inverse demand and supply functions. Students get stuck, especially if they are given the task of determining the supply/demand function (q^D/q^S as a function of p) from a supply and demand schedule, and they then have to make a graphical representation of the inverse supply and demand functions (p as a function of q^D/q^S). It is my firm belief that the tension is created by the fact that students' conceptual understanding of the inverse function is influenced by their procedural knowledge of determining the inverse function.

CONCLUSION

The pursuit of a teaching sequence that could encourage students not to simply swap p and q when they see the word 'inverse' supply or demand function led me on a path of reflection on students' limited understanding of inverse functions. The reasons for my choice of examples during my teaching sequence are twofold. First, it should provide a stimulus for transformation, in that students rethink their prior fixed procedural understanding of inverse functions. Only then can we work towards a more productive understanding of inverse functions that can be applied and useful beyond an x-y function notation context. Secondly, students in this specific service module should have a clear understanding of the different representations of functions that provide different perspectives on this linear economic model of the markets.

Research on students' conceptual understanding of inverse functions is limited. It is particularly so with regards to the mathematical understanding needed for different professional domains. Reflecting on my students' limited understanding of inverse functions was the first stage of the investigation. Implications stemming from this reflection can steer future research. The function concept is defined as a threshold concept in mathematics (Trujillo et al., 2023). We should acknowledge that the lack of understanding among students is complex and linked to the function concept. Other factors that add to the confusion will subsequently be under investigation in future research. These factors also include the notable findings of Basson and Jankowitz (2021) that students at first-year level have trouble handling symbolic language. Notation in which the superscript "−1" denotes the inverse with respect to a particular operation, brings another layer of confusion. Students can be confused by the fact that this notation is used for both reciprocals and inverse functions. A third factor that should be mentioned, is the different meanings assigned to the word 'inverse'. In the inverse function context, the word inverse is used as an adjective, but the adjective 'inverse' could mean several other things. For instance, according to the law of demand, the relationship between price and quantity is described as an 'inverse' relationship, meaning that higher prices result in lower quantity demand and lower prices result in higher quantity demand (Hayes, 2022; The Investopedia team (Boyle, M), 2020).

This paper contributes towards a better understanding of students' learning difficulties, by unpacking the inverse function, reviewing related literature on the development of mathematical knowledge and understanding and reflecting on my experience while working with students studying in an economics context. I gave a possible explanation for why these students have difficulty representing price as a function of quantity graphically after the function $Q = aP + c$ has been determined. I then proposed an alternative approach to determining inverse functions. In my experience prior learned procedure can be very hard to unlearn once it has become entrenched.

We move among strong-minded lecturers of mathematics and equally stubborn mathematics students who do not hesitate to ask why they have to study the subject. There can be a renewed sense of significance when context is considered. We encounter a wide variety of mathematically rich and contextually relevant problems on a daily basis. We have to notice them if we want to expose students to useful mathematical learning experiences.

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PREDICTORS FOR SUCCESS IN FIRST-YEAR UNIVERSITY MATHEMATICS AND STATISTICS

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ABSTRACT

Universities in Australia collect a considerable amount of demographic data about their students. In this paper, such data are analysed in the context of university mathematics and statistics subjects and students' final marks for these subjects. The aim was to determine what, if anything, can be concluded from such data, especially in the context of informing decisions on targeted resourcing or additional support with the goal of improving students' marks. Over 5000 records from ten subjects with high failure rates were analysed.

It was found that most of the demographic information collected by our university is of minimal use for predicting students' final marks in the mathematics and statistics subjects investigated. The demographic information analysed included language spoken at home, age, socioeconomic status, international or domestic status, parents' education, full-time or part-time study, and pathway to university.

The two predictors of most interest are the level of mathematics studied, and the ranking within the state received, in the final two years of secondary school. Gender was the least significant of the demographic factors studied.

INTRODUCTION AND BACKGROUND

Failing a university subject has consequences for students, including increased costs via fees and a later start to a career. Moreover, failing subjects and attrition in higher education have been shown to be linked (e.g., Grebennikov & Skaines, 2008; Harvey & Luckman, 2014). Across Australia, attrition has negative consequences for universities (e.g., loss of students and revenue) and for the country (e.g., loss of potentially skilled workers) (Norton & Cherastidtham, 2018). A report by the Higher Education Standards Panel (2017; p.6) explicitly acknowledges the cost of attrition to both the government and students, stating that 'there should be a sustained effort to improve completion rates'. In Australia, 70% of students who started a bachelor degree in 2009 had completed it by 2014.

Mathematics subjects are reputed to have high failure rates internationally (e.g., DanilowiczGösele, Lerche, Meya, & Schwager, 2017; Lawson, Grove, & Croft, 2020; Saxe & Braddy, 2015; Varsavsky, 2010). Poor mathematical preparation of students is often given as a cause of failure in tertiary mathematics subjects (e.g., Danilowicz-Gösele et al. (2017); Lawson et al. (2020); McMillan and Edwards (2019); Pinxten, Van Soom, Peeters, De Laet, and Langie (2019); Rylands and Coady (2009)). In Australia, the problem of insufficient preparation is getting more acute; for several decades, the proportion of students studying mathematics in the last two years of secondary school has been declining, as has the average level of mathematics subjects studied (Finkel, Brown, Wright, & Wienk, 2020; Nicholas & Rylands, 2015).

Mathematics prerequisites for degrees containing mathematics and statistics subjects would be one way of effecting that students at university are better prepared for study. However, few universities in Australia make use of mathematics prerequisites, with New South Wales (NSW) being the worst state (Finkel et al., 2020). Universities are paid for each student enrolled, so there is a financial disincentive for prerequisites. Besides poor academic preparation, there are a variety of factors affecting failure rates. Baik, Naylor, and Arkoudis (2015) report on data from two decades on the first-year experience in Australia, across all disciplines, and discuss various subgroups of students including:

- students with low admission rank, who were at greater risk of attrition and obtained lower marks in their first semester;
- low socioeconomic status (SES) students (determined in the report by postcode or parents' education levels), who had more financial stress and were less prepared than other students;
- international students, who were more likely to have difficulty with studying;
- women, who reported more stress during the first year, spent less time on campus, were more likely to be studying part time and less likely to have high SES;
- students from regional and remote areas, who reported more financial stress;
- mature age students (defined in the report as age 20 or greater), who were more focused and motivated.

These attributes are intertwined, for example, rural, low SES and international students are more likely to be older. With regards to mathematics, poor study habits were found to be a major factor in a group of 151 students in a mathematics subject in the Philippines (Casinillo, 2019). Snead, Walker, and Loch (2022) used data from six cohorts of students undertaking a first-year mathematics subject and found that age, degree enrolled in and pathway into university were predictors for who was more likely to repeat the subject. Findings on factors that might be predictors of success at university in general may have limited relevance to university mathematics. For example, the gender profile of women in mathematics is far from that of the average student, and a student's first language is likely to have less impact on success in mathematics subjects than it has on success in humanities subjects.

There is a stereotype that men do better in mathematics than women (Girelli, 2023; Lindberg, Hyde, Petersen, & Linn, 2010). Women are underrepresented in the mathematical sciences at university but they tend to complete their courses (Department of Industry, Innovation and Science, 2019). Outcomes for women at university could be influenced by secondary school education, where in Australia, fewer girls than boys study advanced mathematics (Nicholas & Rylands, 2015). Underrepresentation of girls in senior secondary school mathematics subjects is not an Australian phenomenon; in various countries, there is a gender difference in subject choices as soon as students can choose (Girelli, 2023). Much has been written about whether gender has an impact on mathematics performance. Girelli (2023) gives an overview of more than 100 research publications, providing a synthesis of evidence for gender equality in mathematics. Lindberg et al. (2010) performed a meta-analysis of data from 242 articles on mathematics performance at various levels and found strong evidence for gender similarities in mathematics, although they note that a small difference favouring boys appeared in secondary school in more complex problem solving. Akimov, Malin, Sargsyan, Suyunov, and Turdaliev (2023) summarise some work on gender in statistics and found mixed results.

Many other factors have been reported as having an impact on failing a subject and/or retention. An Australian study on attrition across all disciplines (Cherastidtham, Norton, & Mackey, 2018) reported that high impact attributes for degree completion include rank achieved at the end of secondary school, study load (part-time or full-time) and field of education; moderate impact attributes include age and gender; and low impact attributes include SES, attendance type (whether online or on-campus), language spoken at home and highest prior qualification. In mathematics, socio-economic background, age and degree (Finkel et al., 2020) have been reported as impacting attrition. There has been concern for several years in mathematics and computing at Western Sydney University about high failure rates in some subjects. Ten first-year mathematics and statistics subjects for which the failure rates had been around 30% or higher for five years were chosen to be investigated along with the data that the university holds for all students to see if any predictors could be found. Examination reports include reasons for the high failure rates given by the academics in charge of the subjects, with the two reasons 'poor mathematics background' and 'lack of engagement' appearing almost universally. For the 10 subjects analysed, the failure rates remained just as high when the subjects were wholly or partly delivered online in 2020 and 2021 because of COVID-19.

Research aims

Use available data to
(1) identify predictors for success in first-year mathematics/statistics subjects,
(2) discover any differences between subjects or cohorts with regards to predictors for success,
both with the aim of directing action to improve student success.

METHOD

The university, the subjects and the participants

Western Sydney University (WSU) is a large multi-campus university in the state of NSW, Australia with just over 47,000 students. Among the major universities in the Sydney region, there are four other universities that are consistently ranked higher in international rankings. Students can withdraw from a subject without penalty before the semester census date, which is about one month after the start of the semester; no data were obtained for these students. At the end of semester, students are awarded a final mark as a percentage and a grade. Students who do not submit all mandatory assessment tasks receive a special fail grade, Fail Non Submission (FNS). Such students include those who unofficially drop out during the semester and students who make a conscious decision not to submit a mandatory assessment task, perhaps because they know that they cannot pass.

There are no secondary school mathematics prerequisites for entry to WSU. The majority of students studying a first-year mathematics subject are not prepared for university studies in mathematics. The subjects analysed are listed below. In most cases the names of the subjects have been changed to descriptive names.

Basic Mathematics. Basic algebra, functions, graphs, linear and quadratic equations, introductory probability and descriptive statistics. For science students.
Introductory Differentiation. Logarithms, exponentials; mostly differential calculus.
Mathematics 1. Theory and applications of differential calculus, complex numbers.
Discrete Mathematics. Set theory, logic, counting techniques, graph theory.

Engineering Mathematics 0, 1 and 2. A sequence of three subjects for engineering students. The first is a subject for new students not prepared for the second subject. All are deemed to be first-year subjects:

Statistics for Science. A first statistics subject for science students.
Statistics for Data Science. A first statistics subject for data science students.
Statistics. A more theoretical first statistics subject than the two above.

The data

Data were obtained for the Autumn and Spring deliveries of the 10 subjects from 2018 to 2021. Because of the impact of COVID-19 and lockdowns in 2020 and 2021, the university changed the way that many fail grades were recorded. This resulted in marks not being included in the data obtained from the university for many students who failed in 2020 and 2021. Hence the analyses presented here use only the 2018 and 2019 data. There are 5421 records in total.
The data contain for each student, each time they took one of the 10 subjects: the student identifier, the subject, the year and the semester, the final mark (as a percentage), the grade awarded. The other predictors are listed with some explanation of their meaning in this context.

- Age
- Gender. Female or male. There were only four records for two students who identified as non-binary; as we had insufficient data to analyse the performance of non-binary gender students separately, these four records were excluded from the analysis.
- FNS. Boolean: at least one mandatory assessment was not submitted.
- Residency. Domestic (roughly: Australian citizen/permanent resident) or International.
- SES. Low, medium or high: based on the student's postcode of residence.
- Language. Language spoken at home: the categories are (number of students in brackets) English (3066), Other Northern European (14), Southern European (80), Eastern European (80), Iranic (134), Arabic (492), Other Southwest and Central Asian (136), Southern Asian (743), Southeast Asian (26), Mon-Khmer (208), Austronesian (104), Chinese (222), Eastern Asian (18), African (79), Other (19). The categories are based on those from the Australian Bureau of Statistics (2016).
- FirstInFamily. Boolean: neither parent has a qualification at certificate/diploma level or higher, as reported by the student.
- ATAR. Australian Tertiary Admission Rank: a percentile (between 0.00 and 99.95) indicating a student's rank in their age group in the state, based on results from the last two years of secondary school.
- TrueRewards. Boolean: entered through a WSU scheme which offers a university place to secondary students based on some school results before ATARs are announced.
- College. Boolean: the student had previously been enrolled in a WSU College pathway programme.
- HSCMathsLevel. Level of mathematics examined in the last two years of secondary school. The levels are, from lowest to highest, Elementary, Intermediate, Extension 1 and Extension 2. See Finkel et al. (2020) for classification of levels across Australia (Extension 1 and Extension 2 both classified as Higher in this document). A fifth category, None, means that there was no record of an HSC mathematics result. This includes both students who did not study mathematics in the last two years of secondary school in NSW and those who studied mathematics elsewhere.
- Mode. Boolean: the student is enrolled full time (at least three-quarters of a maximum study load).

During 2018 and 2019 there were 32 deliveries of the subjects of interest. The subjects with enrolments are shown in Table 1.

Table 1: The subjects, number of deliveries and enrolments.

Description	**Number of deliveries**	**Number of students**
Discrete Mathematics	2	428
Basic Mathematics	4	1316
Introductory Differentiation	2	99
Mathematics 1	4	272
Engineering Mathematics 0	4	645
Engineering Mathematics 1	4	768
Engineering Mathematics 2	4	872
Statistics	2	477
Statistics for Science	4	517
Statistics for Data Science	2	27
Total	**32**	**5421**

Data analysis

The main aim was to determine important predictors for the final mark, their contribution to the variation in the final mark and to construct a linear model using the significant predictors at 1% level of significance.

The FNS cohort is 13.6% of all records, is very diverse, and could have a large effect on the outcome of the analyses. Therefore the analyses described below were done for all students, referred to as the 'All cohort' and repeated with the FNS cohort removed, the 'non-FNS cohort'. The student cohorts for the subjects are all very different, so it is not surprising that the mean marks for the subjects vary greatly, from 38 for *Statistics* to 59 for *Introductory Differentiation*. The standard deviations were all similar. The final marks for each subject were shifted so that the mean final marks are the same for all subjects; otherwise the effect of the different cohorts would interfere with the analyses.

We evaluated the possible predictors in two dual ways, in each case constructing a linear model with the final mark being the response variable.

(1) Effectiveness of a predictor on its own

- If the predictor was quantitative, it was used as the only explanatory variable.
- If the predictor was categorical, one of the categories was chosen as baseline and the indicator variables for the non-baseline categories were used as explanatory variables.

The significance of the linear model (p-value) and the proportion of the variation of the response variable explained by the model (r^2-value) were computed. For this computation, records for which the predictor variable was missing were removed.

(2) Loss of effectiveness if a predictor is left out/full models

A linear model involving all predictor variables that were significant, the 'full model', was constructed and the proportion of the variation of the response variable explained was computed. Coefficients of explanatory variables reported below refer to the full model.

The full model was then compared to a linear model with the explanatory variable(s) corresponding to the predictor in question being removed, and the resulting reduction in the proportion of the variation of the response variable explained was computed. This was done as a check to detect problems caused by strongly correlated predictors. As this did not reveal any major differences or problems, it won't be commented on further.

For these computations, the following steps were taken for every variable X with missing values:

- An auxiliary variable hasX indicating whether the value of X was available was added and included in the analysis.
- For any record where the variable X was not missing, hasX was set to TRUE and the value of X was not modified.
- For any record where the variable X was missing, hasX was set to FALSE and X was set to the mean of X (for quantitative variables), to FALSE (for logical variables) or to a new baseline category 'Unknown' (for categorical variables).

The auxiliary variables in general turned out to be not significant and/or not contributing much to explaining the variation of the response variable, suggesting that our treatment of missing values is appropriate. Due to the different treatment of missing values, r^2-values computed by the two different modelling approaches cannot be directly compared. The analyses were done using *R* (R Core Team, 2021).

RESULTS

If a predictor explains very little of the variation in the final mark, then from the point of view of students' success it is of little or no use, regardless of its significance. Ten predictors for the final mark were found to be significant at level $p = 0.01$. Both ATAR and HSCMathsLevel are significant, each with negligible p-value.

For the All cohort, the full model using the demographic predictors found to be significant when tested on their own explains 14.3% of the variation in the final mark. A full model using only ATAR and HSCMathsLevel explains 10.1%. Each of the other eight demographic predictors explains much less of the variation in the final mark (see Table 2); even together, the other predictors add relatively little prediction strength. Gender was not significant.

Table 2: Proportion of variability explained by each significant predictor on its own.

	All cohort	**non-FNS cohort**
ATAR	16.7%	19.8%
HSCMathsLevel	9.6%	12.9%
Mode	1.9%	1.1%
College	1.4%	2.3%
Residency	1.3%	1.1%
Language	<1%	1.1%
TrueRewards	<1%	<1%
SES	<1%	<1%
FirstInFamily	<1%	<1%
Age	<1%	<1%

Overall, 13.6% of students were awarded an FNS grade. With the FNS cohort removed, the same ten demographic predictors for the final mark were found to be significant at $p = 0.01$. The proportion of variation explained by a full model using all the significant predictors increased by 3.1% to 17.4%. The proportion of variation explained by a full model using only ATAR and HSCMathsLevel increased by 2.9% to 13.0%. Again, the other demographic predictors added relatively little prediction strength; see Table 2.

In both analyses, ATAR had more of an effect on the final mark than HSCMathsLevel. In each subject ATAR and HSCMathsLevel each explained more variation than any other predictor, except for the *Engineering Mathematics 2* non-FNS cohort, where ATAR explained the largest variation and HSCMathsLevel was not significant. Where numbers are given for the All and the non-FNS cohorts, they will be written as *X* (*Y*), where *X* is for the All cohort and *Y* for the non-FNS cohort.

Table 3 gives the coefficients of predictors for the full models, for predictors which were significant in these models. For binary variables (e.g., Mode) the contribution to the final mark in the model is 0 for the option not in the table. For other categorial variables (e.g., HSCMathsLevel) the contribution to the final mark is 0 for the baseline category 'Unknown' indicating a missing value.

Table 3: Coefficients for each significant predictor in the full models.

	All cohort	**non-FNS cohort**
ATAR: per point	0.51	0.47
HSCMathsLevel: None	−6.50	−7.53
HSCMathsLevel: Elementary	0.65	−0.86
HSCMathsLevel: Intermediate	8.49	6.77
HSCMathsLevel: Extension 1	10.94	9.54
HSCMathsLevel: Extension 2	15.42	14.50
Mode: Full-time	6.65	4.70
College: True	−5.12	−5.47
Residency: International	11.63	9.30
TrueRewards: True	2.61	0.18
SES: Low	−0.20	3.11
SES: Medium	−0.04	3.01
SES: High	1.35	4.24
Age: per year	0.19	0.46

ATAR

ATARs as low as 30 have been reported for students entering university, though more than half of all commencing students in Australia have an ATAR of at least 80 (Cherastidtham et al., 2018). The mean ATAR in NSW is approximately 70 (NSW Department of Education, 2021).

Only 43.9% of all students had an ATAR recorded. The median ATAR was 66.0; the mean ATAR was 64.8, which is well below the mean ATAR of commencing university students in Australia. Across the 10 subjects, the mean ATAR ranged from 61.2 to 83.8 for the smallest subject (27 records), with 74.2 the next highest. The full model predicts an increase in the final mark of 0.51 with each extra ATAR point.

For the non-FNS cohort, 44.8% had an ATAR recorded. The median ATAR was 66.4, the mean was 65.6 and the predicted increase in the final mark per ATAR point is 0.47.

HSC mathematics level

In the full models for both the All and the non-FNS cohorts, the coefficients increase as HSCMathsLevel moves from None, to Elementary, … , to Extension 2 (total increase 21.9 (22.0)). This is mostly true for each subject. In a few cases the coefficient of None was very high, suggesting that while many students with None could well have studied no mathematics in their last two years of secondary school in NSW, a few have very good mathematics.

Other demographic predictors

Apart from ATAR and HSCMathsLevel, the demographic predictors, while explaining little of the variation in the final mark, have some aspects worthy of comment.

Language

We found that 43.4% of students speak a language other than English at home, much greater than the 10% across Australia (Cherastidtham et al., 2018). Language is significant for both cohorts, but explaining only close to 1% of the variation in the final mark. A two-sample *t*-test showed no evidence for a difference between English speakers and speakers of other languages ($p = 0.27$), suggesting that it's not language skills as such that matter but that the cultural background may have an influence. In only two of the individual subjects, Language is significant.

In *Basic Mathematics*, Language is the third most effective predictor for the All and non-FNS cohorts, explaining 4.6% (5.6%) of the variation. It is the largest subject studied (23% of all records), so likely affects the overall results. There are 1316 students with 806 speaking English at home. The differences in marks compared to English speakers predicted by the models for the two cohorts for the next largest language groups are: Arabic −0.2 (−1.0), Southern Asian +2.9 (+2.4), Chinese +6.6 (+6.9).

In *Engineering Mathematics 0*, Language is not significant at 1% for all students, but it is the third most influential predictor for the non-FNS cohort, explaining 5.8% of the variation. The differences in marks compared to English speakers predicted by the model for the next largest language groups are: Arabic −4.8, Southern Asian +1.0.

College

Only a small proportion of students had been enrolled at WSU College, ranging from 5.8% to 15.3% of the subjects. College is significant with negligible *p*-value, explaining 1.4% (2.3%) of final mark for the All and non-FNS cohorts. The coefficients for College in the full models are −5.1 (−5.5).

For most of the individual subjects, College is significant explaining 2%–4% of the variation for all students, and 4%–7% for the non-FNS cohort. The coefficients in the full models are always negative, roughly −4 to −13 marks. Where significant, College predicts a consistent negative effect on the final mark.

Residency

The percentage of international students in the subjects ranges from 3.5% to 17.0%. Residency is significant for both cohorts, explaining just over 1% of the variation in both cases, with the models predicting an extra 11.6 (9.3) marks for international students.

For the individual subjects, Residency is significant only for the engineering mathematics subjects and *Basic Mathematics*, and the coefficients in the full models always predict higher marks for international students.

For *Engineering Mathematics 0* Residency explains 2.6 (2.0)% of the final mark, with the coefficients in the full models +11.2 (+3.5), in *Engineering Mathematics 1* it explains 1.8 (3.4)% with coefficients +15.2 (+14.4) and in *Engineering Mathematics 2* it explains 1.1 (1.1)% with coefficients +8.7 (+8.1).

For *Basic Mathematics* Residency explains 2.7 (2.4)% of the final mark, with coefficients +14.5 (+9.3).

Mode (full-time vs part-time)

Comparing Mode and FNS we find that 22.2% of part-time students drop out during the semester compared to only 12.1% full-time students. A χ-squared test for independence of Mode and FNS showed that this difference is highly significant. For the All and the non-FNS cohorts, Mode explains 1.9% (1.1%) of the variation; the coefficients in the models give an advantage of 6.7 (4.7) marks to full-time study. Where Mode was significant in individual subjects, higher marks were always predicted for full-time study.

Mode was significant in both cohorts only for *Engineering Mathematics 1* and *2*, which had the lowest proportions of full-time students: 80.2% and 77.3%. For *Engineering Mathematics 1*, Mode explained 1.3% (2.7%) of the final mark with coefficients +1.0 (+3.2). For *Engineering Mathematics 2*, Mode explained 3.8% (2.3%) of the final mark with coefficients +8.3 (+5.3).

For *Discrete Mathematics*, *Engineering Mathematics 0*, *Mathematics 1* and *Statistics* Mode was significant only for the All cohort, explaining between 2.0% and 3.6% of the final mark with coefficients between +10.0 and +10.9. Mode was significant only for the All cohort for *Basic Mathematics*, explaining just 0.95% of the variation in the final mark with coefficient +3.7.

Gender

The percentage of females overall is 30.8%. The highest proportion of females was 63.8% in *Basic Mathematics*, the pre-calculus subject taken largely by science students, and the lowest was 8.9% in *Engineering Mathematics 2*. The three engineering mathematics subjects had the lowest proportions of female students, with the proportion decreasing as one moves from the first to the third subject.

Gender was statistically significant in *Engineering Mathematics 1*, explaining just over 1% of the variation in the final mark. The full models give a 7.9 (5.8) mark advantage to being female. Elsewhere Gender was not significant.

The remaining demographic predictors

The predictors Age, SES, FirstInFamily, TrueRewards all show no evidence of practical influence on the final mark.

The subjects

Statistics for Data Science will not be analysed separately, as the number of students is too small for valid conclusions to be made. Some subjects are noticeably different and will be commented on in this section. The percentages of FNS grades ranged from 5.1% for *Introductory Differentiation* to 31.4% for *Statistics*.

Introductory Differentiation

The demographic predictors together explain a huge 48.8% (46.6%) of the variation in the final mark for students in the All and non-FNS cohorts (the next highest is (27.5% (27.7%)). ATAR and HSCMathsLevel alone explain 45.6% (44.8%) of the variation in the final mark. The mean ATAR is 74.2, the highest of all the subjects. Also, the distribution of HSCMathsLevel is shifted towards higher levels compared to other subjects. The subject has the highest mean mark (before shifting) of any of the subjects.

Medical science students and science students (not on a pathway to teaching) make up just over two-thirds of the students, with students planning to be primary or secondary school teachers making up about one-third. Students chose the subject from a pool of mathematics and statistics subjects. Between a third and a quarter of students are enrolled in an advanced degree, and so have previously done much better than average.

Engineering Mathematics 0–2

These three subjects are the only ones in this study that form a sequence which must be followed. Students must either pass *Engineering Mathematics 0* or a readiness test before they can attempt *Engineering Mathematics 1*, which in turn they must pass before attempting *Engineering Mathematics 2*.

In *Engineering Mathematics 0*, the variation explained for the All and non-FNS cohorts is: HSCMathsLevel 16.5% (24.5%) and ATAR 8.8% (6.4%). For *Engineering Mathematics 1*, ATAR explains roughly twice as much of the variation in the final mark as HSCMathsLevel, and for *Engineering Mathematics 2*, in the non-FNS cohort, HSCMathsLevel was not a significant predictor.

For four predictors there appears to be a trend as one moves from *Engineering Mathematics 0* to *Engineering Mathematics 2*; these are shown in Table 4.

Table 4: Changes in HSCMathsLevel**,** ATAR**,** Residency **and** Gender **across the engineering subjects.**

	Eng Mathematics 0	**Eng Mathematics 1**	**Eng Mathematics 2**
None	1.5%	0.8%	0.9%
Elementary	54.8%	18.2%	14.7%
Intermediate	31.9%	49.1%	46.6%
Extension 1	10.8%	27.4%	29.7%
Extension 2	1.0%	4.6%	8.2%
Mean ATAR	61.8	66.6	65.6
International	9.6%	13.5%	17.0%
Females	11.3%	9.4%	8.9%

DISCUSSION AND CONCLUSION

This study was conducted in order to discover, among the demographic data collected by WSU, predictors for student success in mathematics and statistics subjects with high failure rates. Such information might lead to interventions to improve students' performance.

The demographic predictors studied, apart from ATAR and HSCMathsLevel, together explained only a tiny proportion of variation of the final mark, suggesting this is not where resources would be best directed. The predicted increase in final mark of 22 from HSCMathsLevel None to Extension 2 is an excellent reason for universities to promote the higher levels of secondary school mathematics and/or to introduce mathematics prerequisites.

It is disappointing that the proportion of women is only 30.7% overall and much worse in engineering, especially as the results show no difference in performance between women and men. Perhaps the myth that boys and men do better is still believed, and work needs to be done by universities to update beliefs. The decreasing importance of HSCMathsLevel through the engineering mathematics sequence is not surprising as secondary school performance should become less important as students progress, with the earlier university subjects playing a bigger role.

The proportion of international students increases through the engineering mathematics sequence. Indications, where significant, are that international students get higher marks. Visa conditions require such students to study full time, which is another predictor linked to higher marks. University studies are more expensive for international students, which is an incentive to take studies seriously. All these factors could contribute to more international students continuing to *Engineering Mathematics 2*. The modest increase in ATAR along the sequence is unsurprising, but the slight decrease in the proportion of women is concerning especially as in *Engineering Mathematics 1* Gender was significant, with women doing better.

The percentage of students in *Statistics* who were awarded an FNS grade was 31.4%, with 18.1% the next highest. The subject demands more theoretical understanding than the other statistics subjects, and students who don't reach a threshold on the mid-semester test often drop out (recorded as FNS). Students who don't complete the required weekly workshop exercises are also recorded as FNS.

Introductory Differentiation, in some respects, is at the other end of the spectrum. The subject covers secondary school mathematics and so is an easy option for students in advanced degrees. Thus it is unsurprising that this subject has the highest mean mark and lowest FNS proportion. The subject was created for students whose mathematics was not strong enough to take the more difficult *Mathematics 1*; perhaps it only partly serves its purpose.

The 13.0% of all students who were awarded an FNS grade paid fees and did not pass their subject. Adverse circumstances might have forced some to drop out, but not all students. Much money has been paid in fees, and also by the government for domestic students. This is not a good use of resources; finding such students and providing interventions and support where possible could lead to more students passing their subjects.

The main finding with regards to language, as measured by the language spoken at home, is that it is a poor predictor of success. This aligns with the findings of Cherastidtham et al. (2018). With 43.4% of students speaking a language other than English at home this is heartening. Note that many of these students are not international students. The findings is that a significant, number of students who have attended WSU college on average get lower marks while international and full-time students get higher marks.

Actions to improve students' performance

Of the predictors investigated, the previous academic background is the most important, suggesting that resources be used to improve students' knowledge and skills in mathematics. This includes advocating students taking higher mathematics subjects at secondary school and having prerequisites. At university, extending mathematics and statistics academic support would have positive effects as demonstrated by Rylands and Shearman (2018) at WSU and internationally by Lawson et al. (2020). Poor study and learning skills could be a factor, so resources and support could be applied to this.

Limitations and further research

Some predictors were missing from some records, for example, language spoken at home. Also, the latter is not the same as English proficiency. The language groupings might hide relevant results, but for a meaningful analysis of all languages a much larger dataset would have been needed. Some data were reported by the students, and so we are reliant on them providing correct information (e.g., parents' education level). There are many directions for further research. What is the make up of the FNS cohort? Amounting to 13.6% of all students this group is of considerable interest. It's highly likely that much better performance predictors exist than those used here. For example, a quantitative analysis of the effect that student attendance at lectures and tutorials/workshops has on final marks would be interesting.

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THE MATHEMATICAL BELIEFS OF FIRST-YEAR CALCULUS STUDENTS AT A UNIVERSITY IN SOUTH AFRICA

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ABSTRACT

Mathematics learning is significantly influenced by students' beliefs of mathematics. The importance of students' beliefs of mathematics has been shown repeatedly over many years by numerous researchers. Beliefs are founded on learning experiences and are an individualistic interpretation. This paper reflects on an investigation aimed at measuring first-year calculus students' beliefs regarding mathematics. Using questions from the standardised Indiana Mathematical Belief Scale (IMBS), a quantitative measure of these beliefs was attained. The questionnaires were administered to 118 first-year calculus students at a South African university and descriptive analysis was performed on the data. These results pointed out that the highest mean value for the belief "Understanding", has considerably more significance than the belief of "Achieving the correct answer". The results also demonstrated that this particular group of students strongly concur that making an effort and working hard can have an advantageous effect on mathematical abilities. However, it was unsettling to learn that students thought mathematics required step-by-step processes, as observed from the lowest mean for the belief "Steps". Implications stemming from this contradiction are that students do not necessarily understand the difference between step-by-step procedures and conceptual understanding, which might indicate that even their belief systems about mathematics are conflicted.

INTRODUCTION

Learning mathematics can be strongly influenced by students' beliefs, views, or perceptions, of what mathematics, and what learning mathematics is about (Amirali, 2010; Bambang & Mardhiah, 2020; Nurwahyu & Tinungki, 2020; Petocz et al., 2007; Tossavainen et al., 2021). The relationship between mathematical beliefs and learning, according to Spangler (1992), is circular: Students' beliefs of their learning would probably affect their beliefs about what it means for their mathematical learning. On the other hand, how students approach their learning of new mathematical concepts and gaining new knowledge, would depend on their beliefs about mathematics. Additionally, the beliefs mathematics lecturers have about their subject has an impact on how they behave in the classroom, how they view learning, teaching, and evaluation, as well as how they perceive the skills, qualities, personalities, and potential of their students (Barkatsas & Malone, 2005).

For a deeper comprehension of how students learn to problem-solve, Silver (1985) claimed that it is important to study students' beliefs about mathematics. However, there were no tools available to gauge students' beliefs about the discipline of mathematics or the teaching of mathematics (Stage & Kloosterman, 1992). Stage and Kloosterman (1992) subsequently developed an instrument to assess the beliefs of secondary school and college students. In doing so, they gave teachers and lecturers a standardised instrument that enables us to ascertain students' beliefs and, if necessary, alter their education to strengthen those ideas. This study was a pilot study that was conducted to answer the following research question: What are the mathematical beliefs of first-year calculus students at a university in South Africa?

LITERATURE BACKGROUND

Definitions of belief

There has been a great deal of emphasis placed on students' beliefs of both the academic field of mathematics and of themselves as mathematics students. For McLeod and McLeod (2002) there is no single definition of the term "belief " that is universally agreed on, but several types of definitions are illuminative in different situations. Kloosterman et al. (1996) define a belief as an individual's construct that influences behaviour. In other words, a student's belief about learning mathematics is something they are aware of or feel has an impact on their effort. Moreover, a student's perspective about mathematics reflected through his or her beliefs provides a good estimate of the learning and teaching of mathematics that the student experienced (Pehkonen & Törner, 1995). The combination of a diverse range of notions and beliefs as stated by Pehkonen and Törner (1995)**,** will make up someone's viewpoint about mathematics, which can be categorised into four basic groups:
(1) beliefs about mathematics,
(2) beliefs about oneself within mathematics,
(3) beliefs about mathematics teaching, and
(4) beliefs about mathematics learning.

A belief has, according to Nurwahyu and Tinungki (2020), two parts: one is *cognitive* and the other is *motivational.* Students make particular choices about how and when to do mathematics as part of their cognitive activity of motivation in the study of mathematics. Nurwahyu and Tinungki (2020) explain further that mathematical knowledge, assumptions, and other cognitive images are all examples of cognitive components of mathematics, and that a student's attitudes, feelings, and emotions about mathematics are referred to as affective features. Pehkonen and Törner (1995) opine that while the affective component is being highlighted in fundamental beliefs, the cognitive component will be accentuated in the case of conceptions. One's inherent understanding of mathematics and how it is taught and learned is a component of an individual's personal (experience-based) mathematical beliefs. Concepts are distinguishable from so-called basic beliefs, which are frequently unconscious, in that they can be thought of as consciously established beliefs (Pehkonen & Törner, 1995).

Beliefs, as defined by Schoenfeld (1998), are mental constructions that serve as the systematisation of individual beliefs and experiences. To emphasise Schoenfeld's definition of a belief, Ozturk and Guven (2016) went further and stated that beliefs are mental constructions representing the codification of an individual's experiences, behaviours, and understanding in the problem-solving process. As a result, mental processes connected to motivation, performance, and successful learning in mathematics lead to the creation of mathematical beliefs. For the purpose of this paper, the definition of mathematics beliefs, as defined by Kloosterman et al. (1996) will be used.

Beliefs about mathematics

Stage and Kloosterman (1992) identified five beliefs about mathematics when they developed an instrument, the Indiana Mathematics Beliefs Scale (IMBS) to measure students' beliefs about mathematics. These beliefs will now be discussed.

Belief 1: I can solve time-consuming mathematics problems.
The first belief relates to one's perceived competence for resolving time-consuming mathematics problems. College-level mathematics courses will be challenging for students who lack the willingness to tackle challenging topics. It is crucial to consider the beliefs of students on how able they are to solve problems that require more than a few minutes to finish.

Belief 2: Some problems cannot be solved with simple, step-by-step procedures.
The second belief is the belief that some problems cannot be resolved using straightforward, step-by-step instructions. There are rules to always follow for computational procedures, but students must comprehend why those rules are effective. It was crucial to create a scale to gauge students' level of belief in the existence of rules since those who think that all mathematical problems can be solved by applying rules will give up or employ the wrong rule when there isn't one available.

Belief 3: Understanding concepts is important in mathematics.
The necessity of conceptual understanding in mathematics is the third belief. Students who do not believe it is crucial to comprehend the workings of a specific algorithm and who rely on processes they have memorised to solve problems, are setting themselves up for failure in the long run. As opposed to this, students who take the time to comprehend how a process works will realise they can learn mathematics and will be inspired to strive to study.

Belief 4: Word problems are important in mathematics.
The importance of word problems compared to computational skills was the fourth belief that the researchers chose to investigate. Students who think computation is the key to learning mathematics will be less driven to be good problem solvers than those who think word problems are crucial. It is important to take into account student beliefs on this issue since, in the majority of college-level mathematics courses, problem-solving abilities surpass computational skills.

Belief 5: Effort can increase mathematical ability.
The fifth belief is how much students believe that studying and working hard would improve their mathematical skills. This characteristic is closely related to motivation because many students now think they don't have a mathematical mind and shouldn't be expected to do well in mathematics. However, some students hold the belief that with enough work, anyone can study mathematics and advance their mathematical skills. Students' beliefs regarding the amount to which effort can build mathematical ability were taken into consideration because it is obvious that students who believe they cannot improve their mathematical ability by studying, will not attempt to strengthen their problem-solving skills. This belief is based on Carol Dweck's Fixed/Growth mindset theory (Dweck, 2014). According to this theory, a person with a fixed mindset thinks that mathematical aptitude is hereditary and that if you are not born with it, it is difficult to progress in your understanding of mathematics. A person with a growth mindset, on the other hand, thinks that you can always improve your performance on mathematics problems if you put forth effort and work hard.

Belief 6: Mathematics is useful in daily life.
This belief is part of the Fennema-Sherman Mathematics Attitude Scale (Fennema & Sherman, 1976). Usefulness plays a significant role in mathematical motivation. The belief is intended to gauge students' perceptions of mathematics' current and potential value concerning other areas of their lives, such as their future education and employment.

Although beliefs 4 and 6 were not used in this study, these beliefs were discussed for the sake of comprehensiveness. Belief 6 is not part of the original IMBS, but Mason(2003) added this belief to her study.

Belief systems

Beliefs of mathematics are additionally constructed based on philosophies, orientations, and teaching methodologies.

Ernest (1989) differentiated between three philosophical views or beliefs of mathematics:

- An *instrumentalist/toolbox view of mathematics:* Learning mathematics is viewed as the passive recognition of knowledge and the application of a variety of skills, but teaching mathematics necessitates the emphasis on material and its implementation,
- A *Platonist view of mathematics:* A current knowledge framework should be understood and put into practice when learning mathematics. Focusing on the subject matter and highlighting active comprehension, are also necessary for mathematics instruction,
- A *problem-solving view of mathematics:* The discovery of one's particular interests while learning mathematics is seen as a self-directed process, and education must be centred on the student rather than the subject matter.

Furthermore, Felbrich et al. (2008) formulated and named beliefs regarding the nature of mathematics according to the following four orientations:

- *The formalism-related orientation*: As a precise science with an axiomatic foundation and a deductive development process, mathematics is regarded as such,
- *The scheme-related orientation*: It is believed that mathematics is a set of terminology, laws, and formulae (a ‘toolbox’),
- *The process-related orientation:* One way to think of mathematics is as a branch of science that focuses primarily on finding patterns and structures using problem-solving techniques,
- *The application-related orientation*: One could consider mathematics to be a science that has application to daily life and society.

The method used to teach mathematics in the classroom will gradually shape the students' beliefs of the subject. The latter can be easily understood if we keep in mind that individuals’ beliefs about and viewpoints on mathematics serve as a regulatory system for their knowledge structure. The individual can behave and think in this context. However, this structure has a significant impact on the individual’s mathematical skills. Consider this illustration: One student just perceives mathematics as calculations. His comprehension could frequently have come from one-sided calculations that prioritised education for basic students. He may find it difficult or impossible to complete jobs that need thought and when simple calculations may not produce the desired result (Pehkonen & Törner, 1995).

It is important to emphasize that mathematical belief systems also include a predictive component, as well, according to Pehkonen and Törner (1995). Studying mathematics at higher grade levels, such as in secondary schools or universities, requires additional aspects of mathematics than basic calculations. Students who just see mathematics as a manipulative calculation system have limited possibilities to learn successfully in school and/or university. Even though the influence of mathematics beliefs on mathematics learning are not measured in this study, the influence of mathematics beliefs on mathematics learning are discussed to motivate the significance of this study.

Influence on students' learning

Scholarly literature indicates that research focuses more on assessing how students' mathematical performance is influenced by their beliefs than just describing students' mathematical beliefs. For example, the decisions individuals make when evaluating a problem, the types of strategies they use or don't, whether or not they think the problem is difficult, how anxious they are, and how long it will take them to solve the problem, all depend on their beliefs about mathematics (Nurwahyu & Tinungki, 2020). Gomez-Chacon (2000) acknowledged that a student's beliefs have a significant impact on their ability to learn and apply mathematics effectively and that these beliefs are an essential indicator of whether or not students would succeed in learning mathematics.

The structure, excellence, certainty, and origin of mathematical knowledge are all topics covered by epistemological ideas about the nature of mathematics. There exists proof that these beliefs affect how well a learner actually performs and is competent in mathematics since they have an impact on how well they understand the task and, consequently, how they decide to approach it (Felbrich et al., 2008). Furthermore, these ideas influence how people perceive themselves as mathematics learners. Engineering students, according to Harris et al. (2015), are interested in mathematics usage, but they find that the teaching of mathematics in engineering mathematics courses is decontextualized and does not give students a value-added viewpoint. This correlates with the toolbox view of mathematics of Ernst (1989) and the step-by-step belief of mathematics that students may have.

Students also select new mathematical concepts, integrate them into their existing understanding, and build concepts in their thoughts based on their beliefs and prior experiences. There is evidence that students' views and beliefs about mathematical knowledge are related to their difficulties in understanding mathematical concepts (Nurwahyu & Tinungki, 2020). The types of mathematics learning strategies that engineering students, who are in their first year of college, use, are not usually properly known to educators (Harris et al., 2015). One explanation for this is the varied methods used to teach mathematics in schools versus universities. The differences relate to, among other things, expectations for how students should reason and communicate when working on proving and arguing assignments. Harris et al. (2015) also mentioned the possibility that engineering students' beliefs about mathematics could shift as they progress through their time in university. A student can gain entry to a university with strong mathematics results, yet these grades only have a temporary exchange value in the classroom (Harris et al., 2015).

There is a well-established link between students' beliefs of mathematics and their mathematical learning (Schoenfeld, 2016). On the one hand, students' experiences with mathematics instruction have an impact on how they form their points of view (Wong et al., 2001). Conversely, their beliefs or views have an impact on how they act in educational circumstances, which influences how they learn mathematics (Spangler, 1992). Numerous mathematics educators have repeatedly emphasized how important beliefs are to learning mathematics successfully (Garofalo, 1989; Memnun et al., 2012; Mkomange & Ajagbe, 2012; Ozturk & Guven, 2016; Schoenfeld, 2016). Beliefs may have a considerable impact on how students learn and utilise mathematics;

consequently, they may also create an obstacle to good mathematics learning. Students who have strict and negative beliefs of mathematics and how it is learned are more likely to become learners with no interest and who prioritise memorisation over comprehension (Pehkonen & Törner, 1995). According to Boaler et al. (2018), students who treat mathematics as a topic of memorization perform less well academically than those who view it as a subject of concepts that need them to think critically.

The notion that rapid thinkers are required for success in mathematics is a misunderstanding that students hold, even though some of the world's top mathematicians are slow thinkers (Boaler, 2015). With the help of motivation and self-control, mindsets, or self-beliefs direct our behaviour. The idea that our abilities may always be improved is known as a growth mindset or developmental mindset. (Dweck, 2014). A fixed mindset, on the other hand, is the idea that the ability we have, has an innate limit and that we cannot do much about it to adjust. We could have diverse perspectives on our capacity in various branches of learning. For instance, we can believe we can improve our mathematics skills with the appropriate effort and enough time, but we might also believe we will never be able to learn to play the piano.

As stated by Tossavainen et al. (2021), the 'toolbox' belief of mathematics is the best indicator of task performance; the stronger this belief, the less favourable the task performance. It has been found that emphasising the 'toolbox' approach of mathematics while teaching mathematics does not enhance learning outcomes and may even be detrimental. Students' requests for specific examples of how to apply mathematical findings in various contexts rather than an interest in exploring the theoretical underpinnings of these findings are a sign that the 'toolbox' viewpoint is being adversely emphasised (Tossavainen et al., 2021). Wong et al. (2001) claim that a person's beliefs of mathematics serve as a framework for their knowledge structure, which in turn has a significant effect on how well they accomplish mathematical tasks. For instance, a lecturer-dominated learning environment with a focus on calculations sometimes leads to a student's grasp of mathematics being reduced to simple calculations. Pehkonen and Törner (1995) are of the opinion that there are other cultural mathematical notions, maybe misconceptions, such as the idea that mathematics just entails calculation tasks, that have an impact on a student's mathematical behaviour through his or her belief system. There exists a misconception that learning mathematics is entirely about memorizing formulas and rules rather than thoughts, ideas, and creativity. In previous research, Lam et al. (1999) illustrated that students frequently thought of mathematics as a subject of "calculables," which might be the discipline's most real component. When facing something they could manage step by step, young students felt confident. However, if this viewpoint continues to prevail and the student comes to believe that this is the only element of mathematics, it may be difficult for them to have a deeper comprehension of the subject.

The idea that mathematics requires thinking is another aspect of how students perceive the subject. They see mathematics as a form of "thinking exercise", strengthening the intellect in the same way that physical exercise does for the body. The use of mathematics in daily life is especially beneficial. As a driving force behind how students think and behave, beliefs are crucial. The student's beliefs in mathematics serve as a filter, affecting practically all of his thoughts and deeds concerning mathematics (Lam et al., 1999). A student's belief is entirely influenced by their earlier mathematical experiences, generally unknowingly. Pehkonen and Törner (1995) state that students' beliefs are heavily entwined with how they apply their mathematical experience. The authors contend that a student's belief of mathematics may serve as a sign of how well they have been taught in academic settings, such as colleges and universities.

In the next section, the participants in the study will be discussed, followed by the data collection, instrument used, and analysis.

METHODOLOGY

Participants

A quantitative study was conducted at a university in South Africa. 118 first-year calculus students participated. These students were registered for the first-semester introductory algebra and calculus module in the first semester of 2023. The participants included engineering, natural science, business mathematics, and informatics (BMI) undergraduate students, and also students from certain commerce programs. The selection of the students was purposive sampling from the 700 students registered for this module. The 700 registered students are divided into 5 groups. Since the researcher taught only one of these groups, only the students in that specific group were the participants.

Data Collection, Instrument, and Analysis

Stage and Kloosterman (1992) developed and validated an instrument, the Indiana Mathematics Beliefs Scale (IMBS), to assess the mathematical beliefs of secondary school and college students. The IMBS was designed to be used by researchers, who want to ascertain students' beliefs. Items from the beliefs should be randomly dispersed inside a single questionnaire when administering the beliefs, with the exception that questions from the same belief should not be consecutive (Stage & Kloosterman, 1992). There is no total score; instead, each belief is scored independently.

According to Stage and Kloosterman (1992), it is not necessary to use every belief at once. The importance of the word problem belief should only be used in situations where the phrase "word problem" has been introduced to students or where word problems are an ongoing component of the curriculum due to the possibility of confusion about the term "word problem". Because of the content of the mentioned module and the importance of conceptual understanding in this module, the term “word problem” does not apply to this study, and only beliefs 1, 2, 3, and 5 of the belief scales from the IBMS were used ***to measure students’ beliefs of mathematics.*** The researcher used her own discretion when selecting ***two items each from the chosen belief scales.*** The questionnaires were given to the participating students during a contact session. Item 2 was negatively worded, however, for the statistical analysis, reverse coding was applied.

The chosen items from the original IBMS are the following:

1. I feel I can do mathematics problems that take a long time to complete. (From Belief 1)
2. Memorizing steps is not that useful for learning to solve mathematics problems. (From Belief 2) [Reverse coding was applied]
3. Time used to investigate why a solution to a mathematics problem works is time well spent. (From Belief 3)
4. By trying hard, one can become smarter in mathematics. (From Belief 5)
5. Most mathematics problems can be solved by using the correct step-by-step procedure. (From Belief 2)
6. Working can improve one’s ability in mathematics. (From Belief 5)
7. I feel I can do hard mathematics problems if I just hang in there. (From Belief 1)
8. In addition to getting the right answer in mathematics, it is important to understand why the answer is correct. (From Belief 3)

Students' answers to the items were coded for data analysis. According to the Likert-type scale used to evaluate each item, the following numbers show the respondents' level of agreement or disagreement with each statement: 1 = strongly disagree, 2 = disagree, 3 = agree, and 4 = strongly agree. A higher score always indicates a more positive mindset. For analysing the coded data, the program SPSS was used. The mean and standard deviation are descriptive statistics that were utilized for describing the data.

RESULTS AND DISCUSSIONS

The discussion will concentrate on the findings that emerged from the use of IMBS items to gauge students' beliefs. The results of each of the belief scales were investigated since the statistical analysis uncovered noteworthy results regarding such beliefs.

Since 2.5 is the midpoint on a 4-point Likert scale, it was determined that the responses, which were obtained using a Likert scale, should be interpreted as inclined to agree or strongly agreeing for means of 2.5 and above. The participants tended to agree and strongly agree with the questions since this was the case for all the measured beliefs.

In Table 1, the mean and standard deviation results of each item are recorded, while Table 2 illustrates each described belief's mean and standard deviation. Note that each mean score is out of a maximum of 4.

Table 1: Mean and standard deviation of each item

Question	Mean	Standard Deviation
1 (Belief 1)	3.14	0.494
2 (Belief 2)	2.58	0.902
3 (Belief 3)	3.49	0.519
4 (Belief 5)	3.50	0.596
5 (Belief 2)	3.28	0.678
6 (Belief 5)	3.59	0.510
7 (Belief 1)	3.36	0.517
8 (Belief 3)	3.85	0.361

Table 2: Mean and standard deviation of each belief

Belief	Mean	Standard Deviation
1 (Difficult Problem)	3.25	0.432
2 (Steps)	2.93	0.613
3 (Understanding)	3.67	0.359
5 (Effort)	3.55	0.443

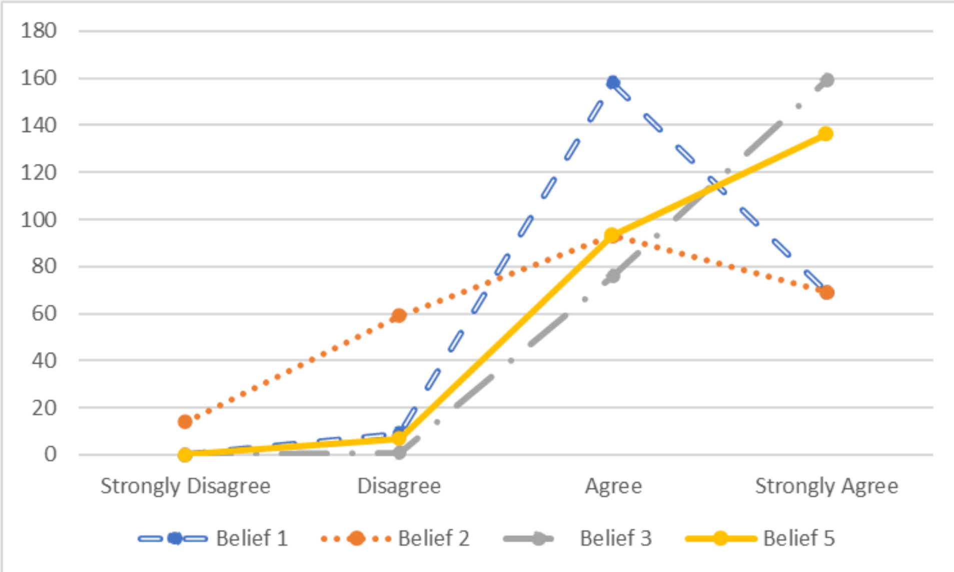

Figure 1: Beliefs of mathematics

Belief 1*: Do students believe they can solve time-consuming problems?*
Student assumptions regarding time-consuming mathematics problems are recorded in the first belief. The mean score for this belief was 3.25 out of 4 (see Table 2), which was the third highest-scoring belief. Most of this group of students agreed that they can solve time-consuming problems (see Figure 1), which implies that they believe they can do mathematics problems that take them a long time to complete, and with this, they can also do hard problems if they persevere, and show grit.

Belief 2*: Do students believe that there are problems that cannot be solved with simple, step-by-step procedures?*
The first item which is part of this belief is negatively worded: "Memorising steps is not that useful for learning to solve mathematics problems." To ensure that a higher score still indicates a positive way of thinking, reverse coding was applied. As seen from Table 2, this belief had the lowest mean value of 2.93 out of 4. Considering the use of step-by-step procedures by these students and the occurrence of rote learning within mathematics, it is concerning that the majority of the students agreed with these items. Step-by-step procedures exhibit negative impacts on students' ability to solve everyday problems (Prendergast et al., 2018), and may also be a sign that students believe that memorising formulas and rules would be enough to solve mathematical issues. This correlates with the toolbox approach (Felbrich et al., 2008), where students believe that mathematics is a set of terminology, laws, and formulae. Since the standard deviation of this belief is the highest at 0.613 (see Table 2), it appears as if there is not a lot of consensus among this group of students about the solving of problems with step-by-step procedures. This group of students indicates a high dependence on rote learning and memorization.

Belief 3*: Do students believe that understanding concepts is important in mathematics?*
According to these students, the mean score for understanding concepts was 3.67 out of 4, the highest-scoring belief, as seen in Table 2 and according to Figure 1. Most of the participants agreed and strongly agreed with the fact that it is important to have a conceptual understanding of mathematics. Thus, for this group of students, comprehending a mathematical concept has much more worth than getting the correct answer.

***Belief 5**: Do students believe that effort can increase mathematical ability?*

This belief, which is about *"Effort can increase mathematical ability",* has the second highest mean of 3.55 out of 4, indicated in Table 2. According to Figure 1, the majority of the students agreed and strongly agreed with the items of this belief. This demonstrates that these students strongly concur that making an effort and working hard can have an advantageous effect on mathematical abilities. This is a promising result since it suggests that these participants show signs of some characteristics of a growth mindset. Dweck (2014) claims that students with a growth mindset have a major advantage over those who have a fixed mindset. She discovered that compared to students with a fixed mindset, those with a growth mindset showed a better commitment to learning and a stronger belief in the value of work in improving grades. Boaler (2015) agreed with Dweck that students must comprehend that problem-solving skills and mathematical reasoning are not instinctive. Hard work and grit are responsible for a lot of this, and therefore a growth mindset is crucial while attempting arduous mathematical problems.

CONCLUSIONS

This study measured the mathematical beliefs of first-year calculus students, by drawing from the IMBS, developed by Stage and Kloosterman (1992). In this study, there were positive and negative results. It can be seen as a positive result that the students had a strong belief that comprehending a mathematical concept has much more worth than getting the right answer. In contrast, the less positive belief is the students' belief that memorising formulas and rules would be enough to solve mathematical problems. Some students can place more emphasis on memorising steps and algorithms than grasping the underlying ideas to solve difficult problems. These results are contradicting. Implications stemming from this contradiction are that students do not necessarily understand the difference between step-by-step procedures and conceptual understanding, which might indicate that even their belief systems about mathematics are conflicted.

Students are generally satisfied with the way they see mathematics and themselves as mathematics learners. Students must have the following beliefs: that they can solve time-consuming problems; that mathematical step-by-step procedures have their confines; that time spent understanding concepts is time well spent; and that effort will improve their mathematics proficiencies. These beliefs must be nurtured by lecturers. Once beliefs have been measured, what should lecturers and students alike, do to nurture positive beliefs? These beliefs are not fixed and can change over time with exposure to different teaching methods, experiences, and interactions, however, changing beliefs is difficult. A growth mindset can however help students develop a more productive approach to learning mathematics, especially first-year calculus. Educators can strive, after measuring mathematical beliefs, to address, and shape students' beliefs to foster a more positive and effective learning environment. Further research is required to investigate the influence of students' mathematics beliefs on their learning of mathematics, and also how positive beliefs can be nurtured in students. Using qualitative measures, such as observations and interviews, may acquire deeper insights into this matter.

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THE ROLE OF NUMBERS IN TEACHING MATHEMATICS AT UNIVERSITY LEVEL

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KEYWORDS: numbers, procepts, Calculus

ABSTRACT

The paper considers different representations of numbers and investigates how these representations may influence the way mathematical courses are taught at university level. The theory of procepts provides a theoretical framework for our consideration. Being analysed as procepts numbers can be seen as a flexible didactical tool. Several examples with focus on mathematical constants are discussed in detail to show how mathematical knowledge and understanding of the Calculus content can be enhanced within the educational theoretical framework.

INTRODUCTION

For any individual the first acquaintance with numbers begins in the form of counting. The further progress is being made almost permanently when learners become more and more aware of different operations with numbers involved, and as a result of that different relationships between numbers. Or, more generally, between mathematical objects where numbers are just some of them. By the time students enter universities they are familiar with several 'special' numbers which their school teachers may have called *mathematical constants* and possibly described them as more important numbers than others. While any number can be considered as having something special in relation to other numbers (or, more generally, to some mathematical objects, e.g. functions) and be treated, therefore, to some extent as a mathematical constant, not so many numbers are famously well-known and only several of them are mentioned in mathematical courses at university level – in particular, in Calculus.

Over the years, I have been asking hundreds of the first-year university students about mathematical constants they know, and almost everyone's response included π. Indeed, there are many references to the most famous transcendental constant in mathematics literature (Finch, 2003, 2018; Hardy & Wright, 1985; Polya & Szego, 1998). However, the question of what π means has predictably narrowed students' responses to its numerical value π=3.14… depending on a number of digits quoted after the point. Only several students have been able to describe π as the ratio of circumference to diameter which remains constant for any circle. The numerical value of π prevailed over any other possible representation. Surprisingly, the same tendency dominated in the views of the third-year university students. The list of mathematical constants has been extended to include e, γ, logarithmic constants, constants that contain π, and even $\zeta(3)$. However, answers about numerical values exceeded other forms like

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

or

$$\frac{\pi}{4} = 4\arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{239}\right)$$

where expressions in the right-hand sides provide much more information than their numerical counterparts in the left-hand sides respectively.

In this paper I focus on the differences in the meaning of numbers and investigate how these differences may influence the way mathematical constants can be taught at university level and how it may affect students' understanding. The theory of *procepts* developed by Gray and Tall (1991, 1992, 1994) provides a theoretical framework for our consideration. Being analysed as procepts mathematical constants and their different representations can be seen through their teaching as a flexible didactical tool to improve students' understanding of basic (and more advanced) techniques in Calculus and other areas of higher mathematics. Finch (ibid.) classifies mathematical constants associated with number theory, analysis, geometry and combinatorics along with well-known constants, e.g. π, that can be found anywhere in mathematics. Mathematical constants contribute to the comprehension of fundamental ideas in mathematics in a much broader way than just the area of application of any constant. In this paper I analyse several teaching examples on mathematical constants in detail to show the ways how mathematical knowledge and understanding can be enhanced and further developed within the educational theoretical framework.

Mathematical Constants as Procepts

I begin with the notion of *procept* as it is defined by Gray and Tall (1992, p.7).
The idea of a process giving a product, or output, represented by the same symbol is seen to occur at all level of mathematics. It is therefore worth giving this idea a name: We define a procept to be a combined mental object consisting of a process, a concept produced by that process, and a symbol which may be used to denote either or both. We do not maintain that all mathematical concepts are procepts. But they do occur widely throughout mathematics, particularly in arithmetic, algebra, calculus and analysis. We may consider number as a procept.
To give an example of mathematical constant as a procept I consider the Euler-Mascheroni constant γ which is defined as follows:

$$\gamma = \lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k} - \ln n\right) = 0.5772156649\ldots$$

Indeed, it has all attributes of the procept. It represents the *process* of tending to a limit and *concept* of the amount by which the partial sums of the harmonic series differ from the logarithmic function, and it has the value which is also represented by a symbol γ. If we have a closer look at mathematical constants as procepts, we can notice that most of them have a composite structure. For example, in the case of γ the process of converging to a limit involves another process – the addition of the first n terms of the harmonic series and the concept of the amount of difference also includes the concept of the partial sums of the harmonic series. Furthermore, a procept of the logarithmic function should be taken into account too.

Other representations of γ such as

$$\lim_{n\to\infty}\left(\frac{1}{n}\sum_{k=1}^{n}d(k)-\ln n\right)=2\gamma-1=0.1544313298\ldots$$

where $d(k)$ denotes the number of distinct divisors of k (Hardy & Wright, ibid.), and

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left\{\frac{n}{k}\right\}=1-\gamma=0.4227843351\ldots$$

where $\{x\}$ denotes the fractional part of x (Polya & Szego, ibid.), bring to the scene other procepts of γ. Altogether they contribute to the variety of procepts defined by the same mathematical constant. I will call all of them *generating procepts* of γ.

A number of procepts is also generated by π. I will discuss some of them in the paper. The area enclosed by a circle of radius 1 is

$$\pi=4\int_0^1\sqrt{1-x^2}\,dx=\lim_{n\to\infty}\frac{4}{n^2}\sum_{k=0}^{n}\sqrt{n^2-k^2}=3.1415926535\ldots,$$

while the length of its circumference is

$$2\pi=4\int_0^1\frac{dx}{\sqrt{1-x^2}}=4\int_0^1\sqrt{1+\left(\frac{d}{dx}\sqrt{1-x^2}\right)^2}\,dx=6.2831853071\ldots.$$

The former procept is based on the concept of area in terms of Riemann integral, i.e. a limit of Riemann sums, the latter uses the concept of arc length, given a continuously differentiable curve.

Another procept of π can be given through the famous Wallis' formula

$$\frac{\pi}{2}=\lim_{n\to\infty}\frac{2\cdot2\cdot4\cdot4\cdot\ldots\cdot2n\cdot2n}{1\cdot3\cdot3\cdot5\cdot\ldots\cdot(2n-1)\cdot(2n+1)}.$$

A procept of π with a probabilistic flavour can be seen through Buffon's needle problem. If a needle of length 1 is thrown at random on a plane marked by parallel lines of distance 1 apart, then the probability that the needle will intersect one of the lines is

$$\frac{2}{\pi}=0.6366197723\ldots$$

Generating procepts, if considered together, bring the opportunity to deal with different processes and concepts through their relationships with each other. They form an overall picture of what is happening around a particular mathematical constant with a number of mathematical ideas available for teaching and learning purposes along with directions for further investigation. The next example shows how a procept of π together with a procept of an infinite product can generate another procept of π with a more composite structure.

Taking

$$\varphi = \frac{\pi}{2} \text{ in } \prod_{n=1}^{\infty} \cos\frac{\varphi}{2^n} = \frac{\sin\varphi}{\varphi}$$

we obtain

$$\frac{2}{\pi} = \cos\frac{\pi}{4} \cdot \cos\frac{\pi}{8} \cdot \ldots \cdot \cos\frac{\pi}{2^{n+1}} \cdot \ldots$$

Since

$$\cos\frac{\pi}{4} = \sqrt{\frac{1}{2}}$$

and

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1}{2} + \frac{1}{2}\cos\alpha}$$

a procept of π as an infinite product can be obtained in the following way

$$\frac{2}{\pi} = \sqrt{\frac{1}{2}} \cdot \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}} \cdot \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}}} \cdot \ldots$$

This formula for representation of π was discovered by Vieta who investigated a limit of areas of Archimedean polygons (Phillips, 1984).

Teaching Mathematical Constants and Cognitive Conflicts

The divergence between those who interpret processes only as procedures and those that see them as flexible procepts (Gray & Tall, 1992) is sometimes called the *proceptual divide*. Gray and Tall (1994) pointed out that procepts in higher mathematics may cause cognitive conflicts, and even those who implicitly sense the flexibility of a procept may experience difficulties. While the process is clear, it is not easy to perform it. The following example demonstrates how a cognitive conflict may influence the growth of the proceptual divide.

An undergraduate student with higher than average performance in mathematical courses was asked to evaluate a limit

$$\lim_{x\to 0} \frac{6^x - 2^x}{x}.$$

The student's suggestion was that the best way to evaluate this limit is numerically, either in *MATLAB*, or using a calculator and taking x in small increments either side of $x = 0$. The *MATLAB* code below was written with no difficulties and the student's comments followed that we can repeat these calculations for *h*=0.001, *h*=0.0001, etc. to estimate the limit at $x = 0$ should get an answer around 1.0986. (The middle two numbers are at *−0.01* and *0.01*, closest to $x = 0$.)

MATLAB Code

```
h=0.01;
x=[-3*h:h:-h,h:h:3*h];
y=(6.^x-2.^x)./x
y = 1.0585 1.0717 1.0851 1.1124 1.1263 1.1404
```

Thus, the exact value of $\ln 3$ represented in the form of a limit has been replaced with its approximation with no idea about the level of accuracy such an approximation may have. As a result, $\ln 3$ as a mathematical constant for the given limit has been lost and the student was unable to recover the exact value from approximation. This process can be interpreted as a procedure while proceptual thinking would include both, the use of procedures, where appropriate, and the use of symbols, as mathematical objects for manipulations, where appropriate, as follows.

$$\lim_{x\to 0}\frac{6^x-2^x}{x}$$

$$=\lim_{x\to 0}\frac{2^x\left(3^x-1\right)}{x}$$

$$=\lim_{x\to 0}\frac{2^x x\ln 3}{x}=\ln 3$$

where both functions $a^x - 1$ and $x \ln a$ behave in the similar way in the given direction

$$a^x-1\approx x\ln a\ (x\to 0),\ a>1.$$

The latter fact comes to the scene as the simplest representation of Maclaurin formula for a^x. Alternatively, students can use L'Hospital's theorem to obtain the answer $\ln 3$. For most mathematical constants, questions related to their approximations are of great importance. In terms of proceptual thinking approximation is not only a procedure. It is also the product of a certain level of accuracy with respect to the exact value defined by that process. Those who use proceptual thinking may succeed in a higher level of mathematical skills and understanding – to investigate how fast or slow convergence is to allow a mathematical constant be evaluated through its approximation with the given accuracy. For example, the definition of the Euler-Mascheroni constant γ converges too slowly to be numerically useful. It is illustrated by the following inequality

$$\frac{1}{2(n+1)}<\sum_{k=1}^{n}\frac{1}{k}-\ln n-\gamma<\frac{1}{2n}.$$

In fact, if we wish K digits of accuracy, then $n \geq 10^{K+1}$ suffices in the summation. The Euler-Maclaurin summation (Finch, 2003) gives an improved estimation

$$\gamma=\sum_{k=1}^{n}\frac{1}{k}-\ln n-\frac{1}{2n}+\frac{1}{12n^2}-\frac{1}{120n^4}+\frac{1}{252n^6}-\frac{1}{240n^8}+\frac{1}{132n^{10}}-\frac{691}{32760n^{12}}+O\left(\frac{1}{n^{14}}\right).$$

Finally, I give two more examples, for e and π respectively, where proceptual thinking may enhance students' knowledge to deal with mathematical constants at a higher level of understanding.

The series,

$$e = \sum_{k=0}^{\infty} \frac{1}{k!} = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \cdots$$

is rapidly convergent, while the following radical representation of π is not suitable for numerical calculations (Finch, ibid.)

$$\pi = \lim_{n \to \infty} 2^n \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots \sqrt{2}}}}}},$$

where the right-hand side has n square roots.

Concluding Remarks

Mathematical constants e, π, γ and others associated with them are important components of any course on Calculus at university level. In this paper I made an attempt and outlined the answers to the three fundamental questions related to mathematical constants as both educational research and teaching domain.

- What is a mathematical constant?
- How much information does any mathematical constant bring to us?
- What are the effective ways of teaching mathematical constants at university level?

The theoretical framework of the theory of procepts helped to advance our knowledge on these questions and make further progress in that direction. If we consider a mathematical constant as a procept, i.e. a combined mental object, it gives us the process and the concept of where that particular value comes from and under which conditions. This knowledge can provide access to more information about mathematical constants, i.e. information about their properties. In other words, it can bring into consideration more procepts, many of them often generated by the same mathematical constant. This situation has a great potential to develop students' understanding of Calculus content to a more comprehensive level. For example, instead of a general question about convergence, students can work on the questions of how fast or slow the given object (function, sequence, etc.) may converge.

Undoubtedly, the teaching benefits of mathematical constants being considered as procepts are very promising. This area has great teaching potential and perspectives, though it is quite complicated for investigation. This paper is one of the first steps in this direction. It is expected that research into teaching of mathematical constants will bring further development of theoretical constructions and practical implications with more details, more clarity and more ideas to come. This will contribute to teaching mathematics in general, and teaching Calculus, in particular.

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ABSTRACTS

PRE-SERVICE PRIMARY TEACHERS' ENGAGEMENT WITH MEASURES OF CENTRAL TENDENCY

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KEYWORDS: mean, median, pre-service teachers; statistical literacy, primary teachers

ABSTRACT

Developing statistical literacy skills is an important outcome of mathematics curricula in helping learners meet the demands of the 21st century and teachers need to be well prepared to achieve these objectives with their learners. The purpose of this study was to explore primary pre-service teachers'(PSTs) understanding of measures of central tendency. The research questions were: 1) To what extent can the PSTs carry out calculations relating to the concepts of mean and median? 2) To what extent are they able to identify and respond to learner errors in these areas?

There were 1647 written responses generated by 183 PSTs written responses to nine items that were analysed using the Mathematical Knowledge for Teaching (MKT) framework. The results showed that 95% of the participants were able to carry out simple computations of the mean and median. Furthermore, more than 90% of the group were able to identify errors or underlying misconceptions in learners' responses. However, the PSTs did not find it as easy to provide feedback to learners about how the learners could correct or recognize the errors related to the mean or the median. In terms of the higher-level cognitive demands related to the Specialised Content Knowledge (SCK) subdomain for the mean, 61% were able to solve the problem requiring them to unpack the calculation of the mean. Effective teaching requires useful feedback to learners on how their misconceptions can be addressed and PSTs need support in developing these skills. To help PSTs to develop statistical literacy skills, it is important for them to solve problems with high levels of cognitive demand so teacher preparation programmes must work towards developing the competence of primary school PSTs in these areas.

DIGITAL STORIES OF STUDENTS' JOURNEYS WITH MATHEMATICS

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KEYWORDS: digital stories, graduate attributes, learning trajectory

ABSTRACT

The objective of this research study is to explore students' mathematics learning trajectories and experiences from prior university participation to how they moved on to further their studies or to employment. Through mapping and sharing, the study focuses on selected digital stories to capture the rich day-to-day experiences of students and graduate academics. The study uses the framework of graduate attributes to characterize the students' mathematics learning trajectories and experiences. The analysis intends to range across the whole dataset, draw on individual experiences' synopses and break up the stories through selected excerpts.

This project is being conducted under the auspices of the UWC Research Chair in Mathematics and Applied Mathematics. While the research chair is focused on topology – an extensive branch of mathematics that studies geometric properties which are preserved under continuous deformation – the central theme is that a sustainable future for mathematics is built on relevant, fundamental mathematics research. Thus, the chair is also concerned with how being a beacon for mathematics impacts students' aspirations and futures. Not every student of mathematics will do research in topology, but every student deserves to be inspired in a research active mathematics environment. Hearing and documenting students' stories of their walk with mathematics is part of understanding this dynamic.

ACKNOWLEDGEMENTS

This is a collaborative project with current and past students who organized themselves in a "Science Research Society" to pursue this research study. It is their initiative and ownership of this project that is motivating and guiding the study.

A CHALLENGE-BASED LEARNING TEACHER SUPPORT TOOL

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KEYWORDS: challenge-based learning, interdisciplinary, student-centred approach

ABSTRACT

Challenge-based learning (CBL) is a student-centred pedagogic approach in which students work in interdisciplinary teams to find solutions to societal challenges. In CBL, the student teams are assisted by and present solutions to a real-world stakeholder. CBL has several dimensions, including flexible learning paths, 21st century skills, flexible teacher roles, stakeholder involvement and flexible assessment. A CBL approach has been productively employed in technical subjects such as robotics (Félix-Herrán et al., 2019) but also has shown promise in mathematics (Pepin & Kock, 2021) and mathematical modelling (Félix-Herrán, 2021). The interdisciplinary nature of CBL accentuates the importance of mathematics.
Despite obvious strengths to CBL, teachers find designing courses or programmes that employ a comprehensive adherence to CBL difficult. An alternative is to identify courses or programmes that already exhibit some CBL characteristics and strengthen those. We recognise that CBL can be present at three different levels, Mild, Moderate and Intense. For example, the teacher role could be learning supervisor and/or field expert (mild), coach as learning guide (moderate), or co-researcher and co-learner (Intense) (Imanbayeva, 2022).
We present an interactive online tool that draws from an evidence-based database to provide practical advice on how to transition from current ways of teaching towards CBL ways of teaching (Imanbayeva, et al., 2023). The design of the tool is grounded in Van den Akker's model (2003) and the content is drawn from the literature and local experience. The tool is unique and has drawn interest as being particularly effective for helping teachers focus on those areas of their courses most amendable to a CBL approach (Poster, 2023). To illustrate how to use the tool, we shall use as a demonstration case a Statistics course offered to second-year bachelor students. We shall provide a link to a beta version of the tool for those who attend the presentation.

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STUDENTS CREATING, USING, AND REVISING NOTES IN CLASS FOR USE DURING ASSESSMENTS

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KEYWORDS: learning moments; note-taking; revision

ABSTRACT

Mathematics and Statistics are disciplines that many would say are complicated formulae or funny symbols and it is all too confusing for many students. Educators have tried to reduce this confusion by allowing students to have their own notes during tests and examinations. According to Charles P. Corcoran, in 2020, the use of notes had no significant effect on the learning outcomes, and he noted there was very little literature available on this topic. At Curtin College, which is pathway college to university, students hope to obtain entry to the university at either first or second year. Most classes during the trimester are interactive, with small groups of students collaborating and creating solutions on whiteboards, facilitated by the educator. Last trimester, a reverse in lecture style was trialed with a cohort of engineering students at Curtin College. The minor twist is students will create notes suitable for the upcoming assessment on whiteboards, with only minor prompts by educator, then use the notes to answer a question in class and finally revise and complete notes. There are many benefits for both student and educator, not only how time-consuming writing notes can be for the student but for the educator you can see the misconceptions and failure to recognise key points. It is hoped that this experience may help students provide better notes in all disciplines. The paper will elaborate on the experience in the classroom setting.

INTEGRALS IN MATHEMATICS AND PHYSICS

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KEYWORDS: integrals, calculus, physics

ABSTRACT

Mathematics and physics are two disciplines separated by a common language. Since the 1990s, we have been developing curricular materials that attempt to bridge this gap, as part of the Paradigms in Physics and Vector Calculus Bridge projects (Paradigms Team, 2019–2023), leading also to the development of an online textbook (Dray and Manogue, 2009–2023). Most of our work focuses on the transition from multivariable calculus to upper-division physics courses such as electromagnetism.

Vector line integrals are an important mathematical object occurring in multiple subdisciplines throughout upper-division physics. Our research question is to compare standard textbook treatments of vector line integrals in lower-division mathematics and physics to identify and compare the disciplinary approaches using *representational transformation diagrams* (Bajracharya et al., 2019). We identify two principal approaches that are loosely correlated with these two disciplines, and which differ primarily in how they treat the dot product. These textbook approaches are compared to existing characterizations for (single-variable) integration in the theory literature (Ely & Jones, 2023), including our characterization of *chop, multiply, add*. Finally, we present a hypothetical learning trajectory for vector line integrals (Dray & Manogue, 2023) designed to scaffold student acquisition of rich concept images that bridge the approaches of these two disciplines. We conclude with a discussion of possible implications for the teaching of single-variable calculus.

ACKNOWLEDGEMENTS

This work forms an integral part of the Paradigms in Physics project (Paradigms Team, 2019-2023), incorporating also the originally separate Vector Calculus Bridge project. These projects have been supported by NSF grants 0088901, 0231032, 0618877, 1023120, 1256606, 1323800, 1836603, and 1836604.

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CONTRASTING ROLES OF MULTIPLE-CHOICE QUESTIONS IN ASYNCHRONOUS AND SYNCHRONOUS LEARNING

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KEYWORDS: synchronous, asynchronous, assessment

ABSTRACT

The recent pandemic caused a brutal upheaval in approaches to learning and teaching and our ability and capacity, in adverse circumstances, to engage with students and help them develop important skills and understanding. Necessity is the mother of invention and there may be silver linings emanating from lessons learnt by us, as educators, when forced to make abrupt changes or rethink, with little notice, our teaching approaches and preferences, which may be long-held and ingrained. Assessment and the protection of academic integrity have always been thorny and controversial issues, but even more so during a pandemic. This talk focuses on contrasting roles taken by the extensive use of multiple-choice questions and aims to tease out important underlying principles in using them for formative and summative assessment tasks, separately and in combination. Functionality may change, depending on whether the learning environment is synchronous (in-person, face-to-face) or asynchronous (online, typically remote, using a learning management system). A variety of examples will be presented. If there is time, connections may be made to models of learning, such as constructive alignment and the role of SOLO (Biggs, 2003), the theory of threshold concepts (Meyer & Land, 2005) and navigation of liminal space (Cousins 2006), and ways and means of engaging students and moving them through the passive-active interface (Easdown, 2007).

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REMOTE TEACHING AND LEARNING DURING COVID-19 ERA: CHALLENGES FACED BY PRESERVICE MATHEMATICS TEACHERS

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KEYWORDS: challenging factors, Covid era, preservice teachers, remote teaching

ABSTRACT

This paper seeks to explore the effect of remote teaching and learning on the challenges faced by preservice mathematics teachers when learning mathematics education during Covid 19 and remote teaching limited to four universities two in South Africa, one in Ghana, and one in New Zealand university. The study followed a quantitative research approach to select a survey research design. A total of 95 preservice mathematics teachers from four universities were randomly assigned to participate in the completion of an online survey. The participants signed the informed consent forms before completing the survey to comply with the ethical requirements. The study revealed two preservice mathematics teachers challenging factors during remote teaching and learning, data, and technological devices for learning. The findings revealed that there is no significant difference $p < .000$ between remote teaching and learning and the challenges faced by preservice mathematics teachers when learning during the Covid era, Therefore, the null hypothesis is rejected since $p < .05$ level of significance. The study concludes that the challenges were more on technological devices for learning. The recommendation is that relevant technological devices need to be provided for the learning of preservice mathematics teachers.

RETHINKING STATISTICS COMPUTER LAB CLASSES

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KEYWORDS: digital stories, graduate attributes, learning trajectory

ABSTRACT

In teaching large undergraduate statistics courses, we had previously followed a standard model of Lectures supplemented by Tutorials and Computer Lab classes. We undertook an extensive re-design of these classes to improve groupwork and collaboration, and to better utilise computing power for producing graphs and conducting analysis. Our re-design aimed to combine the strengths of both types of classes, while removing this artificial separation between types of tasks and building a more authentic learning experience. This was achieved as part of a pilot program trialling our Next Generation tutorial rooms - small-group teaching spaces trialling new electronic whiteboard technology equipped with a fully-featured Windows PC. While this technology has its advantages, most of the benefits are possible within a standard computer lab setting. We will discuss the materials/tasks themselves, what we learned through the process of developing them, and the evaluation of the materials for student learning.

EVALUATING STUDENT ENGAGEMENT WITH AND PERCEPTIONS OF A FLIPPED CLASSROOM DESIGN FOR A LARGE STATISTICS SUBJECT

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KEYWORDS: student engagement, quantitative research, large classes, evaluation

ABSTRACT

We present the preliminary results from a project investigating a large statistics class designed for and taught using a flipped classroom model, with pre-recorded videos. Previous research on flipped learning had focused on measuring the overall effectiveness of the method (with mixed results), we wanted to understand which activities and resources were most beneficial to learning. The study, undertaken in 2021 during the Covid-19 pandemic, utilized student surveys in conjunction with metadata on their engagement with electronic resources. The wider research project, under a constructivist framework, is focused on hypothesis generation, through incorporating student perceptions of activities ("I found the tutorials helpful" and "Please describe one aspect of the teaching in this subject that you liked") with a range of quantitative measures. This preliminary investigation focuses entirely on the quantitative aspects of the metadata, final grades, and modes of learning; future research will evaluate other aspects of the project. We aimed to identify which particular resources or activities were most beneficial to student learning, as measured by final grade. Engagement in activities was measured both by percentage completion and frequency of interaction, depending on the type of activity. We found no differences in final grade between different modes of teaching (online/in-person; U=4284.5, P=0.79) or between those who completed a survey and those who did not (U=3383.5, P=0.27). This gives confidence that the results are not particular to the Covid-19 pandemic or survey participation. All predictors were positively correlated to the final grade (correlations from 0.211 to 0.763) and each other (correlations between 0.199 and 0.986). Overall, all engagement with the subject, whether passive or active, assessed or not assessed, contributed positively to their learning. While this may indicate that any engagement with the subject is beneficial, the varied correlations indicate that some activities and resources are more useful.

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BRIDGING THE GAP: AN ANALYSIS OF THE TRANSITION FROM SCHOOL TO UNIVERSITY MATHEMATICS IN SOUTH AFRICA

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KEYWORDS: School-to-university transition, trigonometry, calculus, mathematics education

ABSTRACT

The transition from high school to university has long been a challenge for mathematics students, with a widening gap in content comprehension and application. This study provides a comprehensive exploration of this gap, focusing on trigonometry and calculus, two pivotal areas in the mathematical curriculum. By analysing the National Senior Certificate (NSC) examination reports and first-year university mathematics results over a span of five years, the research delves deep into the challenges and opportunities presented in this educational transition. In this study we answer the following question, "What should be done to improve and align the mathematical content and teaching of calculus and trigonometry, to ease the transition to first-year university mathematics?". The study is framed by the needs assessment theory and the Cultural-Historical Activity Theory (CHAT) and employs a mixed-methods approach to data collection. A mixed-methods approach is used to collect quantitative data from surveys and qualitative data from interviews. A sample of South African university lecturers teaching students who make the school-to-university transition is surveyed, to gather data on their perceptions of the transition gap. Additionally, interviews with the mathematics educators provide insights into their perspectives on the challenges and strategies for addressing the gap. The findings reveal specific challenges faced by students in trigonometry and calculus, with underlying factors like prior educational experiences, socio-economic conditions, and cultural nuances playing significant roles. The study concludes with the emphasis on targeted interventions to bridge the transition gap, paving the way for a more equitable higher education landscape in South Africa.

THE FIRST YEAR MATHEMATICS PROJECT

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KEYWORDS: community of practice, intervention, scoping review, themes

ABSTRACT

The First Year Mathematics Project is one of the research foci of a tertiary mathematics community of practice in South Africa. The project's main focus is to research the use of interventions to improve the first-year undergraduate mathematics. The first part of the programme involved the study of published literature in this area. The research question guiding the study was: *What was the nature of first year undergraduate mathematics interventions at South African universities?* We conducted a scoping review of the relevant literature from 2015 to 2022, using several databases, in particular the SABINET African Journals database which covers all South African online publications. We preferred a scoping review, because, unlike a systematic review, it has less stringent criteria for inclusion and exclusion of articles, as well as a broader scope, providing a better option at this exploratory stage of the study. Three researchers assessed 30 pre-selected articles independently and 14 of the articles met the inclusion criteria. These criteria were: 1) the article must involve first year undergraduate mathematics students of science or engineering and 2) the research must be an intervention aimed at improving student performance. We studied the articles without any pre-assumptions to identify the main themes from the review following grounded theory techniques. Each article was analysed based on the following: the research design, research sample, aims of the study, nature of the intervention, and findings. We identified four main themes; the use of tutors and teaching assistants; 2) feedback from academics and students; 3) teaching approaches; and 4) the use of technology. We presented these preliminary findings to the community of practice. The community decided to set up subgroups whose task was to conduct further research into each of the themes. Their work in ongoing.

A MATHEMATICS COURSE FOR UNIVERSITY BRIDGING STUDENTS WHO WILL STUDY ARTS DEGREES

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KEYWORDS: bridging, academic numeracy

ABSTRACT

An initiative to provide university bridging students on non-STEM pathways with an appropriate mathematics course is described. Each student selected on the programme must pass one maths course. Prior to 2019, most took a standard course, Maths X, that is pitched at upper secondary levels. The increase in programme numbers introduced a broader range of abilities, with several students deemed to not (yet) be at X level. So, a second course, Y, is offered to meet the learning needs of the lower entry-scoring candidates, many of whom still struggle with fundamental concepts of place value, and multiplicative and proportional thinking. However, after five years of the course being offered it is timely to ask how worthwhile a course like Y is in the learning of these students.

Examining results shows that cohort pass rates have been fair (50%) to good (72%), while Did-Not-Complete student rates ranged from 16% to 28%. Going back over student evaluations exposed cohort variations with most being satisfied, and some even liking mathematics for the first time. Still a few others wanted less group work and suggested better resources (coursebook and online). Then a separate written class survey of percentage tasks showed that many struggled with and desired more on proportional thinking. So, although course Y attempts to connect contexts (e.g., false positives and negatives, local political systems, and loan sharks) to mathematics, fundamental maths concepts have needed to become more visible throughout the course. While courses such as Y seem to meet the needs of most students, deliberate acts of teaching with several underlying themes needs to be implemented regularly.

CONCEPTUALISING LANGUAGES AND LITERACIES TOWARDS UNIVERSITY MATHEMATICS CURRICULUM FOR ACCESS AND TRANSFORMATION

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KEYWORDS: access, languages and literacies, transformation

ABSTRACT

Languages and literacies are central to meaning making of knowledges, cultures, identities, social relations, and values. In undergraduate mathematics, we traditionally value formal mathematics register, written and symbolic modes, and discourses of defining, proving, etc., in an institution's medium of instruction. The University of Cape Town (UCT) is an elite, historically 'white', English-medium university, in a diverse, multilingual context. UCT mathematics students report difficulties accessing knowledge in lectures conducted in English, asking questions, understanding assessment items, and linking school and university mathematics, with implications for their performance, and sense of belonging and being (le Roux, 2017; Shay et al., 2020). In this postcolonial context, mathematics educators are challenged to answer questions of access and transformation: How to use languages and literacies towards equity of student access to and success in a locally relevant curriculum in a globally connected world?

I offer conceptual tools that educators can use for thinking about languages and literacies for mathematics curriculum design and pedagogy towards tackling this challenge. This was developed using scholarship and policy at UCT and elsewhere (e.g., le Roux et al., 2022; Prediger & Hein, 2017; DHET, 2020). *Languages* are conceptualised broadly as registers, modes, genres, discourses, language codes, dialects, and accents. *Literacies* are actions with languages in context: writing, reading, talking, listening, drawing, and using information and technologies. A student uses these: to *access* mathematics (in a lecture, textbook); to *learn* mathematics in formal and informal spaces; to *communicate understanding* in assessments; to *demonstrate as an outcome* their use of formal written mathematics. I elucidate these tools with examples from a first-year mathematics course at UCT. However, they can be adapted for other contexts characterised by asymmetries in what knowledges, languages and literacies are valued, and in which mathematics lectures and students bring diverse language and literacy experiences to the classroom.

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LIMITS AND ONE-SIDED LIMITS AS A TOOL IN MODELING AND APPLICATIONS

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KEYWORDS: limits, one-sided limits, theoretical tool

ABSTRACT

The concept of limit is a common topic in mathematics education papers and books (see for example Juter (2005), Kidron & Zehavi (2002), Szydlik (2000)). It is not so common to find works about one-sided limits. However, this fact does not make this topic less important (Fernández-Plaza et al., 2015).
Many students, and not a few teachers, see limits as a theoretical tool, very useful for the definition of other concepts such as derivatives, integrals, and series, but not as a useful tool for mathematical modeling and applications.

After analyzing some elements of the theoretical framework of modeling and applications, we will see some examples that show how limits and one-sided limits are also important due to their applications to other disciplines. Particularly, we will analyze examples related to chemistry, pharmacokinetics, physical chemistry, and reactor design, among others (Martinez-Luaces, 2017). Finally, examples of non-convergent series will be analyzed, and also how through the concept of limit it is possible to assign a value to those series, with applications in quantum mechanics and optics, among other disciplines (Herman, in press).

Based on the above, some suggestions are proposed, mainly oriented to mathematics courses for other sciences and engineering.

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MATHEMATICAL THINKING WORKSHOPS: BRIDGING THE GAP FROM HIGH SCHOOL TO UNIVERSITY

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KEYWORDS: mathematical thinking, transition courses, at-risk students

ABSTRACT

Many first-year students find the transition from high school to university mathematics a challenging experience. First-year university mathematics students are expected to demonstrate a higher level of mathematical thinking and problem-solving skills than in high school, and many are unfamiliar with the new learning environment.

This presentation highlights preliminary findings from an ongoing study investigating the effectiveness of Mathematical Thinking Workshops in bridging the gap between high school and university mathematical thinking practices, focusing on at-risk students. Two distinct, self-selected cohorts participate in the study: a general group of first-year university mathematics students, which may include those identified as at-risk, and a group specifically identified as at-risk.

The workshops, whose design and implementation are informed by theoretical frameworks such as APOS theory, ZPD, cognitive load theory, and constructivism, are implemented twice a week throughout the academic year.

We qualitatively gathered insights through focus group interviews with students about the workshops. Quantitatively, we built a predictive model using students' pre-university data (such as high school results and NBTs) to forecast university performance. We then used this model to predict the performance of workshop attendees and the findings indicated that the workshop students performed better than expected with statistical significance.

Preliminary findings are in line with the enhancement of students' understanding of and the ability to apply mathematical concepts, with notable improvements in their confidence and metacognitive skills. Although our quantitative data revealed promising trends, it's important to note that students' outcomes are shaped by a multitude of unaccounted-for factors (which does not form part of this research).

As part of the larger research project, data is being analysed in real-time to provide a more comprehensive ongoing evaluation of the workshops' impact. This study highlights the value of pedagogical interventions in supporting at-risk students in their transition to university-level mathematics.

TEACHING MATHEMATICS IN THE DIGITAL AGE: REFLECTIONS ON USING TECHNOLOGY DURING AND AFTER THE COVID-19 PANDEMIC

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KEYWORDS: mathematics education, mathematics educators, technology integration

ABSTRACT

The accelerated adoption of technology in mathematics education has been a focal point in recent educational discourse. As educators increasingly recognise technology's transformative potential, its integration offers a promising avenue to enrich the academic journey. Yet, seamlessly weaving technology into teaching presents multifaceted challenges. This reflective study delves into technology's pivotal role, especially under the unique conditions precipitated by the Covid-19 pandemic. Engaging in thoughtful introspection, the authors traverse the experiences of participants navigating emerging technologies and pedagogical innovations. The Technological Pedagogical Content Knowledge (TPACK) framework underpins this exploration. It emphasises the vital expertise educators must cultivate to holistically integrate technology into their instructional strategies. Drawn from a diverse pool, the study encapsulates perspectives from 20 first-year mathematics educators affiliated with Universities of Technology, Comprehensive Universities and Traditional Universities in South Africa. The study employed a qualitative research method to gather and analyse data. The target population consisted of lecturers responsible for instructing first-year mathematics courses across 26 universities in South Africa. To collect data, the researchers utilised semi-structured interviews, a technique that allows for open-ended responses and provides the flexibility to explore emerging themes. Following the data collection phase, the information was systematically analysed using ATLAS.ti software, a renowned tool for managing and examining qualitative data. Preliminary findings accentuate technology's indispensable role in amplifying student engagement and fortifying learning outcomes. Moreover, the study sheds light on the tangible benefits and potential of hybrid or online teaching models. The insights gleaned serve as a beacon for educators striving to seamlessly fuse technology into their teaching matrix. To culminate, the onus is on educational institutions to ensure equitable technological access. This entails prioritising financial incentives, offering intensive training, and fostering a robust infrastructure to buttress hybrid pedagogical models, ensuring academic continuity amidst disruptions.

HOLISTIC APPROACH TO LEARNERS' SUPPORT IN MATHEMATICS: A RAPID LITERATURE REVIEW

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KEYWORDS: holistic support, mathematics education, articulation gap

ABSTRACT

This paper underscores the significance of adopting a holistic approach to support mathematics learners across both basic and higher education levels. While some may view this approach as a luxury, particularly in low-income settings, its pivotal role in promoting both academic achievement and overall student well-being is undeniable. Through a comprehensive rapid literature review, this paper delves into various studies concerning the holistic approach in mathematics education a subject that often induces apprehension among many learners. The aim of this holistic approach is to inspire students, consequently altering their perceptions and attitudes towards learning mathematics. A rigorous methodological approach was utilised to guarantee integrity and transparency. Initially, the study followed the Preferred Reporting Items for Systematic Reviews (PRISMA) guidelines. This procedure was augmented by Rayyan, an Artificial Intelligence (AI) tool tailored for collaborative systematic literature reviews. With Rayyan, authors were able to blindly screen titles and abstracts of prospective papers. Literature review was procured from three databases: Engineering Village, ERIC via EBSCOHost, and Scopus. From these sources, 95 papers were initially identified based on specified search keywords. Of the 95, a mere 30 met the exacting empirical research criteria. These selected papers then underwent categorisation via coding analysis, capturing all relevant themes. Among the key findings, this paper highlights the indispensability of academic support services, psychosocial interventions, and meeting basic needs such as housing and food security to create an environment conducive for effective learning. Drawing on these insights a conceptual framework aimed at enhancing the transition of students from school was formulated. It is recommended that by addressing the full spectrum of learner needs, potential academic setbacks can be circumvented, laying a solid foundation for the development of non-academic skills. This comprehensive approach resonates perfectly with the overarching goals of both basic and higher education institutions.

EXPLORING THE TRANSITION TO ONLINE TEACHING IN MATHEMATICS EDUCATION: AN AUTOETHNOGRAPHIC STUDY

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KEYWORDS: online teaching, mathematics teacher educators.

ABSTRACT

Universities around the world were suddenly confronted by the Covid 19 pandemic lockdown in 2020. Higher education students and teachers were thrust into an environment of emergency online teaching and learning. Within days higher education shifted to a new mode of delivery from in person face-to-face to an online platform in an attempt at connecting students, teachers, and content via learning management systems. In this study we explored what the sudden shift meant for each of us as mathematics teacher educators and how each of us transitioned from face-to-face teaching to emergency online teaching. Our individual documented reflections on our experiences framed this autoethnographic study. As higher education mathematics teacher educators from Ghana, South Africa and New Zealand, we offer insight into how we adapted our practice and navigated this new teaching space during this period of change. Online teaching is not a new discourse. However, our narratives showed that having digital tools does not necessary equip one with expertise to mediate online content delivery. Our responsiveness to emergency online teaching necessitated an institutional shift and personal shift in our practice as mathematics teacher educators. We aver that the pandemic has forced us to change our teaching approach and welcome a changed pedagogy as mathematics teacher educators. For us mathematics teaching and learning is no longer confined by the on-campus boundary. Could the boundaryless potential of online teaching have implications for mathematics educators and researchers to reflect, connect and be inspired across country borders? A few years on since the pandemic has allowed us to see the potential of online teacher education, not for the purpose of replication of teaching, but reimagining teaching and learning in higher education.

RE-THINKING TEACHING AND LEARNING IN A SOUTH AFRICAN EXTENDED DEGREE PROGRAMME

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KEYWORDS: grounded theory, care, learning communities

ABSTRACT

Engineering programs in South African universities struggle with large diverse classes; diverse in student abilities, prior knowledge and learning styles. This results in some students failing courses, feelings of alienation and low throughput rates. Academic development initiatives, like extended degree programs, aim to redress but face hurdles due to students' hidden abilities and backgrounds. To enhance teaching and learning, understanding the current environment and designing effective interventions is crucial.

This study explores what pedagogical views educators hold of teaching for promoting learning in the first year of the degree program at university, how students experience learning events designed to promote learning in an extended engineering degree program and what the structures or mechanisms are that influence student learning in this program?

Grounded Theory methodology guided this research, which encompassed two cycles involving educators (including distinguished teacher award recipients and extended degree educators) and two cycles involving students (2nd year mechanical engineering students and 1st year mathematics students, both in the extended degree program). The theory is presented using qualitative systems dynamics modelling, shedding light on potential structures that align with empirical findings.

Two primary findings emerged: Firstly, educators can enhance student-focused teaching through responses to diversity, employing care and effort in teaching, fostering quality engagement opportunities, and creating meaningful learning experiences. Secondly, students experiencing alienation tend to either disengage or proactively seek or establish learning communities, ultimately improving engagement quality. The identified structure highlights the significance of care in teaching and the role of learning communities as key drivers and intervention focus points.

CROSS-DOMAIN MAPPING IN DIAGNOSTIC ASSESSMENT: ENHANCING INTERPRETATION FOR STUDENT SUPPORT

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KEYWORDS: mathematics, academic literacy, relative importance analysis, student academic profile

ABSTRACT

The significance of cross-domain mapping in the context of diagnostic assessment lies in its potential to promote the accuracy of interpretation of performance data and any underlying traits, and its ability to illuminate curricular associations and gaps. The university readiness in South Africa is assessed by the National Benchmark Testing (NBT) instruments: the Academic and Quantitative Literacy (AQL) and the Mathematics (MAT). The test scores (IRT-based) place candidates in four benchmarks (Proficient, Intermediate Upper and Lower, Basic) and provide an indication of the level of support that will be required once a student is placed in a university programme (Degree or Diploma).

One of the important uses of NBT data is the provision of additional diagnostic information. This information must be evidence-based, and consider various perspectives, including subject-specific skills, cross-domain skills, and cognitive demands involved in the acquisition of these skills. From the relative importance analyses performed for multiple disciplines and courses based on NBT data, it is evident that the patterns of the predictor variables (NBT subdomains) explaining variance (R-squared) in the course scores are different for students of differing ability groups. This study explores the relationship between Mathematics and Academic Literacy domains, while supporting the argument for within-class ability-grouping for differentiated instruction/support, considering the skills from both domains.

Using the principles of Cognitive Diagnostic Modelling (CDM) as a framework, we have mapped within-domain and cross-domain associations to two test forms (NBT AQL and MAT) that were administered to 1 050 students on the same day. We conducted various statistical analyses in each of the four performance benchmarks to unpack the combination of attributes (skills measured by two domains) and their impact on the interpretation of the candidates' abilities. This research adds to the body of literature on students' academic profiles by examining their competence across various domains and skill sets specific to their disciplines.

DEVELOPMENT AND VALIDATION OF A DIAGNOSTIC TOOL

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KEYWORDS: diagnostic test, validation, computer adaptive test

ABSTRACT

Before the COVID-19 pandemic the Mathematics Education Support Hub (MESH) at Western Sydney University ran a series of face-to-face refresher workshops for incoming undergraduate students who wanted to improve their mathematics and statistics skills required for university study. With the onset of the pandemic, it became necessary to redesign these workshops as online modules which students could study in their own time. One aspect of the face-to face delivery that was lost was the ability of MESH tutors to direct students to the parts of each workshop where they needed to focus their studies.

To overcome this deficiency MESH developed a series of diagnostic tools to help students to determine which section of each module they needed to study. To make the diagnosis as efficient as possible these tools were developed as computer adaptive tests, meaning that questions asked depended on responses to previous questions and students were not asked questions on concepts which the underlying marking algorithm assumes they had mastered or with which the algorithm assumed they were not conversant.

To develop the tools, we first constructed a “knowledge map” of concepts covered in each module with logical linkages between them. The tools were then built using the Numbas testing system’s Diagnostic test algorithm. Since deployment of the tools in February 2023, the Algebra 1 diagnostic tool has been attempted over 800 times.

The aim of this study is to validate the Algebra 1 tool’s efficacy. To do this we have accessed full details of all attempts and have used item response theory to rank questions using both direct and implied scoring. This has allowed us to identify incorrectly classified questions and will lead to refinement of the tool. In this talk we will discuss development and analysis of the Algebra 1 diagnostic tool.

BALANCING GEOMETRIC AND ALGEBRAIC INTERPRETATIONS OF COMPLEX NUMBERS FOR CONCEPTUAL UNDERSTANDING IN A SMART WORLD

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KEYWORDS: complex numbers, conceptual understanding, transformations

ABSTRACT

To prioritise Science, Technology, Engineering and Mathematics education, the South African Department of Education developed the Mathematics Teaching and Learning Framework (henceforth referred to as the Framework) which prescribes that Mathematics be taught for Conceptual Understanding. Along with the development of the Framework, and in response to the skills required for the Fourth Industrial Revolution, the Department has introduced new subjects. These include Technical Mathematics and Science, Coding and Robotics, Aerospace, Biomedical, Ocean and Marine Engineering. One topic that is indispensable to all these subjects is Complex Numbers.

Now, in cases where teachers must teach Complex Numbers that they never encountered formally begs the question: how can teachers with little or no prior encounters with complex numbers be supported to teach the topic of complex numbers?

While the Framework promotes teaching for conceptual understanding because it 'enables learners to see mathematics as a connected web of concepts' (Coles, 2021, p. 9), my hypothesis is that understanding complex numbers conceptually requires viewing them relationally. A relational approach to teaching complex numbers inevitably involves the use of images to express the actions on or relations between complex numbers. I therefore propose that Transformations Geometry provides Mathematics teachers with, not only a way of visually expressing and interpreting complex numbers, but also with a language that can support students' conceptual understanding of complex numbers. I argue, firstly, that the structure of the South African Mathematics Curriculum is such that the topic of transformations –can serve as a language that can mediate the development of this conceptual understanding in students in (and beyond) the Further Education and Training (FET) phase. Secondly, that for students to develop a conceptual understanding of complex numbers, teacher professional development programmes must support teachers towards coordinating geometric and algebraic interpretations of complex numbers with transformations geometry as a basis.

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INVESTIGATING TUTORS' TECHNOLOGICAL AND PEDAGOGICAL EXPERIENCES FROM AN ONLINE TUTORIAL IN A QUANTITATIVE LITERACY COURSE

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KEYWORDS: online tutoring, quantitative literacy, pedagogical issues, technological issues, online learning

ABSTRACT

In the year 2019, the world was struck with the global Covid-19 pandemic which led to many universities, including University of Cape Town (UCT), adapting to an online mode of teaching and learning. Online learning can take the form of asynchronous or synchronous tutorials and lectures. In this study tutors used MS Teams to run tutorials synchronously - while the Learning Management System (LMS) Vula was used asynchronously for tutorials and class materials. There are benefits and challenges associated with online tutorials, which differ depending on the contextual circumstances of implementation (Motaung et al., 2021). This study aimed to determine the tutors' technological and pedagogical, facilitation, instructional, experiences during an online tutorial in an undergraduate Quantitative Literacy course at UCT. In addition, it considered the contribution of this mode of instruction to the teaching, redesigning of delivery of the tutorial to impact on learning. The research methodology employed in this study is design-based research (DBR) (Doig & Groves, 2011; Hunter & Back, 2011). The experiences of the tutors were obtained by using a questionnaire. In addition, the weekly synchronous online tutorials were recorded for observation and reflection purposes. Tutors and students' participation was voluntary by informed consent. The concerns raised by tutors contained both but were not limited to poor attendance, low participation, poor network/connectivity, and low preparation from students. Based on the concerns raised tutors would then use the recorded sessions to reflect and redesign the way they delivered the tutorial. This would then implement new ways to assess and improve the learning to overcome the concerns mentioned above. As a result, the synchronous tutorial sessions were found to be more engaging than asynchronous tutorials but still lacked the benefits of observing struggles among students that face-to-face tutorials offer.

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WORKSHOPS

KINESTHETIC ACTIVITIES THAT REVEAL THE GEOMETRIC CONTENT OF PHYSICS CONCEPTS

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KEYWORDS: kinesthetic activities, complex numbers, density, line integrals, physics

ABSTRACT

Motivation and Target Audience

Physical scientists and engineers are active and intensive users of mathematics. But the siloed nature of mathematics and applied departments often means that no courses are designed to bridge the gap; mathematics courses teach only the algebraic and calculus manipulations and applied courses assume that students already have experience applying those mathematics skills within scientific models. For over 25 years at Oregon State University, we have been studying student reasoning and designing curriculum to bridge this gap (Paradigms Team, 2019–2023). One of the most powerful approaches that we have discovered is to emphasize the geometry of the physical situation, and one of the most powerful representations we employ is kinesthetic activities that ask students to use their bodies and their embodied cognition to model that geometry. Our target audience is teachers of single- and multi-variable calculus and/or complex numbers and researchers who study student reasoning in these contexts.

Workshop Content

Participants will explore two examples: complex numbers, with applications to both the time evolution of quantum spin systems and to quantum computing, and scalar line and surface integrals, with applications involving density, a concept common to multiple scientific disciplines. The workshop will interweave kinesthetic activities, small group problem-solving, and discussions about implementation strategies. Participants will also learn how physical scientists and engineers use this content in applications.

A brief discussion of equity considerations for kinesthetic activities will be included. How might you accommodate people who cannot physically perform the kinesthetic activity? How might you accommodate people who are reluctant to participate publicly with the whole group?

ACKNOWLEDGEMENTS

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PEDAGOGIES OF/FOR MATHEMATICS PRACTICES IN THE SCHOOL-UNIVERSITY TRANSITION

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KEYWORDS: mathematics practices; pedagogy; school-university transition

ABSTRACT

The differences between schooling and school mathematics on the one hand, and university and undergraduate mathematics on the other, are well recognised. These differences – epistemic, ontological, social, linguistic, pedagogical – prompt multiple, varied responses to support students in the school-university transition, each tailored to the contextual specificities. Our four-year long collaborative, research-led curriculum change project focuses on first-year core mathematics for science students at the University of Cape Town. Not only is first-year mathematics performance generally poor, but also inequitable by proxies for 'race' and language (Shay et al., 2020), signaling the stubborn legacy of social and educational inequality in South Africa. This has considerable implications for a student's sense of being, belonging, learning, and ultimate progression as a 'university science student'. Our multi-level project necessarily attends to micro-level classroom pedagogy and learning resources – the focus of this workshop – as well as degree programme structures, course models, student advising, and data analytics.

Recognising and acknowledging students' schooling experiences, our pedagogy creates opportunities for students to learn and use multiple mathematics practices at the intersection of mathematics knowledges and ways of knowing, mathematics learning, and living. These include metacognition (e.g. sitting with discomfort, risk-taking, posing questions, 'knowing' mathematics); literacies (reading, writing, drawing, using manual and electronic technologies); mathematics discourse practices such as proving, defining, and problem-solving; and working individually and collaboratively. This 90-minute workshop will be experiential, with delegates engaging both as students and metacognitively as lecturers with the material offered, and also discussing the experiences. Specific examples of pedagogies of/for mathematics practices will include storytelling as a means to taking ownership of a learning space; establishing norms for collaborative work; mapping learning of content and process on multiple levels; engaging with different mathematical representations as a route to understanding; and a reading and writing exercise for mathematical proof.

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THE EVOLUTION OF THE NUMBER CONCEPT FROM NATURAL TO REAL NUMBERS

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KEYWORDS: real numbers, number concept, conceptual understanding

ABSTRACT

The understanding of the concept of a real number is one of the most important prerequisites for first calculus modules at tertiary mathematics level and beyond. Calculus in turn is prerequisite for almost all higher mathematics modules,

Working with undergraduate students has demonstrated that school graduates exit school without some conceptual understanding and/or with very little working understanding of the real numbers. This challenge is not particular to South Africa (Bergé, 2008). Locally there are examples of discrepancies between the practice at the exit point of high school and the expected practice at the entry point to tertiary mathematics education (Watson, 2015). I concede that it is probably impossible to get students to some dependable conceptual understanding of the real numbers. However, it is possible to get school graduates to some sound working understanding of the real numbers if the appropriate conceptual imagery of the real numbers and their purposes are revealed to learners. The mentioned working understanding I believe puts, early calculus modules at tertiary squarely within the "*zone of proximal development"* of the school graduates That prepares students for higher mathematics modules and for the various theoretical constructions, of the real number system. Getting the conceptual understanding of the real numbers is evolutionary.

This workshop aims to share ideas and experience, to tap into the participants' experience and start collaborative engagements with peers to alleviate the challenges. It is aimed at initiating some sustained investigation(s) to make sense of the challenges impeding mathematics and sciences from being attractive fields of study. I intend to make a call for some point of convergence, for schools and universities, regarding the use of terminology and notation in mathematics.

The idea is to move in the direction of action research hand in hand with the creation of communities of practice to tackle the challenges (Farnsworth, 2016). The communities of practice need to involve high school and university teachers of mathematics to facilitate a smooth transition, referred to as "collective dreaming" (Barton et al., 2010).

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USING ATLAS.TI TO COLLECT, MANAGE AND ANALYSE LITERATURE IN MATHEMATICS AND MATHEMATICS EDUCATION RESEARCH PROJECTS

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KEYWORDS: mathematics education, literature review, qualitative data analysis

ABSTRACT

A literature review is one of the major pillars of academic work, as it is the basis upon which scholars establish context for their research, identify gaps in the literature, formulate their own research problems and objectives, and develop the research design and data collection tools (Benbellal et al., 2021). ATLAS.ti is a computer-assisted qualitative data analysis software (CAQDAS) that facilitates analysis of textual and media data in any discipline and for diverse research topics. Besides assisting with analysis of data, the tools of ATLAS.ti can also be applied to the literature review process particularly when access to library and university facilities is limited.

The workshop consists of both instruction and hands-on exercises in ATLAS.ti. It is aimed to assist participants:

- Develop the conceptual and practical tools necessary to use ATLAS.ti to organise, manage and analyse literature for research projects;
- Develop a framework for undertaking a literature review and/or document analysis systematically;
- To navigate ATLAS.ti to facilitate literature review and/or document analysis;
- On where to go for more resources and advice in undertaking a literature review and/or document analysis.

It is designed for postgraduate students, researchers and scholars not familiar with Atlas.ti. Two facilitators will run a 3-hour long workshop for a maximum of 20 participants and cover the core topics, as well as specialist needs arising out of individual projects.

Participants will need to bring their own computers. While the workshop will be taught on ATLAS.ti 23, participants running ATLAS.ti 9 will be able to follow the workshop. A free trial version of the software is available from https://atlasti.com/. At the end of the workshop, it is envisaged that participants will have developed a sense of how they can best utilise the software to meet their research needs.

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